Direct spatial motion simulation of aircraft subjected to engine failure

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***Abstract**: This paper presents a simulation of the movement of an aircraft that has an engine malfunction. The nonlinear equations of motion are formulated taking into account the yawing moment which is generated and occurs due to the asymmetry of the thrust line. For this simulation the equations are solved directly to find the trajectory of the aircraft and the variations of angles, angular rates, and velocities with time. An initial conditions is required for the simulation as* $(V, \beta, \delta r, \delta a, T, p, q, r, \phi, \psi, \gamma)$ *and a trim condition is calculated to get the angle of attack. The derivatives of angular, linear velocities, angles, moments and position derivatives are calculated. The derivatives calculated are solved using ordinary differential equations solution 4th order Rung Kutta method to get the yawing, pitching, rolling moments, velocity, angle of attack, side slip angle and trajectory of the aircraft according to control surfaces deflections and thrust as inputs.*

***Key Words**: Direct simulation, Nonlinear, Derivatives, Engine failure*

1. INTRODUCTION

Although aircrafts are now designed for flying even after one or more engines failure during the flight phase, the failure of the second engine on one side for multi-engine aircraft is obviously a serious and dangerous situation.

Losing all engines power is even more serious, as illustrated by the Dominicana DC-9 accident in the year 1970, when fuel contamination caused the failure of both aircraft engines. In these cases to have an emergency landing site is then very important. [1]

The ultimate form of engine failure, physical separation, occurred in 1979 when a complete engine detached from American airlines flight 191 during takeoff on the runway, causing damage to the aircraft and loss of control and the aircraft crashed near the airport, figure (1).[2]. Unfortunately the pilot had only 15 seconds to react before the crash. Basically, two aspects of flight on asymmetric power or engine failure (engine thrust loss) must be considered:

1. How to control the aircraft immediately after the failure of one or more engines.
2. To control the aircraft in steady level flight with one or more engines inoperative.

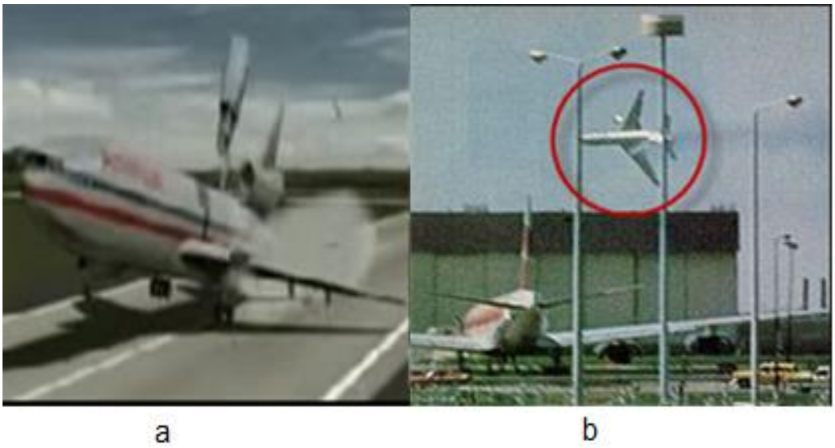

Figure 1: American airlines flight 191: (a) at takeoff, (b) before impact. [2]

2. AERODYNAMICS OF POWER LOSS

When the aircraft is at steady level flight, balance is required for the equilibrium of the forces and moments; this also is considered in case of engine failure. Propulsion systems are not 100% perfectly designed and operated, thus it is found that the engine occasionally fails during any flight phases.

For this reason a multi-engine aircrafts is always designed to be able to continue the flight safely along a prescribed trajectory to predetermined place even when an engine fails or loses an amount of its thrust. When engine fails in a multi-engine aircraft, there are two immediate effects occurring:

1. Yawing moment generated due to the asymmetry of the thrust line as seen in figure (2). This yawing moment causes the aircraft to sideslips towards the filed engine; this side slip has two disadvantages: increasing the drag and decreasing the tail fin's angle of attack.
   The amount of this yawing moment generated depends on the engine thrust from the live engine, the distance between the thrust line, the aircraft’s center of gravity and the aircraft directional stability which tends to oppose the asymmetric yawing moment. The yawing moment is also affected directly by the rate of engine thrust decay.
2. The second effect is roll as in figure (3), which occurs when the aircraft begins to yaw towards the failed engine, and resulted in a decrease of lift generated from the retreating wing and a yaw induced roll towards the failed engine.

