Large caliber blood vessel pseudoaneurysm following prosthetic surgery. Mathematical model and numerical considerations

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***Abstract**: Using a non-Newtonian mathematical model for the blood flow and a generalized Maxwell model for the viscoelastic behavior of the large vessels – elaborated and presented already by us in previous papers [1], [2] we make some remarks on the wall shear stress (WSS) in the case of an artery whose vessel wall is replaced by a vascular prosthesis following a surgical intervention. Considering then a pseudoaneurysm which is located on both genuine blood vessel and prosthesis we analyze the distribution of wall shear stress and taking also into account the viscoplastic behavior of the prosthesis we try to determine the mechanical conditions which would lead to a possible "jerk" ("rupture") of the vascular vessel in the presence of the pseudoaneurysm.*

***Key Words**: non-Newtonian blood flow; generalized Maxwell viscoelastic behavior; viscoplatic behavior of the PFTE prosthesis; pseudoaneurysm; wall shear stress; rupture risk*

1. INTRODUCTION

The main goal of this research is to assess the risk of complication of a new aneurysm (the so called pseudoaneurysm) which could follow a prosthetic surgery of a damaged large blood vessel. We accept that a PFTE prosthesis has been used, and the surgical technique is illustrated in figure 1 [10].

Clinical practice shows that there is a trend for such reconstruction method of this new aneurysm – fact pointed out even by certain necroptic studies (figure 2 [4]).

In what follows, we try to calculate the wall shear stress (WSS) at significant points of this new aneurysm.

These values could be then compared with those of the corresponding forces (stress) (which joins the particles of the vessel walls) in view of the prediction of a possible "jerk" (rupture) of this new aneurysm.

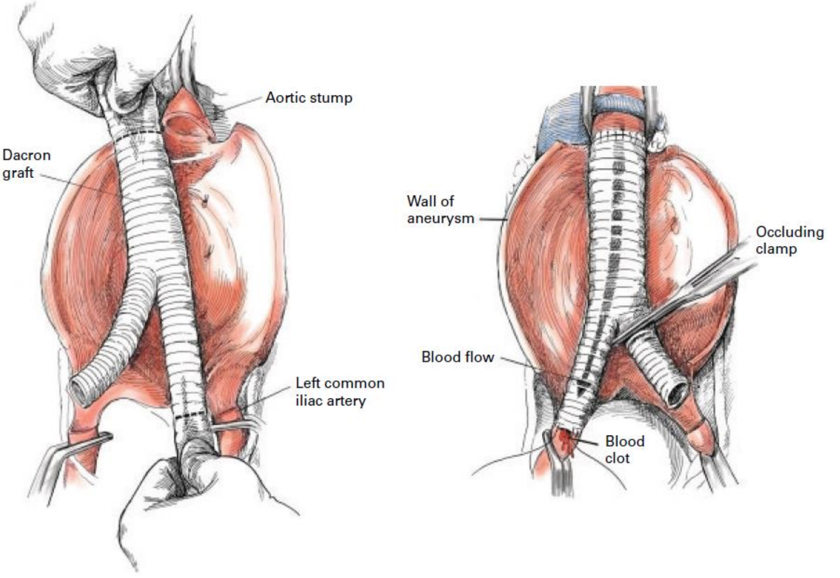

Figure 1. Resection of the abdominal aortic aneurysm by a PFTE graft [10]

It is important to remark that we will consider only prosthesis made by polytetrafluoroethylene (PFTE) – better known as Teflon [3], such a prosthesis observing an elasto-plastic behavior together with the viscous nature of the material.

