Compensation for errors in determining the angle in the flight-navigation complex of the aircraft in case of failure of the satellite navigation system

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Abstract: *Currently, navigation equipment for long-range aircraft, where the flight is characterized by a long duration and the absence of frequent and abrupt maneuvers, continues to be actively improved. Therefore, the main purpose of the work is to analyze the compensation of the error in determining the angle in the flight-navigation complex of the aircraft in case of failure of the satellite navigation system. In order to achieve the goal, inertial navigation methods using an attitude and heading reference system were used. It was determined that when comparing the evolution graphs of the heading and errors in determining the eastern component of the ground speed, it is possible to establish the dependence that at the moments of maneuvers associated with a significant change in the direction of this speed, the phase and amplitude of the error fluctuations are also changed. It was established that today the operational technology is equipped with blocks of an inertial navigation system. It helps to improve the accuracy of the definition of navigation information.*

Key Words: *aviation technology, error evolution, extrapolation, ground speed*

1. INTRODUCTION

The article discusses the approach to solving the problem of deteriorating accuracy of determining the heading in the flight-navigation complex of the aircraft, in the standard mode providing for this purpose the integration of the inertial navigation system (INS) and the global navigation satellite system (GNSS), in the case of temporary loss of the ability to receive navigation signals or their significant distortion and subsequent operation of the INS in the autonomous mode [1], [2], [3].

The key feature of the approach proposed by the authors is the absence of the need for technical or algorithmic modification of the INS block, since the solution of the problem is achieved at the level of secondary data processing of the INS and consumer navigation equipment (CNE) of GNSS. On the basis of the considered approach, an algorithm for data processing and software and mathematical support for modeling the processes of the operation of the aircraft flight-navigation complex using the developed algorithms were created. The computational experiments carried out through software have shown the efficiency of algorithms in terms of performance of INS's autonomous solution and the possibility of practical implementation of the proposed approach.

Currently, navigation equipment for long-range aircraft, whose flight is characterized by a long duration and the absence of frequent and abrupt maneuvers, continues to be progressively improved. One of the most important parameters for the implementation of navigation system in long flights is the heading, defined as the angle between the direction to the north along the tangent to the local meridian at the location of the aircraft and its travel speed vector **W** (1).

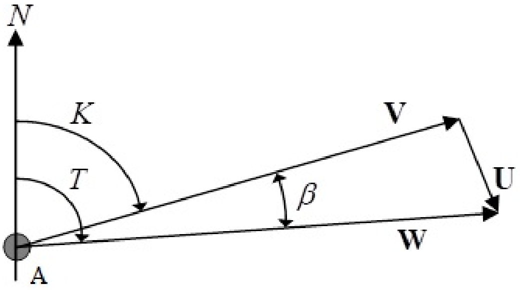

Fig. 1 – Navigation triangle for speeds

Fig. 1 adopted the following notation: ***T*** – heading, ***K*** – course angle (angle between north direction and airspeed vector **V** AC), β – the drift angle of the aircraft (caused by the side wind, characterized by the velocity vector **U**).

2. RESEARCH METHODS

Traditionally, the definition of the heading was carried out using inertial navigation methods using attitude and heading reference system or INS [4], [5], [6], [7]. In the case of INS, the heading can be determined from the relationship:

$$T = arctg\left(\frac{W_e}{W_n}\right) \tag{1}$$

where W_n – the northern component of the ground speed vector **W** AC, W_e – the eastern component of the vector **W**.