There are many forces acting on the aircraft in case of engine failure, namely:

1. Side forces act on the fuselage and vertical tail, due to sideslip. The total force is stabilizing, and will act behind the center of gravity.
2. Side forces act on the rudder hinges, caused by rudder deflection, which pivots the aircraft about its center of gravity.
3. The lateral component of weight produced by banking of the aircraft.
4. Thrust from the live engine(s).
5. Total drag.

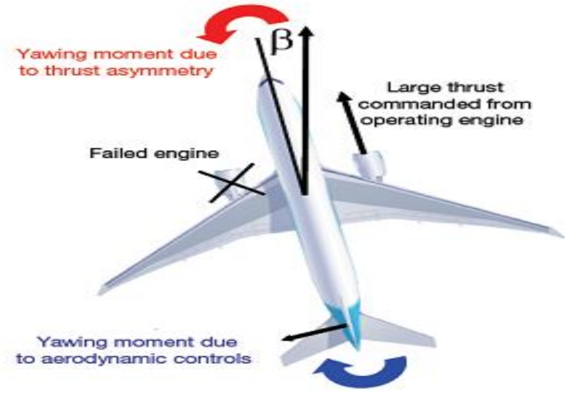

Figure 2: Yawing moment due to asymmetric thrust

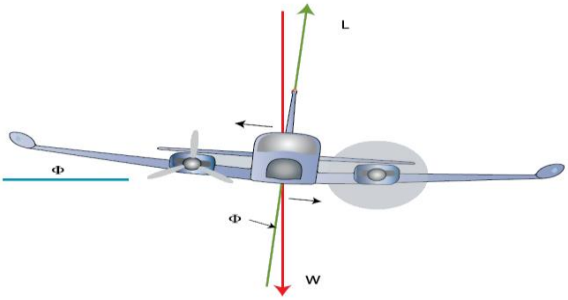

Figure 3: Rolling moment due to asymmetric thrust

When aircraft is in at a sideslip condition, the drag (D) is increased and hence decreases aircraft performance and airspeed. A side force (Y) is generated by vertical tail due to sideslip that is stabilizing because the moment (N_β) generates a force which tends to return the nose of the aircraft back into the relative wind.

When the aircraft is banked a sideslip also generates a rolling moment (L_β) due to the relative wind blowing under the high wing with live engine. The side force will starts accelerate and consequently will drift the aircraft to the failed engine to a descending flight path and nose down to the earth.

Then the relative wind and sideslip angle (β) reverse to the other side and the weathercock stability will starts to turn the nose of the aircraft to the ground as in the American airlines fight 191accident in 1979 after the engine separated from the left wing at takeoff, figure (1). According to the roll due to failed engine the left wing stalled while the right one continued to produce and the aircraft rolled to the left side reaching a bank angle of 112 degrees towards the failed engine (partially inverted) at the moment before impact as seen in figure (2.b).

3. NONLINEAR EQUATIONS OF MOTION

The rigid body equations of motion are obtained by applying the Newton's second law, which states that the summation of all external forces acting on a body is equal to the time rate of change of the momentum of the body, and the summation of the external moments acting on the body is equal to the time rate of change of momentum (angular momentum). [3&4]

The nonlinear forces and moments equations of the six degrees of freedom aircraft model can be written in the body axis by three forces, three moments, three kinematic and three linear position equations. The general configuration of the aircraft can be seen as in figure (4).