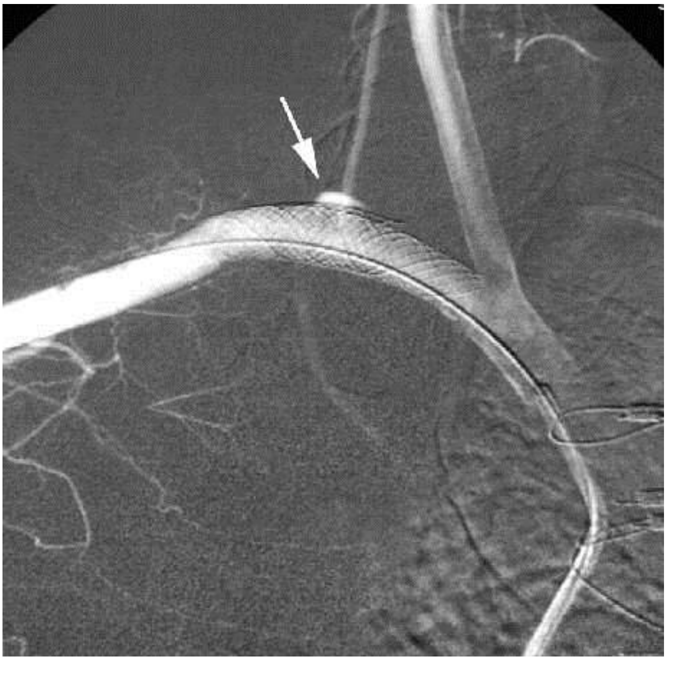

Figure 2. Persistent prosthesis pseudoaneurysm

Fluoropolymers are a class of polymers defined by the presence of carbon and fluoride that have many unique mechanical and chemical properties. For example, most fluoropolymers have a lower friction coefficient than most other materials, and the chemical resistance and thermo-mechanical stability are better than most other polymeric materials.

PFTE – invented by Plucket of DuPont in 1938 [3], first in the fluoropolymer class, shows up to be one of the materials accepted by a human body which gives good results as prosthesis in vascular surgery.

Essentially the model of mechanical behavior for such a PFTE prosthesis is built on the assumption that its deformation can be decomposed into two parts

$$\varepsilon = \varepsilon^{ve} + \varepsilon^{vp},$$

where ε is the total strain, ε^{ve} is the viscoelastic strain and ε^{vp} is the viscoplastic strain.

This model agrees with an elasto-plastic and viscous contribution to stress. In fact the most suitable behavior law seems to be a modified Kelvin stress-strain relation with an added power-law viscous term, namely

$$\sigma = C\gamma^{n} + k\dot{\gamma}^{m},$$

where σ is the shear stress, γ is the shear strain, $\dot{\gamma}$ is the shear rate, k is the consistency index, n and m are power-law exponents.

We remark also that the stress-stain behavior in the viscoplastic component (absent in the case of non-prosthesis blood vessel) could be represented by various hardening laws as Johnson-Cook, Khan-Huang, etc. [9]. At the same time in the early models, prior to pseudoaneurysm for instance, only the viscoelastic part of the stress-strain behavior was taken into consideration. Of course for large deformations, as the pseudoaneurysm implies, the plastic part of the stress-strain behavior should be taken into account [9].

2. CASE OF UNDEFORMED BLOOD VESSEL–PROSTHESIS JUNCTION

This research analysis firstly a large blood vessel which has been surged with PFTE prosthesis. Our model is based on the non-Newtonian Cross law for the blood flow, i.e., $\mu(\dot{\gamma}) = \mu_s + \dfrac{\mu_0^*}{1 + (k\dot{\gamma})^{1-n}}$, $\dot{\gamma}$ being the shear rate, μ_s the constant plasma viscosity, μ_0^* the viscosity coefficient at zero shear rate, k a time constant and n the index for a shear thinning behavior, while the wall vessel (genuine or prosthesis) are considered to observe a viscoelastic behavior of generalized Maxwell type. This approach has been already used by us in several papers [1], [2] and it seems to be in good agreement with the reality. That is why we will keep this mathematical model during this research too.

We will focus on the wall shear stress (WSS) which is at the origin of an irreversible deformation or even at the “jerk” (rupture) of the involved wall. This possible destructive deformation – which could lead to a fatal stroke, may have the origin at an over blood pressure due to the internal physical pressure and to the pressure due to the viscosity forces.

