

Regression Models for Predicting Force Coefficients of Aerofoils

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DOI: 10.13111/2066-8201.2015.7.3.1

Received: 05 June 2015 / Accepted: 13 July 2015

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Abstract: Renewable sources of energy are attractive and advantageous in a lot of different ways. Among the renewable energy sources, wind energy is the fastest growing type. Among wind energy converters, Vertical axis wind turbines (VAWTs) have received renewed interest in the past decade due to some of the advantages they possess over their horizontal axis counterparts. VAWTs have evolved into complex 3-D shapes. A key component in predicting the output of VAWTs through analytical studies is obtaining the values of lift and drag coefficients which is a function of shape of the aerofoil, 'angle of attack' of wind and Reynolds's number of flow. Sandia National Laboratories have carried out extensive experiments on aerofoils for the Reynolds number in the range of those experienced by VAWTs. The volume of experimental data thus obtained is huge. The current paper discusses three Regression analysis models developed wherein lift and drag coefficients can be found out using simple formula without having to deal with the bulk of the data. Drag coefficients and Lift coefficients were being successfully estimated by regression models with R^2 values as high as 0.98.

Key Words: Green Energy, Wind Energy, Regression Model, Vertical Axis Wind Turbine

I. INTRODUCTION

Vertical Axis Wind Turbines (VAWTs) have been receiving renewed interest during the past decade, primarily due to their ability to get easily integrated in to the urban environment.

Companies are emerging globally that manufacture VAWTs for installation into easily bendable areas like roof tops, landmark spots, building facades, etc. There is a dearth of analytical models that can accurately predict the performance of VAWTs.

A good summary of the analytical models developed in the past has been explained in [5]. An important aerodynamic characteristic that is inevitable in all these models is the lift and drag coefficient for the aerofoil. These coefficients depend on the Reynolds number of flow, angle of attack of the wind and shape of the aerofoil.

National Advisory Committee on Aeronautics (NACA) has carried out extensive experiments on these force coefficients and has documented them for 78 aerofoil shapes they were experimenting [2]. However, as the study was aimed at the aircraft industry, the

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Reynolds number of flow for which the experiments were conducted was higher than that experienced by VAWTs. Identifying the void on data for VAWTs, Sandia national laboratories carried out experiments on 7 symmetrical NACA aerofoil shapes and have documented the results [7].

Although, the study is old, the results of the report have been universally accepted and referenced even nowadays [1, 4, 6].

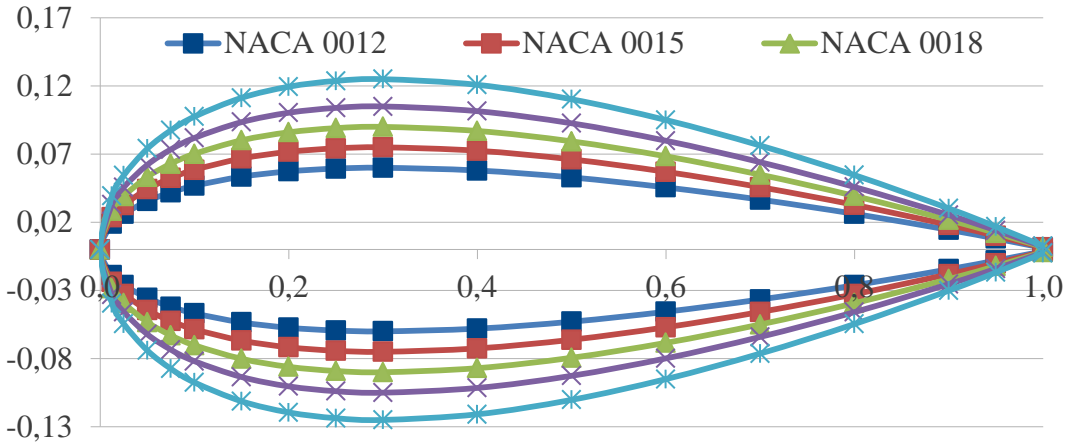


Fig. 1 NACA symmetrical aerofoil

Out of the seven symmetrical aerofoils considered in the Sandia national laboratories report [7], complete tables have been provided for five of them, namely NACA 0012, NACA 0015, NACA 0018, NACA 0021 and NACA 0025 for the entire range of angles of attack (0 to 180 degrees) and Reynolds numbers in the range of 10,000 to 1,00,00,000. The shape of NACA symmetrical aerofoils are plotted in fig. 1.