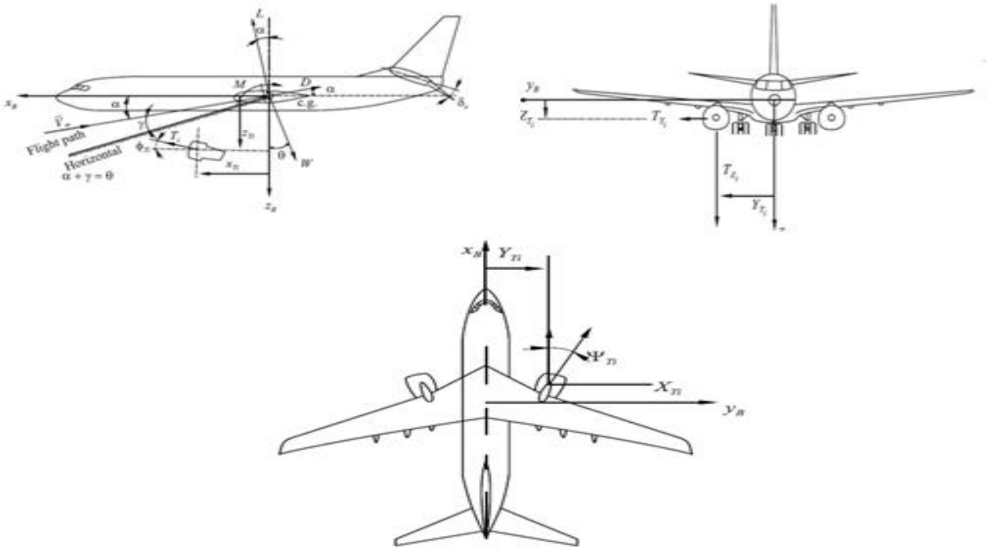

Figure 4: Thrust forces and moments. [5]

The nonlinear equations of motion are written as:

Forces equations:

$$m(u^{\cdot}+qw-rv)=F_{Gx}+F_{Tx}+F_{Ax}=F_x \tag{1}$$

$$m(v^{\cdot}+ru-pw)=F_{Gy}+F_{Ty}+F_{Ay}=F_y \tag{2}$$

$$m(w^{\cdot}+pv-qu)=F_{Gz}+F_{Tz}+F_{Az}=F_z \tag{3}$$

Where $u\,,v\,,w$ are translational velocities and the $p\,,q\,,r$ are the angular rates on body axis.

$$u=V.cos\alpha\,cos\beta \quad , \quad v=V.sin\beta \quad , \quad w=V.sin\alpha\,cos\beta \tag{4}$$

Indices $G, A\ and\ T$ in the above equations represent gravity, aerodynamic and thrust contributions respectively.

Additional source of drag appears in case of engine inoperative due to failure, the fan of the engine inlet produces an extra drag force proportional to engine inlet area (A_{eng}); this drag is known as a wind milling drag and is also a function of the dynamic pressure. Modeling this fan drag is especially important for the study of engine failure. [6]

$$uD_i=0.3A_{eng}q \tag{5}$$

Thus the total drag is:

$$D=C_Dq\,S+D_i \tag{6}$$

The gravitational and aerodynamic forces are obtained as:

$$F_{Gx} = -mg\ sin\theta \quad , \ F_{Gy} = mg\ sin\phi\ cos\theta \ , \quad F_{Gz} = mg\ cos\Phi\ cos\theta \tag{7}$$

$$F_{Ax} = C_x.q.S \ , \quad F_{Ay} = C_y.q.S \ , \quad F_{Az} = C_z.q.S \tag{8}$$

The lift, drag and cross coefficients are:

$$C_L = C_{L0} + C_{L\alpha}.\alpha \tag{9}$$

$$C_D = C_{D0} + k{C_L}^2 \tag{10}$$

$$C_c = C_{n\beta}.\beta \tag{11}$$

The coefficients $C_x, C_y\ and\ C_z$ are calculated from:

$$C_x = -C_D cos\alpha.cos\beta - C_C cos\alpha.sin\beta + C_L sin\alpha \tag{12}$$

$$C_y = -C_D sin\beta - C_C\ cos\beta \tag{13}$$

$$C_z = -C_D sin\alpha.cos\beta - C_c sin\alpha.sin\beta - C_L cos\alpha \tag{14}$$

The total thrust force is the sum of thrust generated from the all engines (n).

$$F_{Tx} = \sum_i^n T_{xi} \ , \qquad T_{xi} = T.cos\phi_{Ti} cos\psi_{Ti} \tag{15}$$

$$F_{Ty} = \sum_i^n T_{yi} \ , \qquad T_{yi} = T.cos\phi_{Ti} sin\psi_{Ti} \tag{16}$$

$$F_{Tz} = \sum_i^n T_{zi} \ , \qquad T_{zi} = -T.sin\phi_{Ti} \tag{17}$$

<u>Moments equations</u>:

$$\dot{p}I_{xx} + qr\left(I_{zz} - I_{yy}\right) - (r^{\cdot} + pq)I_{xz} = L_A + L_T = L \tag{18}$$

$$\dot{q}I_{yy} + pr(I_{zz} - I_{xx}) + (p^2 - r^2)I_{xz} = M_A + M_T = M \tag{19}$$

$$\dot{r}I_{zz} + pq\left(I_{yy} - I_{xx}\right) + (qr - p^{\cdot})I_{xz} = N_A + N_T = N \tag{20}$$