For the sake of simplicity we accepted during all this research an axial-symmetric configuration, Oz being the axis of symmetry.

By assessing of the WSS along the coupled blood vessel and the graft (prosthesis) we anticipate the state of health of the successful surged patient. More precisely for a specific case illustrated in figure 3 we have the evolution of WSS (presented in figure 4). Here the total length of the arterial segment is 1*cm* (from which the length of the genuine vessel is 0.6*cm* and the length of the prosthesis is 0.4*cm*, see figure 3), the internal diameter of the blood vessel is 7*mm* and the thickness of the vessel wall is 0.8*mm*. The mass density of the blood has been fixed at $\rho = 1060 kg/m^3$.

The mesh generated for both the fluid and solid domain consists of 901 triangular elements and 21366 degrees of freedom.

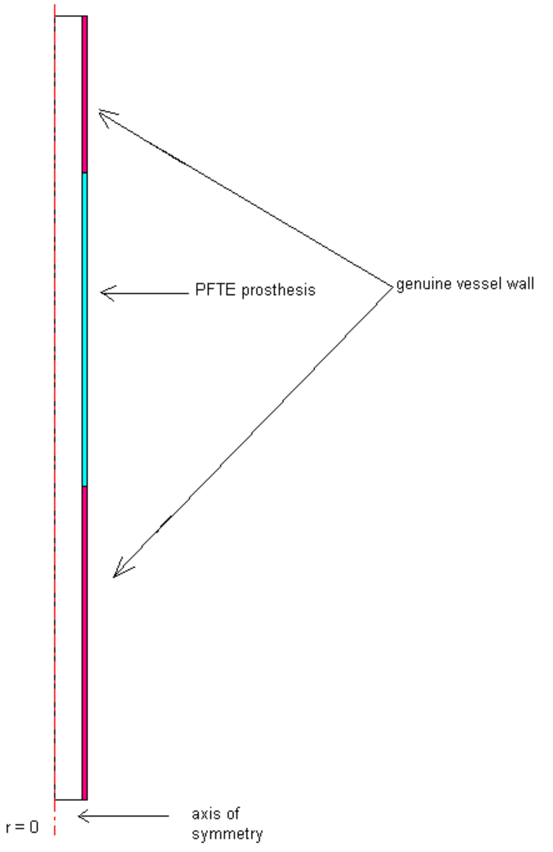

Figure 3. Axial-symmetric geometry for an artery with PFTE prosthesis

To calculate the wall shear stress we use $WSS = \mu(\dot{\gamma})\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}\right)$, where u and v are the velocity components of the blood in the r and z directions respectively [8].

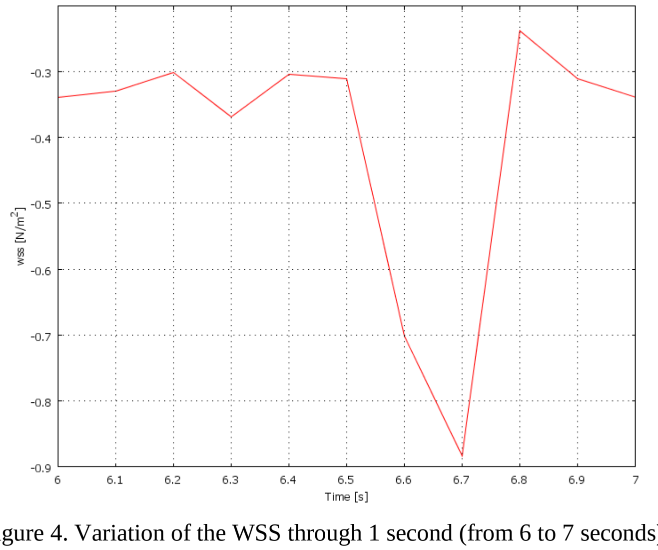

Figure 4. Variation of the WSS through 1 second (from 6 to 7 seconds)

We can remark that these values of the WSS got in our numerical approach are in accord with those presented in the work of Papaioannou & Stefanos [7]. This validates the accuracy of the use of the Cross type non-Newtonian model for the blood flow together with the viscoelastic behavior of the vessel walls and prosthesis.

