Evolutionary, Iterative Optimum-Optimorum Theory

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Abstract: *The performing of the aerodynamical, global optimized (GO) shape of flying configurations (FCs) leads to an enlarged variational problem with free boundaries. The optimum-optimorum theory was developed by the author in order to solve this enlarged variational problem, inside of a class of FCs, with some chosen common properties. This theory was used for the inviscid GO shape of three models with high aerodynamical performances, namely: ADELA (a delta wing alone) and of two integrated wing-fuselage FCs FADET I and FADET II, flying in supersonic flow. The refinement of the optimization strategy, in form of an evolutive, iterative optimum-optimorum theory, is here presented. The inviscid GO shape of FC, represents now only the first step of this iterative method. A computational checking of this shape is made by using new hybrid analytical-numerical solutions for the Navier-Stokes layer. The total drag coefficient (including friction) is computed and a weak interaction aerodynamics/ structure, via new and/or modified constraints is proposed. Up the second step of iteration process, a migration in the drag functional and in the constraints is performed.*

Key Words: enlarged variational method, Navier-Stokes layer, supersonic flow, weak interaction aerodynamics/structure, deterministic optimization, genetic algorithms properties.

1. INTRODUCTION

The classical variational problem, concerning the aerodynamical optimization of the shape of a FC with a given planform, leads to a classical variational problem with fixed boundaries. The author has two times enlarged this variational problem, in order to be able: to determine the GO shape of the FC and to include the friction effect in the computation of the total drag functional and in the optimal design.

The first enlargement consists in the determination of the inviscid GO shape of the FC (namely, the simultaneous optimization of its camber, twist, thickness and also of its similarity parameters of the planform), which leads to an enlarged variational problem with free boundaries. An own optimum-optimorum (OO) theory was developed in order to solve this enlarged variational problem. The inviscid GO shape of FC is chosen in the frame of a class of admissible FCs with good suited common properties. The OO-theory was used by the author for the aerodynamical GO of the shapes of three aerospace models, namely: of ADELA (a delta wing alone) and of two integrated wing-fuselage FCs FADET I and FADET II, optimized, respectively, for the cruising Mach numbers 2.0, 2.2 and 3.0.

The second enlargement of the variational method, used here, consists in the development of an iterative OO-theory, which takes the inviscid GO of the FC's shape, previously deter-mined, as first step of iteration. This shape is checked by using the new developed hybrid analytical-numerical solutions for the Navier-Stokes layer (NSL), which use the analytical hyperbolical potential solutions of the flow over the same FC twice, namely firstly as outer flow and secondly the velocity's components are considered between the corresponding components of the potential velocity and polynomial expansions with

arbitrary coefficients, as in [1]. The friction drag coefficient is computed and this FC's shape is also controlled for the structure point of view. Up the second step of iteration, the total drag coefficient is the new functional and the new and/or modified set of constraints, resulting after a weak interaction aerodynamics/structure, are considered. The evolutive, iterative optimum-optimorum theory can be used for the refinement of GO shape of FC.

2. THE OPTIMUM-OPTIMORUM THEORY

The first enlargement consists in the inviscid GO shape of the FC (namely, the optimization of its camber, twist and thickness distributions and also of its similarity parameters of the planform), which leads to an enlarged variational problem with free boundaries. An own OO-theory was developed in order to solve this enlarged variational problem. The global optimized FC's shape is chosen inside of a class of admissible FCs, which is defined by some common properties. Two FCs belong to the same class, if: their surfaces-expansions are piecewise expressed in form of superpositions of polynomes with the same maximal degree (with unknown coefficients), their planforms are polygones (with unknown similarity parameters of planform), which can be related through affine transformations and they fulfill the same constraints. A lower-limit hypersurface of the drag functional $C_d^{(i)}$ as function of the similarity parameters ν_i is defined, namely,

$$(C_d^{(i)})_{opt} = f(\nu_1, \nu_2, ..., \nu_n) \tag{1}$$

Each point of this hypersurface is obtained by solving a classical variational problem with given boundaries (i.e. a given set of similarity parameters). The position of the minimum of this hypersurface, which is numerically determined, gives us the best set of the similarity parameters and the FC's optimal shape, which corresponds to this set, is in the same time the global optimized FC's shape of the class. The determination of the GO shape of the model FADET I, via OO-theory, is presented here, as exemplification.