The values of $\dot{p}, \dot{q}\ and\ \dot{r}$ are found by using the relations below:

$$T_0.\dot{p} = (BC - D^2)T_1 + (FC + ED)T_2 + (FD + EB)T_3 \tag{21}$$

$$T_0.\dot{q} = (FC + ED)T_1 + (AC + E^2)T_2 + (AD + EF)T_3 \tag{22}$$

$$T_0.\dot{r} = (FD + BE)T_1 + (AD + FE)T_2 + (AB - F^2)T_3 \tag{23}$$

Where $T_0, T_1, T_2\ and\ T_3$ are given by:

$$T_0 = ABC - AD^2 - BE^2 - CF^2 - 2DEF \tag{24}$$

$$T_1 = (B - C)qr + (Eq - Fr)p + (q^2 - r^2)D + L \tag{25}$$

$$T_2 = (C - A)rp + (Fr - Dp)q + (r^2 - p^2)E + M \tag{26}$$

$$T_3 = (A - B)pq + (Dp - Eq)r + (p^2 - q^2)F + N \tag{27}$$

$$\begin{bmatrix} A & F & E \\ & B & D \\ & & C \end{bmatrix} = \begin{bmatrix} I_x & I_{xy} & I_{xz} \\ & I_y & I_{yz} \\ & & I_Z \end{bmatrix}$$

Similarly, Indices $A\ and\ T$ in the above equations represent aerodynamic and thrust contributions, respectively.

$$L_A = C_l qSb \quad , M_A = C_m qSc \quad , N_A = C_n qSb \tag{28}$$

$$C_l = C_{l_\beta}.\beta + C_{l_p}.\frac{pb}{V} + C_{l_r}.\frac{rb}{V} + C_{l_{\delta a}}.\delta a + C_{l_{\delta e}}.\delta e \tag{29}$$

$$C_m = C_{m_0} + C_{m_\alpha}.\alpha + C_{m_q}.q + C_{m_{\delta e}}.\delta e \tag{30}$$

$$C_n = C_n.\beta + C_{n_p}.\frac{pb}{V} + C_{n_r}.\frac{rb}{V} + C_{n_{\delta a}}.\delta a + C_{n_{\delta e}}.\delta e \tag{31}$$

The moments generated in case of inoperative engine are written as:

$$L_T = \sum_i^n \left(-T_{yi} z_{Ti} + T_{zi} y_{Ti}\right) \tag{32}$$

$$M_T = \sum_i^n (T_{xi} z_{Ti} - T_{zi} x_{Ti}) \tag{33}$$

$$N_T = \sum_i^n (T_{yi} x_{Ti} + T_{xi} y_{Ti}) \tag{34}$$

These moments arms $x_{Ti}\ , y_{Ti}\ and\ z_{Ti}$ are the aircraft engine location with respect to its center of gravity. The derivatives of velocity, angle of attack and side slip angle are calculated as:

$$\dot{V} = \frac{1}{m}\begin{pmatrix} F_x cos\alpha.cos\beta + F_y\ sin\beta + F_z\ sin\alpha.cos\beta \\ +mg\begin{pmatrix} cos\theta\ sin\phi\ sin\beta - sin\theta\ cos\alpha\ cos\beta \\ +cos\theta\ cos\phi\ sin\alpha\ cos\beta \end{pmatrix} \end{pmatrix} \tag{35}$$

$$\dot{\alpha} = \frac{1}{m.v.cos\beta}\begin{pmatrix} F_x sin\alpha + F_z cos\alpha \\ +m.g(sin\theta\ sin\alpha + cos\theta cos\phi cos\alpha) \\ -m.V(p.cos\alpha + q.cos\beta - r.sin\alpha\ sin\beta) \end{pmatrix} \tag{36}$$

$$\dot{\beta}=\frac{1}{m.V}\begin{pmatrix} F_x cos\alpha\ sin\beta+F_y cos\beta-F_z sin\alpha\ sin\beta \\ +m.g\cos sin\beta(sin\phi cos\beta-cos\phi\ sin\alpha) \\ +m.V(p.sin\alpha-r.cos\alpha) \end{pmatrix} \tag{37}$$

Kinematics equations:

The three Euler angular velocities are calculated as:

$$\dot{\phi}=p+\frac{r.cos\phi+q.sin\phi}{cos\theta}+sin\theta \tag{38}$$

$$\dot{\theta}=q.cos\phi-r.sin\phi \tag{39}$$

$$\dot{\psi}=\frac{r.cos\phi+q.sin\phi}{cos\theta} \tag{40}$$

Linear position equations:

The three linear coordinates $(x, y\ and\ z)$ with respect to the earth axes can be linked to the translational velocity components via the Euler angles as follows:

$$\begin{aligned}\dot{x}=u.cos\theta.cos\psi+v(sin\theta.sin\phi.cos\psi-cos\phi.sin\psi)+w(sin\theta.cos\phi.cos\psi \\ +sin\phi.sin\psi)\end{aligned} \tag{41}$$

$$\begin{aligned}\dot{y}=u(cos\theta.sin\psi)+v(sin\theta.sin\phi.sin\psi+cos\psi.cos\phi)+w(sin\theta.cos\phi.sin\psi \\ -sin\phi.cos\psi)\end{aligned} \tag{42}$$

$$\dot{z}=-u.sin\theta+v.cos\theta.sin\phi+w.cos\theta.cos\phi \tag{43}$$

4. DIRECT SIMULATION

For this simulation the above equations are solved directly to find the trajectory of the aircraft and the variation of angles, angular rates, and velocities with time. An initial conditions is required for the simulation as $(V, \beta, \delta r, \delta a, T, p, q, r, \phi, \psi, \gamma)$ and a trim condition is calculated to get the angle of attack (α).

At a given altitude and aircraft velocity, the components of velocity along the three axes are calculated using equation (4). Then the lift, drag and cross coefficients are calculated as in equations (9, 10&11). The $x, y, and\ z$ forces coefficients are obtained as in equations (12, 13&14). The forces components acting on the aircraft are calculated using equations (7, 8, 15, 16&17). From equations (29, 30&31) the rolling, pitching and moment coefficients are calculated by using a control surfaces as inputs, and then the moments are calculated by equation (28). The terms (T_0, T_1, T_2, T_3) are calculated from equations (24 to 27) and then the derivatives of angular velocities from equations (38 to 40). The linear velocity, angle of attack and side slip angle derivatives are calculated using equations (35 to 37). By using equations (38 to 40) we can get the yaw, pitch and roll angles derivatives. The linear position derivatives are calculated using equations (41 to 43).

We solve the derivatives calculated above using ordinary differential equations solution 4th order Rung Kutta method to get the yawing, pitching, rolling moment, velocity, angle of attack, side slip angle and trajectory of the aircraft according to control surfaces deflections and thrust as inputs.

5. RESULTS

An Airbus A-300 is selected for this simulation. Two cases are simulated; at cruising with $200\,m/sec$ at $2000\,m$ and with $200\,m/sec$ at $1500\,m$. Figures (5 to 14) expressed the motion of the aircraft in case of inoperative engine. The failure is expressed as a total loss of thrust of one engine after 2 seconds of flying at steady level flight at a certain altitude. The computation of the trim values is by assuming that one engine has lost full thrust and the rudder and aileron deflection is being constant at zero angle deflection during the simulation. Simulation results depict that if there is no change in control surfaces deflection and second engine throttle setting after the failure, aircraft starts to lose altitude rapidly as seen in figure (5) and crashes after 21 seconds in case (1) and after 19 seconds in case (2).

The angle of attack (α) change is unstable; it changes and decreases continuously until reaches the value that the wings cannot produce necessary lift and aircraft goes into stall condition. The angle of attack decreases after failure as a result of the pitching moment and the drag force increases. Figure (14) demonstrates the change of angle of attack when the trim value is applied. As it can also be seen after applying the trim value the angle of attack exhibits small oscillation in the beginning and then it becomes constant at the post failure value. Side slip angle (β) is expected to be zero during normal cruise flight, at engine failure the aircraft can continue the flight by shifting its nose. Figure (13) shows the yaw angle generated due to the aircraft side slipping.