In fact we intend to compare the WSS evaluated on the boundary of the vessel with the internal forces assessed on the same boundary. These last forces are connected with the stress vector $\vec{T}$ from the Maxwell model which governs now the behavior of the involved wall. The expression for the stress vector is $\vec{T} = \mathbf{T}\vec{n}$, where $\mathbf{T}$ is the stress tensor in the Maxwell model while $\vec{n}$ is the normal unit vector at the considered point.

The components of the stress tensor **T**, using the general Maxwell model for viscoelasticity are $T_{ij} = s_{ij} - pI_{ij}$, where s_{ij} are the components of the stress deviator $\mathbf{s} = 2G_0(\eta_0\mathbf{e} + (\sum_{m=1}^{4}\eta_m q_m)\mathbf{I})$, $\mathbf{e}$ being the rate of strain deviator ($\mathbf{e} = \boldsymbol{\varepsilon} - 1/3Tr(\boldsymbol{\varepsilon})\mathbf{I}$), $\boldsymbol{\varepsilon}$ being the rate of strain tensor, G_0 is the shear modulus, $G_0\eta_0$ is the long term shear modulus, $A = \sum_{m=1}^{4}\eta_m q_m$, η_m ($m = 0,1,2,3,4$) being coefficients of the relative rigidity of the wall and q_m are parameters attached to the extension of the wall, ($\sum_{m=0}^{4} q_m = 1$). According to our references the components of the (generalized) Maxwell stress vector are [2]

$$T_r = -\left\{2G_0\left[\eta_0\left(\frac{\partial u}{\partial r} - \frac{1}{3}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z}\right)\right) + A\right] - p\right\}\frac{\frac{dz}{dr}}{\sqrt{1 + \left(\frac{dz}{dr}\right)^2}} +$$

$$+ 2G_0\left[\eta_0\,\frac{1}{2}\left(\frac{\partial v}{\partial r} + \frac{\partial u}{\partial z}\right)\right]\frac{1}{\sqrt{1 + \left(\frac{dz}{dr}\right)^2}}$$

and

$$T_z = -2G_0\left[\eta_0\,\frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}\right)\right]\frac{\frac{dz}{dr}}{\sqrt{1 + \left(\frac{dz}{dr}\right)^2}} +$$

$$+ \left\{2G_0\left[\eta_0\left(\frac{\partial v}{\partial z} - \frac{1}{3}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z}\right)\right) + A\right] - p\right\}\frac{1}{\sqrt{1 + \left(\frac{dz}{dr}\right)^2}}.$$

This vector $\vec{T}$ will be projected on the unit tangent vector $\vec{t}$ of the considered boundary. When the WSS evaluated on the boundary of our pseudoaneurysm overpasses the internal cohesion forces (calculated by $\vec{T}\cdot\vec{t}$) then a possible “jerk” could take place and serious health complication (a stroke, for instance) becomes possible. Accepting that the equation of the vessel wall (with pseudoaneurysm) – in the Cartesian coordinate system of the radial axis *r* and the axis of symmetry *z*, from the plane $\theta = const$, is $z = z(r)$ and the expression for the stress vector is $\vec{T} = \mathbf{T}\vec{n}$ (**T** being the stress tensor in the Maxwell model while $\vec{n}$ is the outward unit vector at the considered point), following the steps of our previous paper [2], we get that the "rupture" condition becomes

$$\left| -\left\{2G_0\left[\eta_0\left(\frac{\partial u}{\partial r} - \frac{1}{3}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z}\right)\right) + A\right] - p\right\}\frac{\frac{dz}{dr}}{1+\left(\frac{dz}{dr}\right)^2} + 2G_0\left[\eta_0\frac{1}{2}\left(\frac{\partial v}{\partial r} + \frac{\partial u}{\partial z}\right)\right]\frac{1}{1+\left(\frac{dz}{dr}\right)^2} - \right.$$