However, the method of determining the heading with an obvious advantage – the autonomy of the definition – has a significant drawback due to the fundamental principles of inertial navigation. This disadvantage is in increasing errors in determining navigation parameters in time and is inherent in any type of inertial navigation systems even with their ideal technical implementation due to the general instability of the algorithm of their functioning [7], [8], [9]. Thus, the accuracy of the solution formed by the INS is determined by a number of factors independent from its constructive implementation. The external disturbing factors causing an increase in errors include the error in determining the initial conditions (coordinates, speed, orientation) made at the stage of preparation of the system, which are necessary for solving the basic navigation equations. The accuracy of determining the initial conditions depends on the implementation of the algorithm of the initial connection and on the technical capabilities of the geodetic binding of the AC launch site to the Earth's surface, or on the accuracy of the carrier's navigation system in the case of motion starting from a moving base. Another source of errors of the navigation solution of the INS is the error of the adopted mathematical model of the gradient of the gravitational field of the Earth. $\mathbf{g}(\mathbf{R})$ relative to the real gravitational field, which, as we know [5], [7], is characterized by significant gravitational anomalies. Positive feedback in the INS, implemented through an algorithm for calculating the gravitational acceleration required for introducing appropriate corrections to the measurement data of accelerometers, depends on the accuracy of information about the current location. Errors in the coordinates and components of velocity when used to calculate the gravitational acceleration vector lead to the appearance of a type of errors, described by a harmonic law with frequency $\omega = \sqrt{\frac{g}{R}}$ where g – the absolute value of the acceleration of gravity, R – Earth radius [9], [10] .The frequency corresponds to the oscillation period of the Schuler pendulum. $\tau_{Ш}$ = 84.4 min. The mentioned deficiency necessitates the implementation of the correction of the INS using external sources of information or other components of the aircraft flight-navigation complex (FNC). The use of the air data computer (ADC) for these purposes is difficult because it requires external information about the wind speed and direction, which is not always available and reliable.

Currently, the FNC of long-range aircrafts includes various radio navigation systems (RNS), including satellite (GNSS) [7], [8], [9]. Due to their integration [11], [12], [13], [14], [15] with the INS, it is possible to obtain more accurate navigation information for controlling the movement of the aircraft. In addition, when using such an aggregation, the errors in determining navigation parameters cease to increase with the flight time, their dispersion decreases [7], [16], [17], [18]. Such a positive effect is provided mainly due to the difference in the spectral characteristics of the errors that are combined (complex) in the composition of the FNS radio engineering and non-radio navigation systems. Thus, GNSS allows determining the horizontal components of ground speed and, on the basis of them, similarly to the INS, calculate the value of the heading. However, the use of GNSS consumer navigation equipment (CNE) as part of the FNS leads to an additional dependence of its operation on external conditions. For example, the aircraft navigation system becomes vulnerable to the effects of electronic warfare (EW), capable of suppressing or distorting the GNSS signals [15], [19]. In addition, in case of accidental failure or malfunction of any GNSS components, the possibility of correcting the INS and the heading value generated in the FNS may be lost, which will lead to errors of the nature described above.

3. RESULTS AND DISCUSSIONS

In researches [10], [20], some approaches were considered that meant to improve the accuracy of INS due to the damping of the Schuler oscillations [9] by increasing the complexity and improvement of the data processing algorithms of gyroscopic and inertial meters applied in the combination with INS. In particular, the use of smoothing filters and nonlinear elements to diminish errors in orientation and navigation parameters. By using mathematical modeling, it has been shown [10], [20] that such measures can improve the navigation solution when using INS autonomously. At the same time, today a large number of aircraft in operation, including for military purposes (and therefore potentially exposed to EW), are equipped with INS systems, the operation algorithms of which do not provide for compensation for Schuler oscillations, or this compensation is not sufficiently effective. In this regard, the task of reducing errors in determining navigation information, in particular, the heading, with relatively short-term GNSS operation failures (and other onboard RNS) and the operation of the INS, built according to the traditional scheme [7], in a completely autonomous mode, becomes important. The solution of the problem will allow to us operate the system without the costly replacement of the equipment of the INS and reduce the cost of upgrading avionics.

3.1 Analysis of INS errors nature

Fig. 2 shows a graph of the evolution of the heading, built on the basis of the FNC data obtained during the actual flight of the aircraft along the given route.

The values of the components of the ground speed, on the basis of which the ground angle was calculated, were formed by strapdown INS (SINS) [21], [22] and NAP GNSS CNE, i.e. possessed the maximum accuracy provided without the use of external monitoring tools. In the future, we will conditionally consider as "true" values of the navigation parameters formed in a similar way.