3. THE GLOBAL OPTIMAL DESIGN OF THE MODEL FADET I

A wing-fuselage FC is here considered as a discontinuous wing along the junction lines wing/fuselage. If, additionally, the wing and the fuselage have the same mean surface and the same tangent plane, in each point of their junction lines, the equivalent wing of the wing-fuselage configuration is here called integrated wing-fuselage. Its mean surface $Z(x_1, x_2)$ is supposed to be continuous, but, for the sake of generality, the thickness distributions $Z^*(x_1, x_2)$ on the lateral sides, corresponding to the wing and $\overline{Z}'(x_1, x_2)$ on the central part, corresponding to the fuselage zone, are different. The computation is made in a dimension-less system of coordinates $\widetilde{O}\,\widetilde{x}_1\widetilde{x}_2\widetilde{x}_3$, as in [1], [2], it is:

$$\widetilde{x}_1 = \frac{x_1}{h_1}\ ,\quad \widetilde{x}_2 = \frac{x_2}{\ell_1}\ ,\quad \widetilde{x}_3 = \frac{x_3}{h_1}$$
$$\left(\ \ell = \frac{\ell_1}{h_1}\ ,\ y = \frac{x_2}{x_1}\ ,\ \widetilde{y} \equiv \frac{\widetilde{x}_2}{\widetilde{x}_1} = \frac{y}{\ell}\ \right) \tag{2a-c}$$

The downwashes $\widetilde{w}$ on the thin component and $\widetilde{w}^*$ and $\overline{w}^*$ on the wing and on the fuselage of the thick-symmetrical component of the integrated FC are expressed in form of superposition of homogeneous polynomes (with unknown coefficients), namely:

$$\widetilde{w} = \sum_{m=1}^{N} \widetilde{x}_1^{m-1} \sum_{k=0}^{m-1} \widetilde{w}_{m-k-1,k} \left|\widetilde{y}\right|^k$$

$$\widetilde{w}^* = \sum_{m=1}^{N} \widetilde{x}_1^{m-1} \sum_{k=0}^{m-1} \widetilde{w}^*_{m-k-1,k} \left|\widetilde{y}\right|^k \qquad \text{(3a-c)}$$

$$\overline{w}^* = \sum_{m=1}^{N} \widetilde{x}_1^{m-1} \sum_{k=0}^{m-1} \overline{w}^*_{m-k-1,k} \left|\widetilde{y}\right|^k$$

The junction lines wing/fuselage are considered as two artificial ridges of the integrated wing. The corresponding axial disturbance velocities u and u^* on the thin and thick-symmetrical components of the FC, obtained by the author, by using the hydrodynamic analogy of Carafoli, are:

$$u = \ell \sum_{n=1}^{N} \widetilde{x}_1^{n-1} \left\{ \sum_{q=1}^{E\left(\frac{n-1}{2}\right)} \widetilde{C}_{n,2q} \widetilde{y}^{2q} \cosh^{-1} \sqrt{\frac{1}{\widetilde{y}^2}} + \sum_{q=0}^{E\left(\frac{n}{2}\right)} \frac{\widetilde{A}_{n,2q} \widetilde{y}^{2q}}{\sqrt{1-\widetilde{y}^2}} \right\} \qquad (4)$$

$$u^* = \ell \sum_{n=1}^{N} \widetilde{x}_1^{n-1} \left\{ \sum_{q=1}^{E\left(\frac{n-1}{2}\right)} \widetilde{C}^*_{n,2q} \widetilde{y}^{2q} \cosh^{-1} \sqrt{\frac{1}{\nu^2 \widetilde{y}^2}} + \sum_{q=0}^{E\left(\frac{n-2}{2}\right)} \widetilde{D}^*_{n,2q} \widetilde{y}^{2q} \sqrt{1-\nu^2 \widetilde{y}^2} \right.$$