$$\left. - 2G_0\left[\eta_0\frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}\right)\right]\frac{\left(\frac{dz}{dr}\right)^2}{1+\left(\frac{dz}{dr}\right)^2} + \left\{2G_0\left[\eta_0\left(\frac{\partial v}{\partial z} - \frac{1}{3}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z}\right)\right) + A\right] - p\right\}\frac{\frac{dz}{dr}}{1+\left(\frac{dz}{dr}\right)^2}\right| <$$

$$< \left|\left(\mu_s + \frac{\mu_0^*}{1 + k\left[2\left(\frac{\partial u}{\partial r}\right)^2 + 2\left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial r} + \frac{\partial u}{\partial z}\right)^2 + 2\left(\frac{u}{r}\right)^2\right]^{\frac{1-n}{2}}}\right)\left(\frac{\partial v}{\partial r} + \frac{\partial u}{\partial z}\right)\right|.$$

Here μ_s and μ_0^* are viscosity coefficients of the blood, *k* a time constant, *n* the index for a shear thinning behavior while *p* is the pressure [2].

3. CASE OF A PSEUDOANEURYSM ON BLOOD VESSEL–PROSTHESIS JUNCTION

The practice shows that there is a trend of a post-surgical reconstruction of a new aneurysm – called also a "pseudoaneurysm". This could be either a consequence of an endoleak in the proximity of the prosthesis (device related issues) or of the existence of a hole/tear of the blood vessel or of the prosthesis itself. The graft embolization and the formation of the blood clots that block the flow of the blood can also determine such a pseudoaneurysm. Of course all these assumptions must be associated with some high blood pressure.

To determine when and how a junction genuine vessel-prosthesis may lead to a pseudoaneurysm, we must investigate first the conditions for starting a plastical deformation.

A yield strength or yield point of a material is defined in material science as the stress at which a material begins to deform plastically. Prior to the yield point the material will deform elastically and will return to its original shape when the applied stress is removed. Once the yield point is passed, some fraction of the deformation will be permanent and non-reversible [11]. Knowledge of the yield point is vital when designing a component since it generally represents an upper limit to the load that can be applied. It is also important for the control of many materials for avoiding catastrophic or ultimate failure.

A yield criterion is a hypothesis concerning the limit of elasticity under any combination of stresses. For an isotropic material as PFTE is, denoting by σ_1, σ_2 and σ_3 the three principal stress directions we have:

Maximum Principal Stress Theory (W.J.M Rankine) states that yield occurs when the largest principal stress exceeds the uniaxial tensile yield strength, i.e., $\sigma_1 \geq \sigma_y$, σ_y being the critical yield stress.

Maximum Principal Strain Theory (St.Venant) which in terms of the principal stresses is determined by the condition $\sigma_1 - \nu(\sigma_2 + \sigma_3) \geq \sigma_y$, i.e., yield occurs when the maximum principal strain reaches the strain corresponding to the yield point during a simple tensile test.

Maximum Shear Stress Theory (Tresca yield criterion) assumes that yield occurs when the shear stress τ exceeds the shear yield strength τ_y, which usually satisfies the condition $\tau = \dfrac{\sigma_1 - \sigma_3}{2} \leq \tau_y$.

The stress at which yield occurs is dependent on both the rate of deformation (strain rate) and the temperature at which the deformation occurs. Early work by Alder and Philips in 1954 found that the relationship between yield stress and strain rate (at constant temperature) was described by a power law relationship of the form $\sigma_y = C(\dot{\varepsilon})^m$, where C is a constant and m is the strain rate sensitivity (m can be found from a log-log plot of yield stress at a fixed plastic strain versus the strain rate, namely $m = \dfrac{\partial \ln \sigma(\varepsilon)}{\partial \ln \dot{\varepsilon}}$). In continuum mechanics a quantity called von Mises stress is commonly used. If J_2 is the second deviatoric stress invariant (of the stress deviator tensor $\boldsymbol{\sigma}^{dev} = \boldsymbol{\sigma} - 1/3(tr\boldsymbol{\sigma})\mathbf{I} = (S_{ij})$ of the stress tensor $\boldsymbol{\sigma} = (\sigma_{ij})$ – denoted also by $\mathbf{T} = (T_{ij})$), we can write

$$J_2 = \frac{1}{2} S_{ij} S_{ji} = \frac{1}{\sigma}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]$$

and the von Mises stress is

$$\sigma_v = \sqrt{3J_2} = \sqrt{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]}.$$