Table 1: Initial values at cruise before the failure

Case	Alt (m)	V (m/sec)	α (deg)	T (KN)	δ_e (deg)	δ_r (deg)	δ_a (deg)
1	2000	200	3.54	240	−7.29	0	0
2	1500	200	3.34	240	−7.46	0	0

Table 2: Final values at crash

Case	V (m/sec)	α (deg)	T (KN)	δ_e (deg)	δ_r (deg)	δ_a (deg)	$Time$ (sec)	$Roll\ Angle$ (deg)
1	281.5	2.75	120	−7.29	0	0	21	250.5
2	262.5	2.5	120	−7.46	0	0	19	207.5

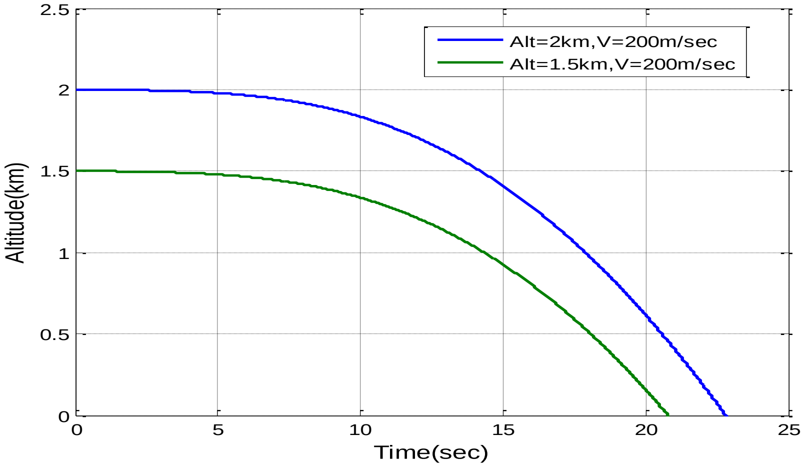

Figure 5: Variation of altitude with time

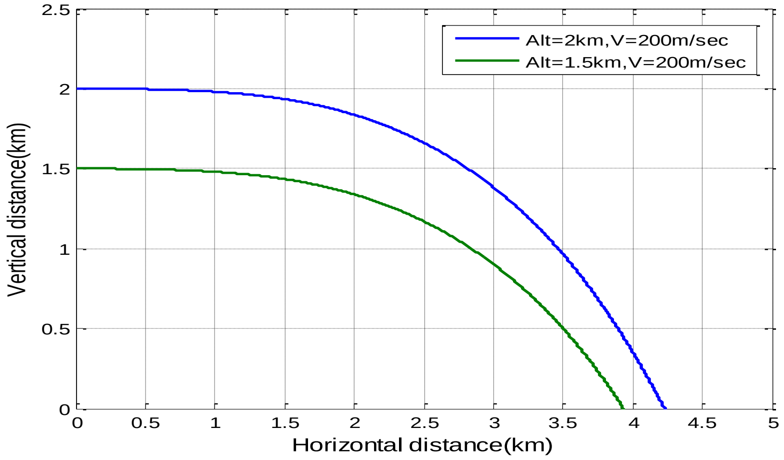

Figure 6: Variation of vertical and horizontal distance

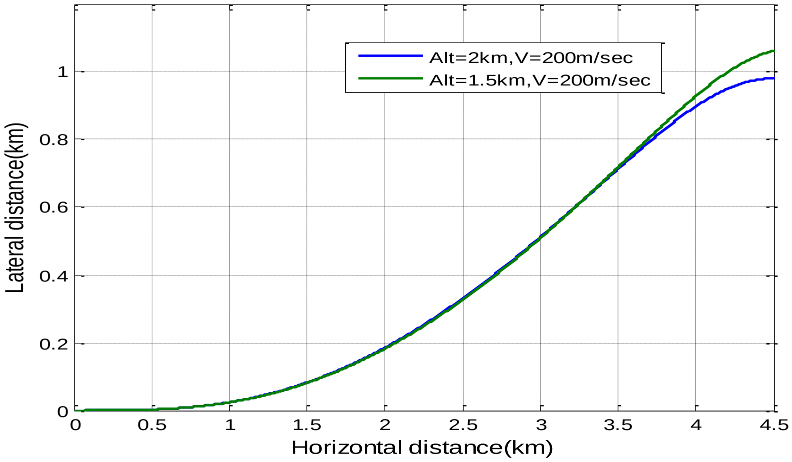