$$\left. + \sum_{q=0}^{n-1} \widetilde{G}^*_{nq} \widetilde{y}^q \left[\cosh^{-1} N_1 + (-1)^q \cosh^{-1} N_2\right] \right\}. \qquad (5)$$

$$\left(M_{1,2} = \sqrt{\frac{(1+\nu)(1\mp\nu\widetilde{y})}{2\nu(1\mp\widetilde{y})}}, \qquad N_{1,2} = \sqrt{\frac{(1+\overline{\nu})(1\mp\nu\widetilde{y})}{2(\overline{\nu}\mp\nu\widetilde{y})}} \right)$$

The parameters of optimization are the coefficients of the downwashes $\widetilde{w}_{ij}$, $\widetilde{w}^*_{ij}$ and $\overline{w}^*_{ij}$ and the similarity parameters of the planform of the entire FC and of the planform of the fu-selage ν and, respectively, $\overline{\nu}$. The quotient $k = \overline{\nu} / \nu$, which depends on the purpose of the FC, is taken constant, during the optimization process.

For a given value of the similarity parameter of the FC's planform, the optimization of the shapes of its thin and thick-symmetrical components can be separately treated.

The initial constraints for the thin FC component are: given lift and pitching moment coefficients and also the own introduced Kutta condition on leading edge in order to cancel the induced drag and to avoid the conturnement of flow on leading edge, at cruise.

The initial constraints for the thick-symmetrical components are: the given relative volumes of the fuselage and of entire FC, the closing condition on leading edges and also the own in-troduced integration conditions along the junction lines wing/fuselage.

For a given value of the similarity parameter of the planform of the FC the corresponding optimal values of the coefficients of downwashes are analytical, uniquely determined by solving of two linear algebraic systems, as in [1], [2].

If the similarity parameter of the planform of FC is sequentially varied a lower limit-line of the inviscid drag functional of optimal FCs, as function of this similarity parameter is obtained by solving the variational problems for the corresponding values of the similarity parameter (for FCs with subsonic leading edges is $0 < \nu < 1$).

The position of the minimum of this limes-line gives the optimal value of the similarity parameter $\nu = \nu_{opt}$ and the corresponding optimal FC is, in the same time, the global optimized FC of the class.

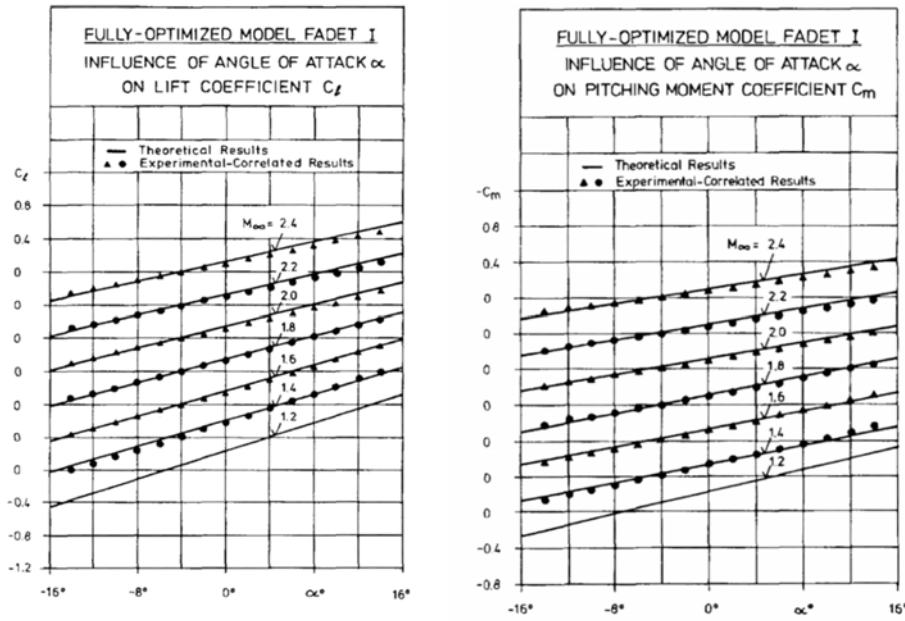
The GO shape of the fully-integrated wing-fuselage model FADET I, designed by the author, for the cruising Mach number $M_\infty = 2.2$, obtained by using the OO-theory, is presented in the Fig. 1 and looks, in transversal sections, like a flying bird!