The resulting tables amount up to a total of 2878 values of lift and drag coefficients for different combinations of Reynolds number, angle of attack & shape of aerofoil. This is a large set of data.

In addition, for purposes of developing algorithms for the analysis of a general 3-D shaped VAWT, these data in the form of equations (as a function of Reynolds number, angle of attack & aerofoil. shape) are needed as they simplify the task.

This paper discusses the regression analysis carried out on the lift and drag values using three methods, namely: Trigonometric fit, Quadrilateral fit & Cubic fit.

The coefficient of determination (R^2), which is a measure of goodness of the regression fit, is reported for each case which gives an indication of the level of confidence that can be placed on the resulting equations.

Section II of this paper deals with the analysis part, section III deals with the results of the analysis and its interpretation and section IV is the conclusion.

II. REGRESSION ANALYSIS

To understand the nature of lift and drag values reported in Sandia report [7], they were plotted on a graph for interpretation of the trend.

The plot of the lift coefficient versus the angle of attack is shown in fig. 2 and the plot of the drag coefficient versus the angle of attack is shown in fig. 3. A legend summary for the plots is given in Table 1.

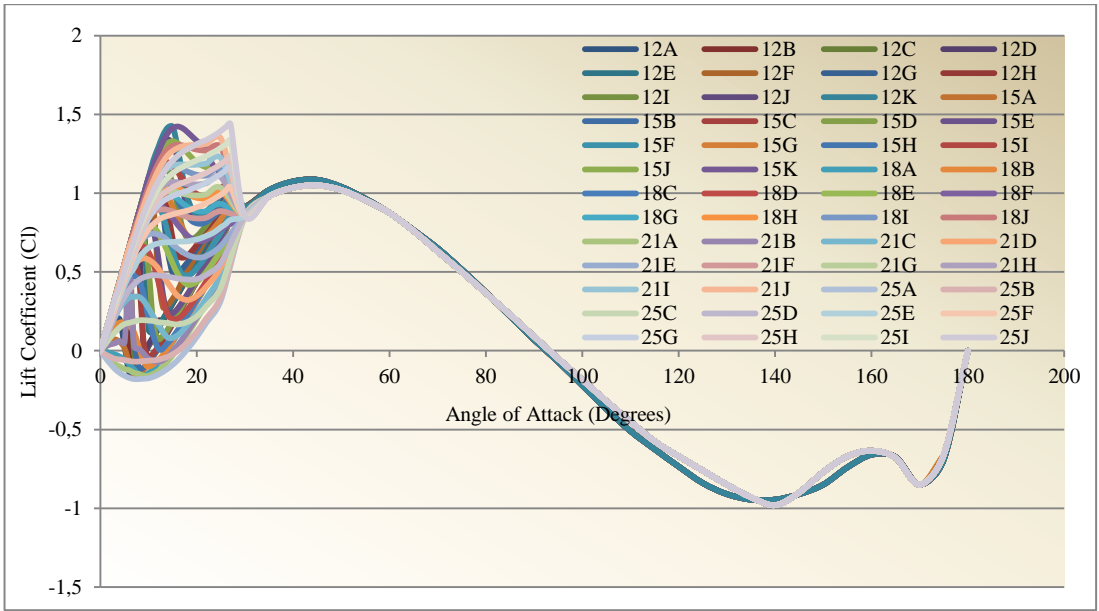


Fig. 2 The Lift Coefficient versus the Angle of Attack for different airfoils and Reynolds Numbers