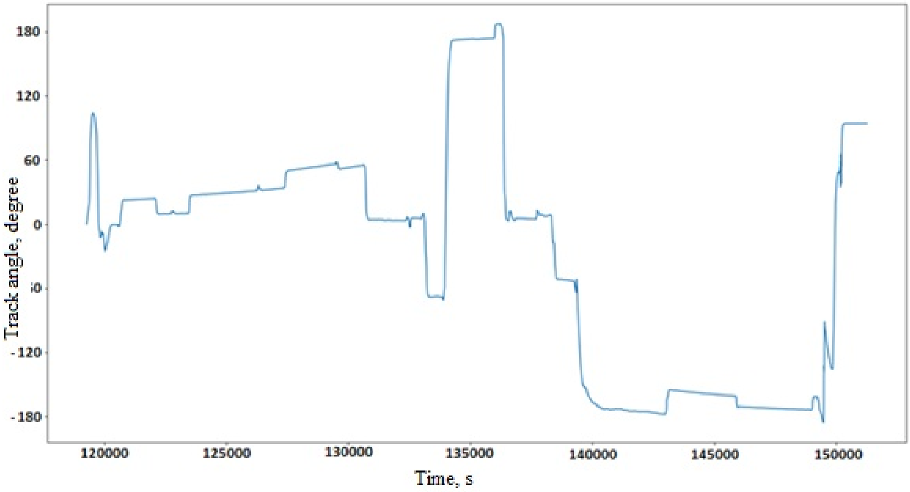

Fig. 2 – The graph of the evolution of the heading of AC

During the flight of the aircraft, an autonomous solution of the SINS was also formed, for which the evolution of errors in the determination of the northern ΔW_n and eastern ΔW_e ground speed components as shown in 3.

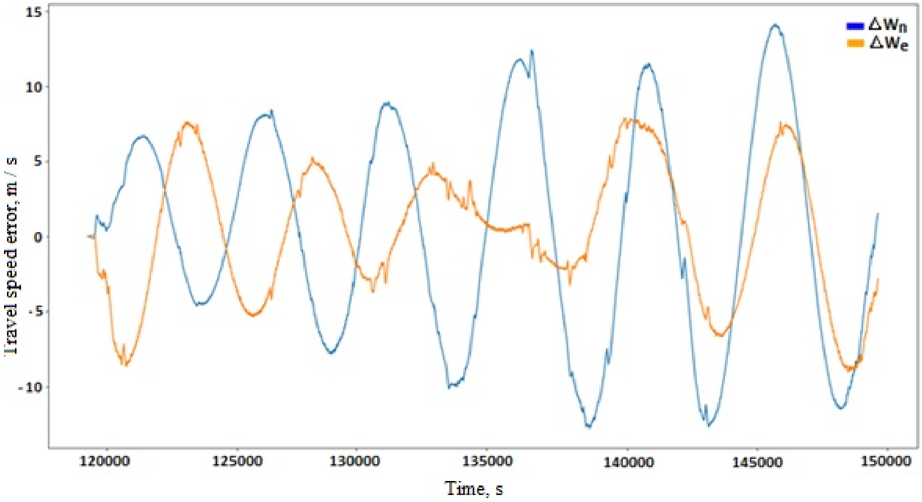

Fig. 3 – The evolution of errors in determining the northern and eastern components of the ground speed of the AC

As it can be seen from the graphs presented, the evolution of errors is a near-harmonic oscillatory process with a close to the Schuler period and an amplitude varying according to a law close to linear.

However, comparing the graphs of the evolution of the heading and the errors in determining the eastern component of the ground speed, it can be noted that at the moments of maneuvers associated with a significant change in the direction of this speed, the phase and amplitude of the error fluctuations also change.

3.2 INS navigation accuracy improving problem solution approach

Understanding the nature of the dynamics of errors in determining the components of ground speed ΔW_n and ΔW_e, the solution of the problem in question may consist in extrapolating the values of these errors in the event of loss of possibility of using other corrective information. To do this, in the process of normal operation of the FNC, the magnitude of the error of the INS should be continuously recorded, so that then on the basis of a certain sample of their values immediately preceding the moment of correction loss, it would be possible to construct approximating dependencies used for extrapolation. The nature of the dependence of the dynamics of errors in Fig. 3 indicates the expediency of approximation on the basis of the following generalized function:

$$f(t) = (k_1 t + b_0)\, sin(\omega_0 t + \varphi_0) + b_1 \tag{2}$$

where: k_1 – the rate of increase of the amplitude of oscillations, b_0 – the initial value of the amplitude of oscillations at the estimated interval, b_1 – shift along the vertical axis, ω_0 – oscillation frequency, φ_0 – the initial phase of oscillation.

Because fluctuations occur at the Schuler period $\tau_S = 84.4$ min. then the value ω_0 can be taken equal $\frac{2\pi}{\tau_S} = 0.00124$ rad/ s.