The von Mises yield criterion suggests that the yielding of materials begins when the second deviatoric stress invariant J_2 reaches a critical value. In other words a material is said to start yielding when its von Mises stress reaches a critical value known as the yield strength, σ_y.

Mathematically the von Mises yield criterion is expressed as $J_2 = k^2$, where k is the yield stress of the material in pure shear (when $\sigma_{12} = \sigma_{21} \neq 0$ while the other components $\sigma_{ij} = 0$). It can be shown that $k = \dfrac{\sigma_y}{\sqrt{3}}$. Combining the above equations, the von Mises criterion can be expressed as $\sigma_v^2 = 3J_2 = 3k^2$, or substituting J_2 with terms of the Cauchy stress components

$$\sigma_v^2 = \frac{1}{2}\left[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{23}^2 + \sigma_{31}^2 + \sigma_{12}^2)\right].$$

In the case of a pure shear stress, when $\sigma_{12} = k = \dfrac{\sigma_y}{\sqrt{3}}$, the von Mises yield criterion, expressed in principal stresses is

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 6\sigma_{12}^2 \text{ [11]}.$$

In what follows we will calculate the wall shear stress (WSS) in the significant points of this new aneurysm (pseudoaneurysm). As the pseudoaneurysm could affect also the prosthesis zone we must take in consideration the special behavior of teflon material which has been now deformed irreversible, i.e., the viscoplastic component of the stress-strain dependence. This becomes dominant in opposition with the case when the pseudoaneurysm is absent and when the viscoelastic feature represents the most important part of the prosthesis behavior.

These values of WSS will be then compared with those which are considered “normal” (determined in the first part of the paper, i.e., without pseudoaneurysm) in view of an attempt of prediction of a possible “jerk” from the wall of the aneurysm due to a high value of the WSS. Such “critical” values for such a “jerk” (rupture) have been already got, by continuum mechanics considerations, in our previous work devoted to AAA (Abdominal Aortic Aneurysm) [2].

We will use the configuration presented in figure 5. The total length of the arterial segment is 1*cm*. The internal diameter of the undeformed blood vessel is 7*mm*, meanwhile the diameter of the deformed vessel at the middle of the pseudoaneurysm is 14*mm* and the thickness of the vessel wall is 0.8*mm* as in the previous case. The mesh generated for both the fluid and solid domain in the case of the artery with pseudoaneurysm consists of 1060 triangular elements and 19346 degrees of freedom.