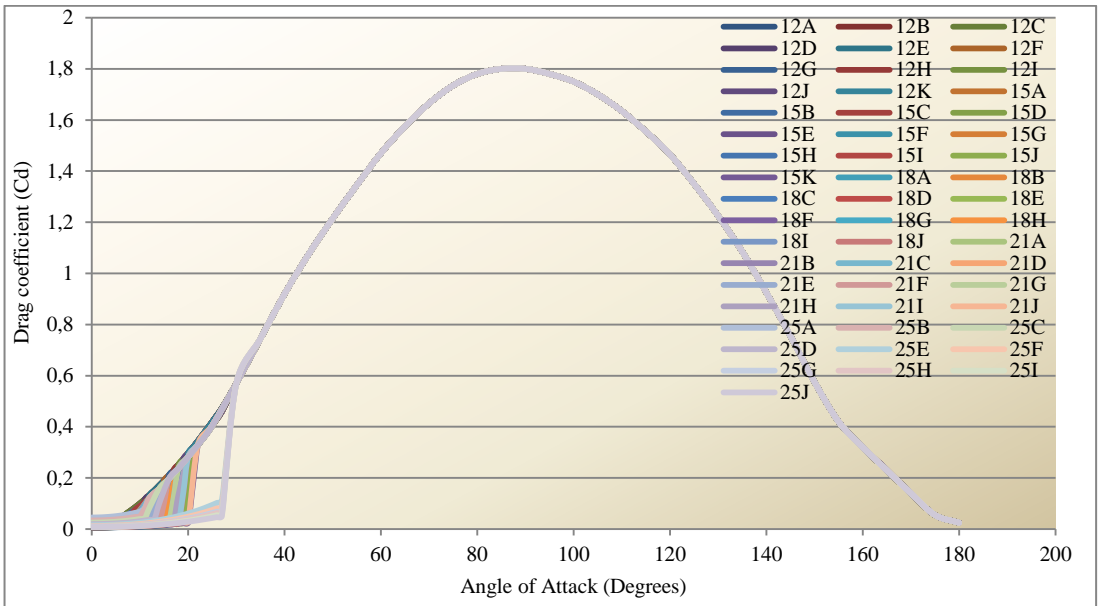


Fig. 3 The Drag Coefficient versus the Angle of Attack for different airfoils and Reynolds Numbers

Table 1: Legend summary for interpretation of graphs

Aerofoil Shape		Reynolds Number	
12	NACA 0012	A	10000
15	NACA 0015	B	20000
18	NACA 0018	C	40000
21	NACA 0021	D	80000
25	NACA 0025	E	160000

Example: "18F" denotes analysis for the case where the shape is NACA 0018 and the Reynolds Number is 360000	F	360000
	G	700000
	H	1000000
	I	2000000
	J	5000000
	K	10000000

From fig. 2 and fig. 3, it can be clearly seen that beyond 30 degrees, there is no effect of Reynolds number and aerofoil shape on lift and drag coefficients.

For the values of lift and drag coefficients for an angle of attack less than 30, there is a function of all the three variables, namely the shape of the aerofoil, the Reynolds Number and the Angle of Attack.

Further from fig. 2 and fig. 3, we can see that there is a distinctive angle of attack above which the lift to drag ratio decreases. This point is known as "Stall" [3] and it finds application in all areas of fluid dynamics.

Based on the above revelations; for the purposes of regression analysis, the lift and drag coefficient data were divided into four zones.

- Zone 1 - Coefficients up to the value of Stall
- Zone 2 - Coefficients at the value of Stall
- Zone 3 - Coefficients beyond the value of Stall and up to 30°
- Zone 4 - Coefficients equal to and beyond 30°

Three methods were used for the regression analysis and the findings of each were compared. A description of these methods will be given in the following sub-section. They are as follows:

- Quadratic polynomial fit
- Cubic polynomial fit
- Trigonometric fit

Quadratic polynomial fit

In this method, Zone 4 which is dependent only upon α , was approximated by a quadratic polynomial and resulting values of R^2 and coefficients of polynomial were noted down.

For the remaining three zones (1, 2 & 3), the lift and drag coefficients were fitted with linear regression models with independent variables as Reynolds number, angle of attack & aerofoil shape.

Through trial and error, it was found that taking logarithm of Reynolds number yielded better results than using Reynolds number alone as the variable. This was found to be true for all the regression fits in this study and hence logarithm of Reynolds number was used wherever Reynolds number variable appeared.

Further, it may be noted that the shape of the aerofoil, namely NACA 0012, NACA 0015, NACA 0018, NACA 0021 and NACA 0025 are not continuous values as desired by a regression analysis fit, However, upon using values of 12, 15, 18, 21 and 25 respectively for the aerofoils mentioned above, it was found out that a good fit was achieved.

Cubic polynomial fit

This fit is same as the quadratic polynomial fit except that Zone 4 regression analysis was carried out using a cubic polynomial rather than a quadratic polynomial which gives more accurate results.

Trigonometric fit

Upon close examination of the graphs of lift and drag coefficients beyond 30^0 , it was seen that in addition for them being independent of Reynolds number and shape of aerofoil, they approximately resembled the trigonometric curves of sin and cos. The graph of the drag coefficient approximately resembles the sine curve and the graph of the lift coefficient resembles the cosine curve.