Fig. 1 The global optimized shape of the fully-integrated wing–fuselage model FADET I

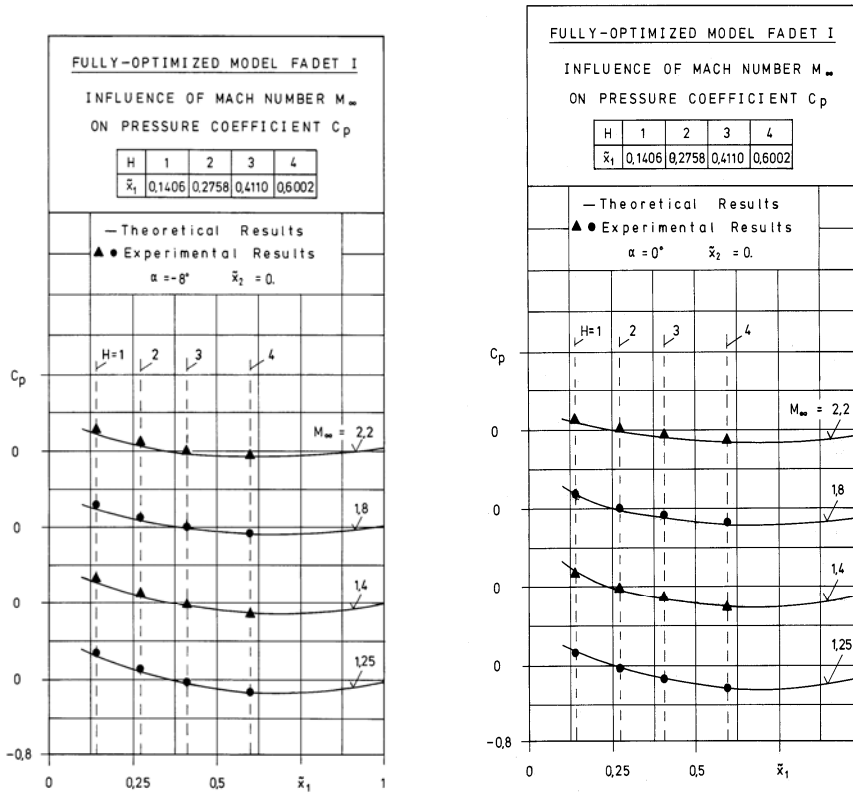
The aerodynamical characteristics of the integrated, GO shape of the model FADET I was checked in the trisonic wind tunnel of the DLR-Cologne, in the frame of the author's research project, sponsored by the DFG.

A very good agreement between the theoretical and experimental results, obtained by using inviscid analytical, hyperbolical solutions for its lift and pitching moment coefficients, is presented in the Fig. 2a,b.

In the Figs. 3a-c is compared the theoretical predicted distribution of pressure coefficient with experimental results, in the longitudinal central section of the upper side of model FADET I, at the angles of attack $\alpha = -8^\circ,\ 0^\circ,\ 8^\circ$ and for the range of Mach numbers $M_\infty = (1.25 - 2.4)$. For this range of Mach numbers, the model FADET I has subsonic leading edges.



Figs. 2a, b The lift and the pitching moment coefficients of the global optimized model FADET I