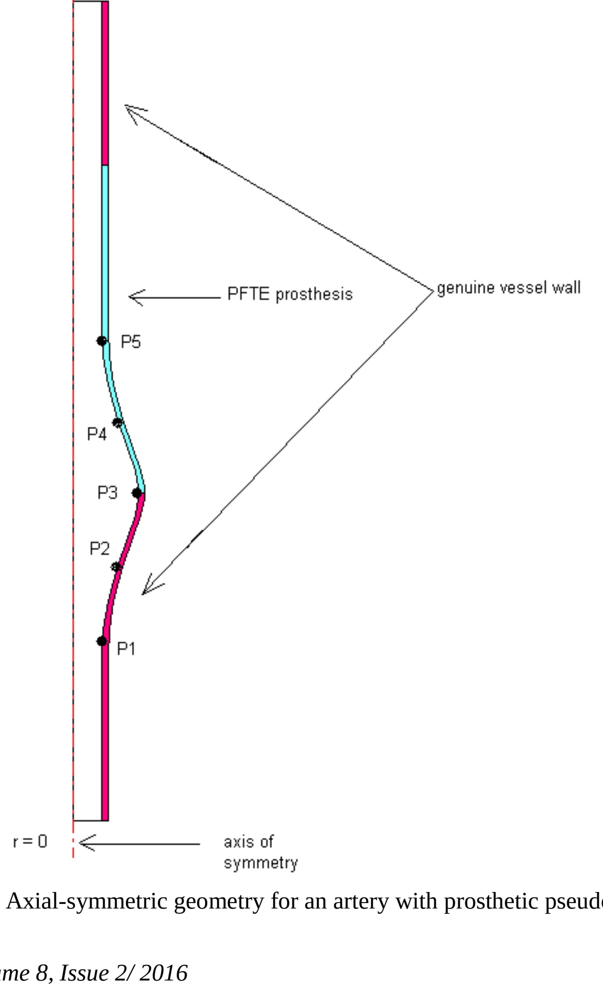

Figure 5. Axial-symmetric geometry for an artery with prosthetic pseudoaneurysm

Again we intend to compare the WSS evaluated on the boundary of the pseudoaneurysm with the shear stress assessed on the same boundary. For practical reasons we will use for WSS an appropriate interpolation (for instance a cubic spline as we have already considered for the AAA case [2]) together with the subprogram obtained with COMSOL 4.3 package.

For the affected prosthesis zone we must take into account also the specific behavior of the prosthesis material which now has been plastic deformed and the corresponding stress T should be determined (now T is not of Maxwell type anymore).

For getting an evaluation of the viscoplastic behavior of our PFTE prosthesis we will use a curve which expresses stress versus strain in tension and compression (ASTM D695). This curve (figure 5 of the reference paper [3]) shows that the stress (expressed in MPa) takes the (approximate) values -25, -17, 0, 10, 13 corresponding to the strain values (respectively) -0.2, -0.1, 0, 0.1, 0.3 in both situations of compression and tension, at 23^0C.

Using this data $T^1 \equiv T(-0.2) = -25$, $T^2 \equiv T(-0.1) = -17$, $T^3 \equiv T(0) = 0$, $T^4 \equiv T(0.1) = 10$, $T^5 \equiv T(0.2) = 13$, accepting that $T(\varepsilon) \in C^2(\varepsilon)$ we will construct the corresponding cubic spline interpolation $S(\varepsilon) \in C^2(\varepsilon)$. This type of approximation made by cubic function "preserves" the shape of the exact $T(\varepsilon)$ and it will be the function we may deal with for assessing the behavior of the PFTE prosthesis.

Precisely, denoting by $M_i = S''(\varepsilon_i)$, $i = 1,2,3,4,5$, the spline interpolation function (if $h_i = \varepsilon_i - \varepsilon_{i-1}$) joined to the "$i$" subinterval, is given by [5]

$$S_i(\varepsilon) = \frac{M_i(\varepsilon - \varepsilon_{i-1})^3 + M_{i-1}(\varepsilon_i - \varepsilon)^3}{6h_i} + \left(T^{i-1} - \frac{M_{i-1}h_i^2}{6}\right)\frac{\varepsilon_i - \varepsilon}{h_i} + \left(T^i - \frac{M_i h_i^2}{6}\right)\frac{\varepsilon - \varepsilon_{i-1}}{h_i}$$

$$i = 2,3,4,5.$$

Concerning the constants M_i they can be obtained by solving the following algebraic linear system [5].

$$a_0M_0 + c_0M_1 = d_0$$
$$b_iM_{i-1} + a_iM_i + c_iM_{i+1} = d_i,\ i = 1,2,3,4,5$$
$$b_6M_5 + a_6M_6 = d_6,$$

where $M_0 = M_6 = 0$, $b_i = \dfrac{h_i}{h_i + h_{i+1}}$, $c_i = 1 - b_i$, $d_i = \dfrac{6}{h_i + h_{i+1}}\left(\dfrac{T^{i+1} - T^i}{h_{i+1}} - \dfrac{T^i - T^{i-1}}{h_i}\right)$,

$i = 1,2,3,4,5$;

$b_6 = 1$, $c_0 = 1$, $d_0 = \dfrac{6}{h_1}\left(\dfrac{T^1 - T^0}{h_1} - T'^0\right)$, $d_6 = \dfrac{6}{h_6}\left(T'^6 - \dfrac{T^6 - T^5}{h_6}\right)$, $a_i = 2$, $i = 1,2,3,4,5$.