Hence, the lift coefficients values for Zone 4 were subjected to linear regression with cosine of the α values and drag coefficients for Zone 4 with sine of α .

Reasonably good fit was obtained (the details of the fit to be presented in section IV) and thus included in this study.

With the values of regression coefficients obtained by Zone 4 analyses, the values of the lift and drag coefficients for other zones (zone 1, 2 & 3) were estimated.

Understandably, the values in these zones were giving poor estimates (as they are functions of all the three variables).

However, it was found out that the difference in the actual values of the lift and drag coefficients (in these zones) and that estimated by the trigonometric functions in these zones were giving reasonably good fit when resorted to simple linear regression analysis using the three variables (α , Reynolds number & aerofoil shape).

The rest of the analysis was the same as what was done for the quadratic and cubic polynomial fits.

If Loop

An IF loop was built for the data set that would identify the zone to which the combination of Reynolds number, angle of attack and aerofoil shape belongs to (Table 2). The Calculation of stall angle (α_s) precedes this step and is carried out for each set of Reynolds number & aerofoil shape based on regression analysis coefficients and then rounded off to the nearest whole number (to enable the IF loop comparison).

Table 2: Logical sequence of the IF loop

“AND” Conditions		Zone
$\alpha < \alpha_s$	$\alpha < 30^0$	Zone 1
$\alpha = \alpha_s$	$\alpha < 30^0$	Zone 2
$\alpha > \alpha_s$	$\alpha < 30^0$	Zone 3
$\alpha > \alpha_s$	$\alpha = 30^0$ OR $\alpha > 30^0$	Zone 4

III. RESULTS & DISCUSSIONS

The results of the regression analysis are presented in Table 3, 4 & 5. Table 3 is for the quadratic polynomial fit, Table 4 is for the cubic polynomial fit and Table 5 is for the trigonometric fit.

The symbols used in the Tables are,

α = Angle of attack

α_s = Angle of stall

N = Numerical equivalent of shape of aerofoil (for e.g.: NACA 0018 is 18)

log (Re) = Logarithm of Reynolds number

Table 3: Quadratic Fit Results

Zone	Variable	Equation ($m_1*v_1 + m_2*v_2 + \dots b$)							R ²
		m ₁	v ₁	m ₂	v ₂	m ₃	v ₃	b	
4	Lift			0.000123697	α^2	-0.04036	α	2.572181	0.880
	Drag			-0.000262917	α^2	0.048193	α	-0.516846	0.951
3	Lift	0.199321	Log(Re)	0.042823	α	-0.00834	N	-1.17889	0.853
	Drag	-0.00139	Log(Re)	0.021844	α	-0.00185	N	-0.1056	0.991
2	Lift	0.69588	Log(Re)	-0.04274	α	0.042027	N	-3.32006	0.890
	Drag	-0.02614	Log(Re)	0.013576	α	-0.00204	N	0.092617	0.929
	α_s	4.56746	Log(Re)	0.29483	N			-15.79263	0.906
1	Lift	0.195874	Log(Re)	0.049428	α	-0.0137	N	-0.66326	0.817
	Drag	-0.01489	Log(Re)	0.002151	α	9.36E-05	N	0.087071	0.502
Lift Values - Overall									0.898
Drag Values - Overall									0.976

Table 4: Cubic Fit Results

Zone	Variable	Equation ($m_1*v_1 + m_2*v_2 + \dots b$)							R ²
		m ₁	v ₁	m ₂	v ₂	m ₃	v ₃	b	
4	Lift	3.29166578*10 ⁻⁶	α^3	-0.000913177	α^2	0.056683	α	0.004188	0.980
	Drag	1.58568950*10 ⁻⁶	α^3	-0.00076241	α^2	0.094939	α	-1.75392	0.988
3	Lift	Same as that given in the table for Quadratic polynomial fit (Table 3)							
	Drag	"							
2	Lift	"							
	Drag	"							
	α_s	"							
1	Lift	"							
	Drag	"							
Lift Values - Overall									0.962
Drag Values - Overall									0.991