To determine the remaining coefficients of approximating dependence, it is suggested to use the traditional approach – the method of least squares (OLS) [7], [11], [23]. The vector of estimated parameters shall be:

$$\mathbf{X} = (k_1, b_0, b_1,\ \varphi_0) \tag{3}$$

Let's mark the dependence (2), parametrized by the components of the vector (3) as $f(\mathbf{X}, t)$. The recurrence relation of Gauss – Newton method as an implementation of the OLS [24] for obtaining an estimate of the vector (3) will be as follows:

$$\hat{\mathbf{X}}_{i+1} = \hat{\mathbf{X}}_i + \left(\mathbf{H}_i{}^T \mathbf{H}_i\right)^{-1} \mathbf{H}_i{}^T \left(\tilde{\mathbf{Y}} - \mathbf{F}\left(\hat{\mathbf{X}}_i\right)\right) \tag{4}$$

where $\hat{\mathbf{X}}_i$ and $\hat{\mathbf{X}}_{i+1}$ – are the estimates of the condition vector (3) at the i -th and i + 1 points in time, respectively, the matrix $\mathbf{H}_i$ consists of partial derivatives $f(\mathbf{X}, t)$ on the components of the vector (3) and has the following form:

$$\underset{[N\times 4]}{\mathbf{H}_i} = \begin{pmatrix} \left.\frac{\partial f(\mathbf{X}, t_1)}{\partial k_1}\right|_{\mathbf{X}=\hat{\mathbf{X}}_i} & \left.\frac{\partial f(\mathbf{X}, t_1)}{\partial b_0}\right|_{\mathbf{X}=\hat{\mathbf{X}}_i} & \left.\frac{\partial f(\mathbf{X}, t_1)}{\partial b_1}\right|_{\mathbf{X}=\hat{\mathbf{X}}_i} & \left.\frac{\partial f(\mathbf{X}, t_1)}{\partial \varphi_0}\right|_{\mathbf{X}=\hat{\mathbf{X}}_i} \\ & & \cdots & \\ \left.\frac{\partial f(\mathbf{X}, t_N)}{\partial k_1}\right|_{\mathbf{X}=\hat{\mathbf{X}}_i} & \left.\frac{\partial f(\mathbf{X}, t_N)}{\partial b_0}\right|_{\mathbf{X}=\hat{\mathbf{X}}_i} & \left.\frac{\partial f(\mathbf{X}, t_N)}{\partial b_1}\right|_{\mathbf{X}=\hat{\mathbf{X}}_i} & \left.\frac{\partial f(\mathbf{X}, t_N)}{\partial \varphi_0}\right|_{\mathbf{X}=\hat{\mathbf{X}}_i} \end{pmatrix} \tag{5}$$

vector $\widetilde{\mathbf{Y}}$ is represented as a set of error values for determining the northern or eastern components of the ground speed of the aircraft, referred to the points in time $t_1, t_2, \ldots, t_N$ where N – is the number of measurements that are approximated.

Considering the type of approximating dependence interval $t_N - t_1$ can be limited to one period of oscillation, the frequency of the measurements should be chosen not less than $10/\tau_S$ 0.002 Hz (with normal operation of the INS in the FNC, this frequency is much higher).

Vector function $\mathbf{F}(\widehat{\mathbf{X}}_i)$ looks as follows:

$$\underset{[N\times 1]}{\mathbf{F}(\widehat{\mathbf{X}}_i)} = \begin{pmatrix} f(\widehat{\mathbf{X}}_i, t_1) \\ \cdots \\ f(\widehat{\mathbf{X}}_i, t_N) \end{pmatrix} \tag{6}$$

Received using the described parameters $\widehat{X}_n$ and $\widehat{X}_e$ dependencies $f(\widehat{X}_n, t) = \Delta W_n^*(t)$ and $f(\widehat{X}_e, t) = \Delta W_e^*(t)$ for errors of the northern and eastern components of the ground speed, we can extrapolate the values of these errors and correct the measured INS values and the ground angle:

$$T^*(t) = arctg\left(\frac{W_e^{AINS}(t) - \Delta W_e^*(t)}{W_n^{AINS}(t) - \Delta W_n^*(t)}\right), \tag{7}$$

where $T^*(t)$ – is the corrected value of the track angle, $W_e^{AINS}(t)$, – are components of ground speed, calculated by the INS. $W_n^{AINS}(t)$

The success of such a correction, obviously, would depend on the stability of the parameters of INS error, which, as it was shown above, can change when making significant aircraft maneuvers.

The circumstance imposes a restriction on the use of the considered approach to compensate for the error in determining the heading during active maneuvering of an aircraft during flight.