Figure 7: Variation of lateral and horizontal distance

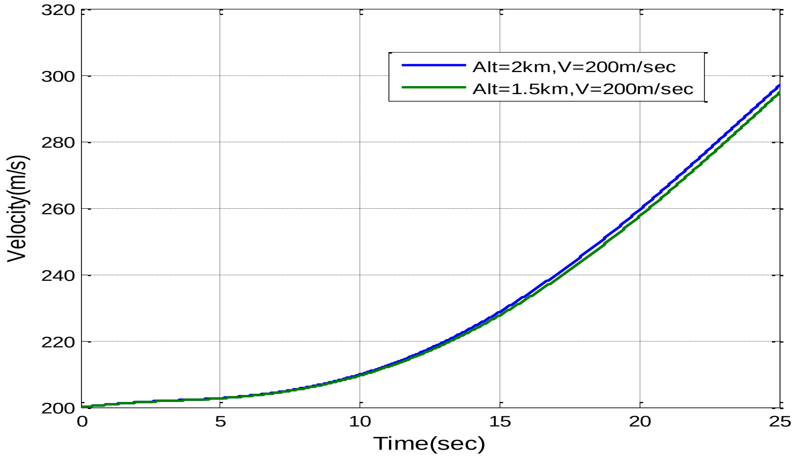

Figure 8: Variation of velocity with time

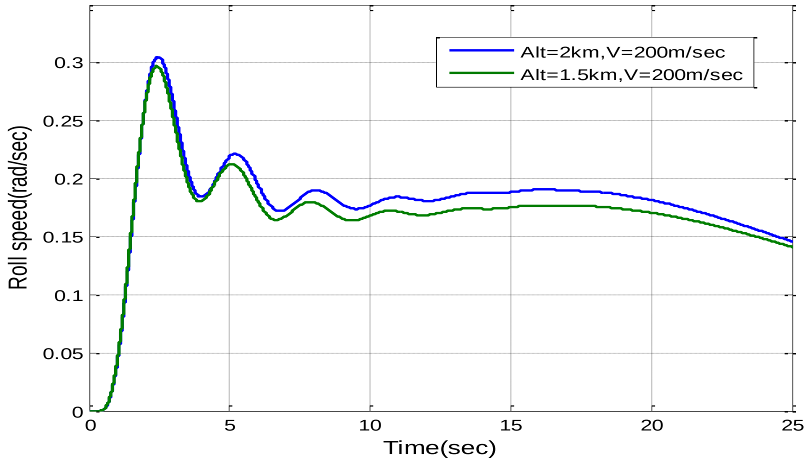

Figure 9: Variation of roll speed with time

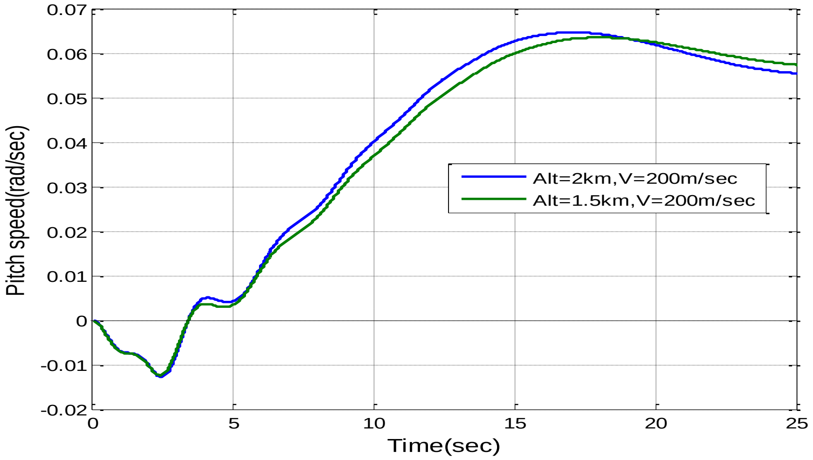

Figure 10: Variation of pitch speed with time

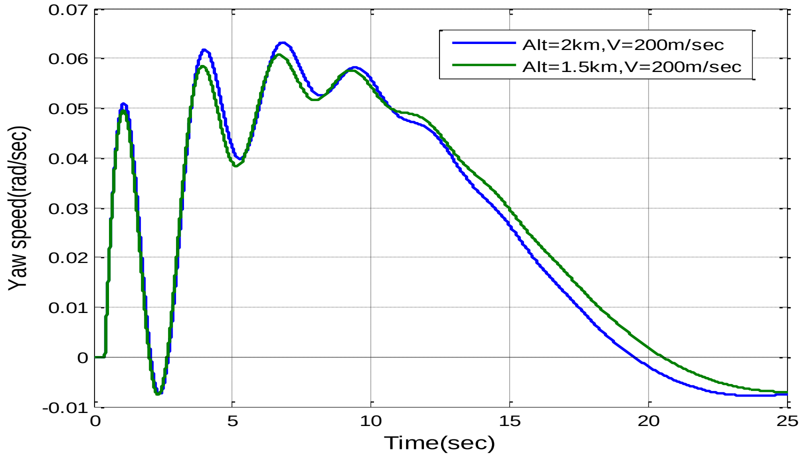

Figure 11: Variation of yaw speed with time

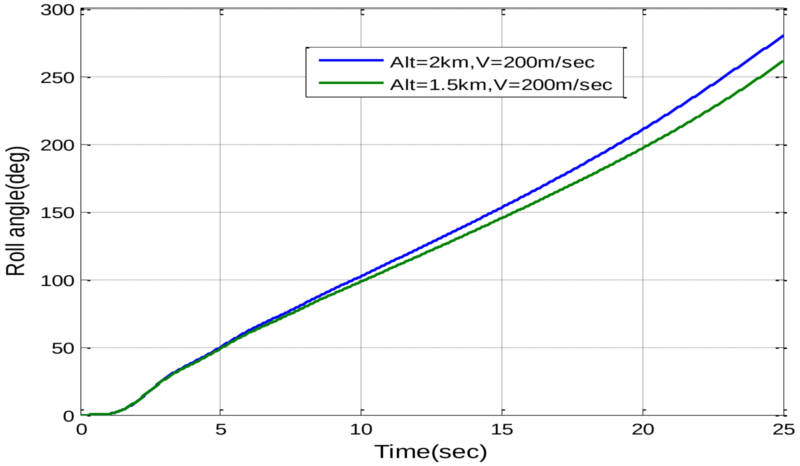

Figure 12: Variation of roll angle with time

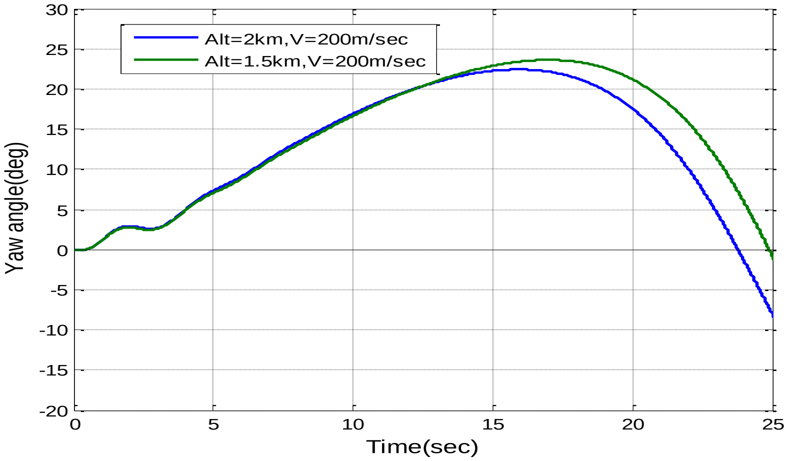

Figure 13: Variation of yaw angle with time

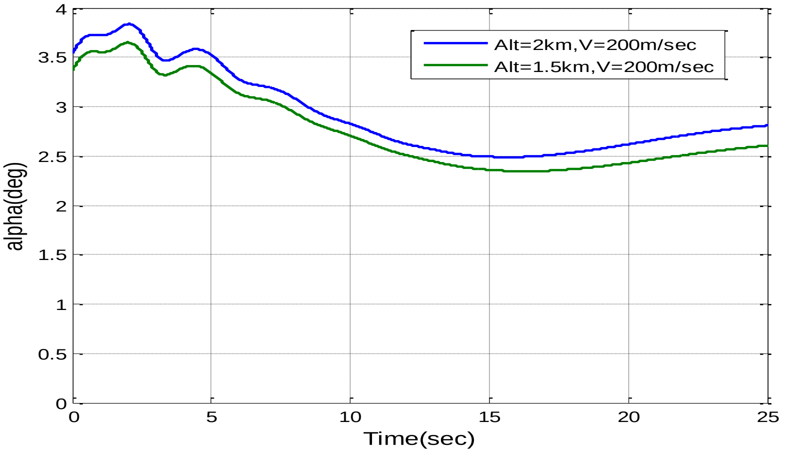

Figure 14: Variation of angle of attack with time

6. CONCLUSION

It is clear that the aircraft equations of motion are highly nonlinear and coupled equations; any force and moment exerted on the aircraft has its influences on the other axes. The engine thrust loss during flight is a critical failure which has strong influences both in lateral and longitudinal motions. Future studies may investigate the use of inverse simulation to accounts for the control surfaces deflection and throttle setting for the live engine to stabilize the aircraft and continue to flight and lands safely.

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