Table 5: Trigonometric Fit Results

Zone	Variable	Equation ($m_1*v_1 + m_2*v_2 + \dots b$)							R ²
		m ₁	v ₁	m ₂	v ₂	m ₃	v ₃	b	
4	Lift	1.07718	Cos(α)					0.14734	0.877
	Drag	2.001954	Sin(α)					-0.2773	0.972
Regression analysis on Force coefficients of Zone 1, 2 & 3 after taking the difference of that estimated by trigonometric functions									
3	Lift	0.198799	Log(Re)	0.048797	α	-0.00858	N	-2.44796	0.871
	Drag	-0.00172	Log(Re)	-0.01118	α	-0.002	N	0.15377	0.954
2	Lift	0.698349	Log(Re)	-0.03894	α	0.041895	N	-4.57436	0.896
	Drag	-0.0251	Log(Re)	-0.02058	α	-0.00209	N	0.360303	0.991
	α_s	4.56746	Log(Re)	0.29483	N			-15.79263	0.906
1	Lift	0.195406	Log(Re)	0.052953	α	-0.01343	N	-1.90187	0.837
	Drag	-0.01522	Log(Re)	-0.03197	α	0.000201	N	0.360509	0.993
Lift Values - Overall									0.897
Drag Values - Overall									0.984

For better comparison of the three methods, the R² values for the lift coefficient, drag coefficient and the combined case of all the three fits are plotted on a bar chart as shown in fig. 4 (note that Y-axis has been started from 0.85 for the clarity of viewing).

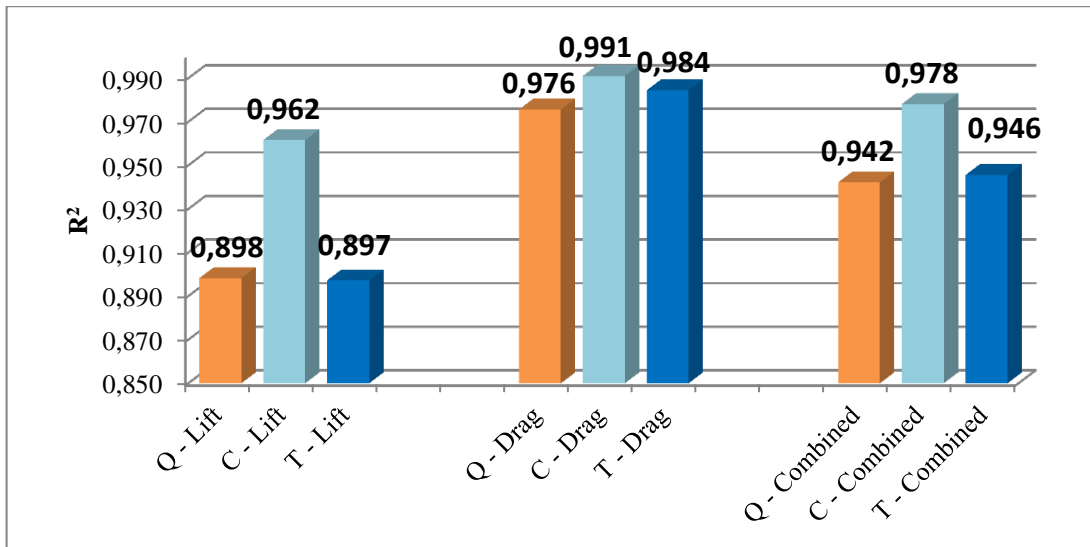


Fig. 4 R^2 for lift & drag coefficients (Q – Quadratic, C – Cubic & T – Trigonometric)

As clearly evident from the chart, the cubic polynomial gives a better approximation overall when compared with the other two methods. The quadratic & trigonometric approximations for drag case are almost as good as that estimated by the cubic polynomial.

However, the approximation of the lift coefficient by both methods lags far behind than the approximation obtained by the cubic polynomial method resulting in a reduction in the combined case approximation.

In this study, Zone 4 approximations using quadratic & cubic polynomials were only presented as higher order polynomials; they did not present a significant improvement in goodness of fit.

The R^2 values for the fourth, fifth & sixth order polynomial were 0.981, 0.981 & 0.982 respectively which were not significantly higher than that obtained with the cubic polynomial case (0.978).

IV. CONCLUSIONS

The lift & drag coefficients for five symmetrical aerofoils. established experimentally have been fitted with equations using the regression analysis by three methods namely quadrilateral fit, cubic fit & trigonometric fit and their R^2 values were found to be 0.942, 0.978 & 0.946 respectively. As in regression analysis any fit which gives an R^2 value of 0.95 and above is considered excellent; the cubic fit which is the best of the three and quadrilateral and trigonometric fits which are bordering 0.95 can be considered suitable for the purpose of implementation in algorithms.

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