3.3 Computational experiments and INS accuracy improving developed algorithms efficiency evaluation

To check the efficiency and effectiveness of the error compensation algorithm based on the extrapolation, mathematical software was developed [25], the input data for which were samples of the true component velocity values determined by the ISS.

As a criterion for the effectiveness of the algorithm under discussion, it is proposed to consider the ratio of the integral values of the errors of heading using compensation δ_Σ^* and without compensation δ_Σ:

$$E = \frac{\delta_\Sigma^*}{\delta_\Sigma}, \tag{8}$$

where:

$$\delta_\Sigma^* = \int_{t_1}^{t_2} |\delta^*(t)| dt \tag{9}$$

$$\delta_\Sigma = \int_{t_1}^{t_2} |\delta(t)| dt \tag{10}$$

$$\delta\,(t) = T^{AINS}\,(t) - T^{IST}\,(t), \tag{11}$$

$$\delta^{*}(t) = T^{AINS}\,(t) - T^{*}\,(t) - T^{IST}\,(t), \tag{12}$$

t_1, t_2 moments of the beginning and end of the application of compensation, $T^{AINS}\,(t)$ – the value of the track angle formed by the INS without correction, $T^{IST}(t)$ – the true value of the track angle.

Fig. 4 shows the evolution of the observed errors in determining the northern ΔW_n and eastern ΔW_e components of the ground speed of the aircraft, approximating their curves, as well as the evolution of errors in determining the angle $\delta(t)$, $\delta^{*}(t)$ and their absolute values.

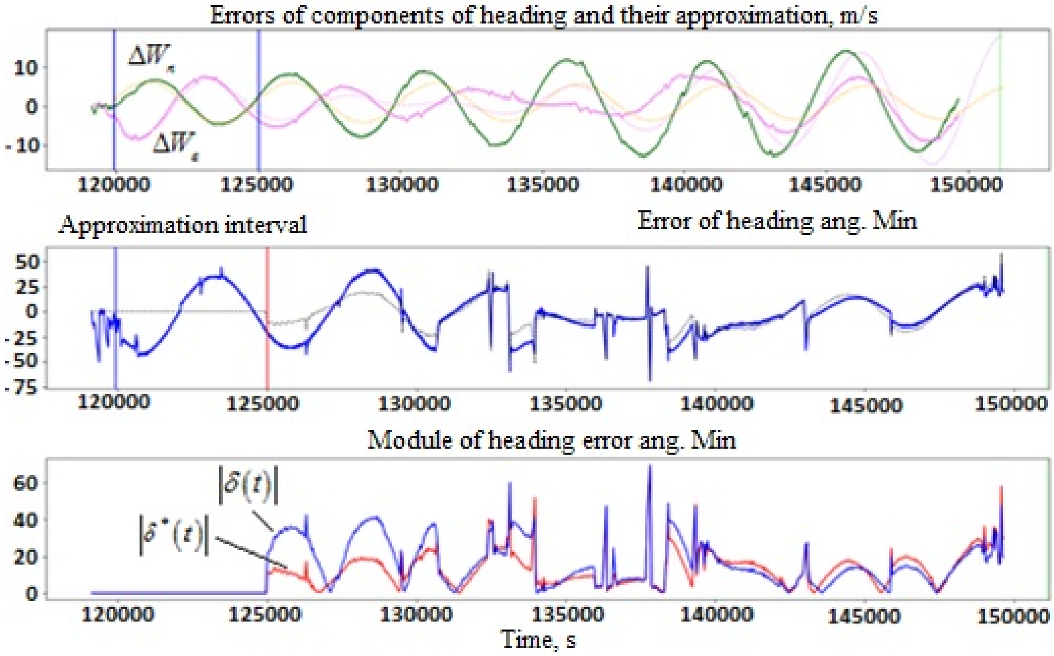

Fig. 4 – Error compensation, experiment 1

The approximation of errors was made on the basis of data about their evolution over the interval of 120,000-125,000s, which approximately corresponds to the Schuler period. The period between measurements was 1s.

The dependency analysis in fig. 4 shows that the absolute values of the compensated error do not exceed its original values in the interval 125,000– 135,000s, and in the interval 125,000– 1,300,000s they are significantly smaller.

However, extrapolation over longer intervals leads to the fact that the corrected errors become bigger then original ones.

Fig. 5 shows the results of the experiment, similar to the conditions of the previous one, but at the interval of approximation of errors of the components of the ground speed: 125000 – 130,000s.