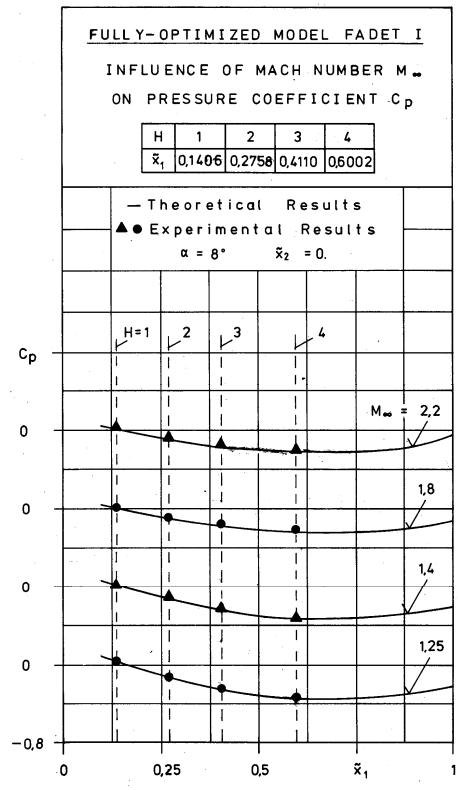


Fig. 3a-c The distribution of pressure coefficient on the central cut at the upper side of the model FADET I, for the angles of attack $\alpha = -8^\circ,\ 0^\circ,\ 8^\circ$

A very good agreement between experimental and theoretical results is obtained by using of the own analytical hyperbolical potential solutions for the lift, pitching moment and pressure coefficients on FCs with subsonic leading edges, at moderate angles of attack, in supersonic flow, as it can be seen in the Figs. 2a,b - Figs. 3a-c. This agreement leads to the remarks:

- The validity of the three-dimensional analytical hyperbolical potential solutions for the axial disturbance velocities with the chosen balanced singularities and the corresponding developed software for the above aerodynamical coefficients are confirmed;
- The influence of friction upon these coefficients is neglectable;
- The flow is laminar, as supposed here and remains attached in supersonic flow, for larger angles of attack than by subsonic flow;
- Hybrid solutions for the Navier-Stokes layer (NSL) with important analytical properties are proposed here, for the computation of the total drag (including friction) and of all the aerodynamic characteristics, at higher angles of attack.

4. HYBRID SOLUTIONS, FOR NAVIER-STOKES LAYER

The analytical, hyperbolical, potential solutions are used in the first, inviscid step of the evolutionary, iterative OO-theory. These solutions are replaced by own developed reinforced hybrid solutions for the NSL, obtained by crossover of analytical with numerical solutions in

order to obtain 'talented' hybrid NSL's solutions, which have the generality of the numerical solutions, fulfill the non-slip conditions and have also important analytical properties. Let us introduce a spectral variable η , it is:

$$\eta = \frac{x_3 - Z(x_1, x_2)}{\delta(x_1, x_2)} . \quad (0 < \eta < 1)$$

Hereby $Z(x_1, x_2)$ is the equation of the surface of the flattened FC and $\delta(x_1, x_2)$ is the NSL's thickness distribution. The spectral forms of the axial, lateral and vertical velocity's components u_δ, v_δ and w_δ, the density function $R = \ln \rho$ and the absolute temperature T are here proposed, as in [1]-[3], namely:

$$u_\delta = u_e \sum_{i=1}^{N} u_i \eta^i , \qquad v_\delta = v_e \sum_{i=1}^{N} v_i \eta^i , \qquad w_\delta = w_e \sum_{i=1}^{N} w_i \eta^i ,$$

$$R = R_w + (R_e - R_w) \sum_{i=1}^{N} r_i \eta^i , \tag{6a-e}$$

$$T = T_w + (T_e - T_w) \sum_{i=1}^{N} t_i \eta^i$$

Here R_w and T_w are the given values of R and T at the wall , u_e, v_e, w_e, R_e and T_e are the values of u, v, w, R and T at the NSL's edge, obtained from the outer inviscid hyperbolical potential flow, and u_i, v_i, w_i, r_i and t_i are their free spectral coefficients, which are obtained by fulfilling of the NSL's partial differential equations (PDEs). The physical equation of ideal gas for the pressure p and an exponential law of the viscosity μ versus T are used:

$$p \equiv R_g \rho T = R_g e^R T$$

$$\mu = \mu_\infty \left[\frac{T}{T_\infty} \right]^{n_1} . \tag{7a-b}$$

Here are: R_g and T_∞ the universal gas constant and the absolute temperature of the undisturbed flow and n_1 is the viscosity exponent. The use of the density function $R = \ln \rho$ (instead of the density ρ), proposed here, combined with the formulas for R, T, u, v, w, p and μ, and with the collocation method allow the determination of the spectral coefficients of all the physical entities, namely: R, T, p and μ only as functions of the spectral coefficients of the velocity's components, as in [1]-[3]. The spectral forms (6a-e) automatically satisfy the boundary conditions at wall $(\eta = 0)$. The boundary conditions at the NSL's edge are eliminated by fixing seven spectral coefficients of the velocity's components. If the spectral forms given in (6a-c) are introduced in the NSL's PDEs of impulse and the collocations method is used, the spectral coefficients of the velocity's components u_i, v_i

and w_i are obtained by the iterative solving of a linear algebraic system with slightly variable coefficients, which values are taken for the precedent iteration. The analytical properties of the hybrid numerical NSL's solutions proposed here are the following: they have correct last behaviours, they have correct jumps along the singular lines (like subsonic leading edges, junction lines wing/fuselage) and the singularities are balan-ced), are accurate because they are meshless and the derivatives can be exactlly computed. Further, by using of a logarithmic density function $R = \ln \rho$, a splitting of the NSL's PDEs is realized, which speeds up the computation time.

5. EVOLUTIVE, ITERATIVE OPTIMUM-OPTIMORUM THEORY

The second enlargement of the variational method consists in the development of an iterative OO-theory, in order to introduce also the influence of friction in the drag functional and in the aerodynamical GO shape of FC. An intermediate computational checking of the inviscid GO shape of the FC is made with own hybrid solvers, for the three-dimensional Navier-Stokes layer (NSL). The friction drag coefficient $C_d^{(f)}$ of the FC is determined. The iterative OO-theory uses the inviscid hyperbolical potential solutions as start solutions and the inviscid GO shape of the FC, only in its first step of iteration. The inviscid GO shape is checked also for the structural point of view. Additional or modified constraints, introduced in order to control the camber, twist and thickness distributions of the aerodynamical, global optimized FC's shape, for structural reasons, are here proposed. In the second step of optimization, the predicted inviscid optimized shape of the FC is corrected by including these supplementary constraints in the variational problem and of the friction drag coefficient in the drag functional. The iterative optimization process is repeated, until the maximal local modification of the shape in two consecutive optimization steps presents no significant change. The chart flow of the evolutive, iterative OO-theory is given in the Fig. 4. A weak interaction aerodynamics/ structure, via new and modified constraints, introduced for the structure reasons, in the process of the determination of aerodynamical GO shape, is proposed.

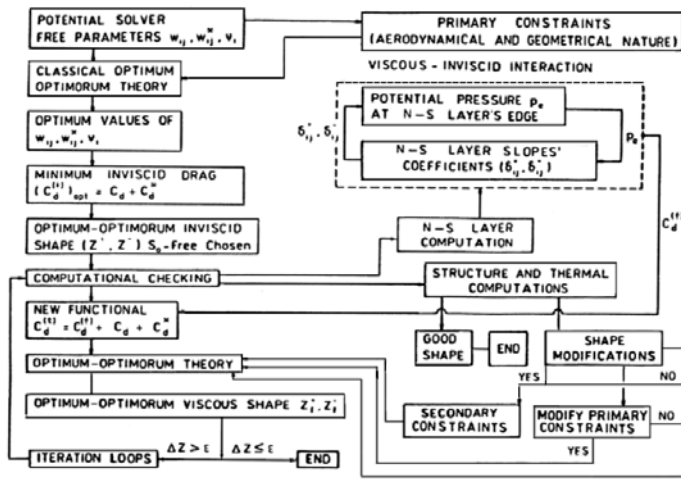


Fig. 4 The chart flow of the evolutive, iterative optimum-optimorum theory

6. CONCLUSIONS

The evolutionary, iterative optimum-optimorum theory is a deterministic theory, which has almost all the attributes of the genetic algorithms like migrations, mutation, crossover, multiple selections, allows the multidisciplinary optimal design by using additional and/or modified constraints, request from the structure purposes, allows the multipoint design by morphing, is flexible, accurate, economic and competitive.

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