Just like in the past experiment, an increase in the value of the corrected error is observed with an increase in the extrapolation duration.

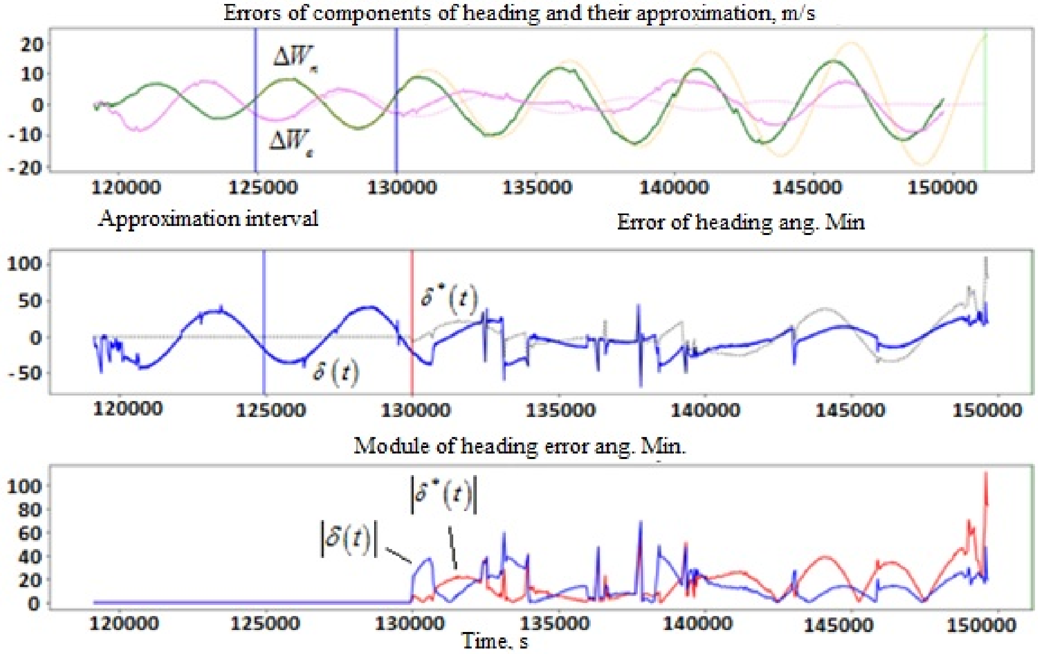

Fig. 5 – Error compensation, experiment 2

Table 1 describes the values of the criterion of the efficiency of the compensation algorithm obtained for different positions of approximation interval on the time axis and the extrapolation duration.

Table 1. – Efficiency of compensation algorithm for determining heading

t_2, s t_1, s	$E = \frac{\delta_\Sigma^*}{\delta_\Sigma}$			
	t_1+5000	t_1+10000	t_1+15000	t_1+20000
125000	0.446	0.830	0.823	1.251
130000	0.665	0.619	2.252	2.477

Criterion values over 1 indicate that it is inappropriate to apply the compensation algorithm in these conditions.

4. CONCLUSIONS

The article describes the approach proposed by the authors to improve the INS solution accuracy in the autonomous mode without integration with other FNC navigation subsystems. The main idea of the proposed approach is the application of the algorithm for estimating and leveling Schuler oscillations, which makes it possible to improve the accuracy of the navigation solution generated by existing INS equipment without its hardware modification. To estimate the effectiveness of the proposed approach and the developed algorithms, simulation using real data was performed.

The simulation results showed that in the absence of significant aircraft maneuvers on an interval equal to the Schuler period, after losing the corrective information from GNSS, the accuracy of heading calculation due to the use of error compensation algorithm increases by 35-72%, and up to 35 % at double interval. Further use of the algorithm is inexpedient, since the values of the corrected error will be closed to or exceeded the original. It is also worth noting that the duration of effective use of the algorithm significantly depends on the nature of the aircraft's movement – intensive maneuvering after loss of GNSS operability will lead to deterioration of parameters of Schuler oscillations and will not allow using the results of the approximation to compensate for the error in determining heading. To restoration of possibility for compensation, corrective information should be restored from GNSS or other sources during mostly straight flight. $\frac{\tau_S}{2} - \tau_S$.Turns along the course of the movement of the aircraft will lead to the need to reset and update the accumulated information about the heading's errors evolution.

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