Here we accept that $T^0 = -35$, $T^6 = 23$ and $h_1 = h_6 = 10$. The additional constants $M_0 = M_6$ which would vanish (a hypothesis acceptable due to the trend of the dependence curve which changes its concavity at "far field") have been introduced to assure the uniqueness of this spline interpolation $S(\varepsilon)$.

Concerning the error of the approximation it is of the same order as of certain powers of $h = \max_i(\varepsilon_i - \varepsilon_{i-1})$, the degree of accuracy increasing together with the regularity of $T(\varepsilon)$ [6]. This approximation gives the dependence (law of behavior) between the magnitudes of stress versus the strain. Once $T(\varepsilon) \approx S(\varepsilon)$ is got for the prosthesis material we can immediately compare its values with those of WSS on the boundary.

When WSS overpasses $T(\varepsilon)\,(S(\varepsilon))$ there is a possibility of a "jerk" (rupture) of the prosthesis.

If we need a similar approximation for stress versus strain in shear we have to follow the same steps by using the curve of dependence illustrated in the figure 6 of the same reference [3]. In fact all the above considerations remain valid by replacing $T_{sh}^1(0)=0$, $T_{sh}^2(4)=3.7$, $T_{sh}^3(6)=4.2$, $T_{sh}^4(8)=5$, $T_{sh}^5(10)=5.2$, $T_{sh}^6(0)=5.5$, with $M_0=M_6=0$ and the spline approximation $S_{sh}(\varepsilon)$ will be obtained by solving the equivalent similar algebraic system.

For sake of simplicity, it is possible to build also a linear interpolation $S_L(\varepsilon)$ for the behavior of the PFTE prosthesis.

More precisely if we consider data $T^1(-0.2)=-25$, $T^2(-0.1)=-17$, $T^3(0)=0$, $T^4(0.1)=10$, $T^5(0.3)=13$ completed with $T^0(-10.2)=-35$, $T^6(10.2)=23$ a linear interpolation leads to

$$S_L(\in)=\begin{cases} T^j\,\dfrac{\varepsilon-\varepsilon_{j-1}}{\varepsilon_j-\varepsilon_{j-1}}+T^{j-1}\,\dfrac{\varepsilon_j-\varepsilon}{\varepsilon_j-\varepsilon_{j-1}} & \text{for}\,\varepsilon\in(\varepsilon_{j-1},\varepsilon_j)\\[2ex] T^{j+1}\,\dfrac{\varepsilon-\varepsilon_j}{\varepsilon_{j+1}-\varepsilon_j}+T^j\,\dfrac{\varepsilon_{j+1}-\varepsilon}{\varepsilon_{j+1}-\varepsilon_j} & \text{for}\,\varepsilon\in(\varepsilon_j,\varepsilon_{j+1})\end{cases}$$

(ε_j, T^j) being a matching point, or in other words

$$S_L(\in)=\sum_{j=0}^{6}T^jS_L^j(\varepsilon),$$

with

$$S_L^j(\varepsilon)=\begin{cases}\dfrac{\varepsilon-\varepsilon_{j-1}}{\varepsilon_j-\varepsilon_{j-1}}, & \text{for}\,\varepsilon\in(\varepsilon_{j-1},\varepsilon_j)\\[2ex] \dfrac{\varepsilon-\varepsilon_{j+1}}{\varepsilon_j-\varepsilon_{j+1}}, & \text{for}\,\varepsilon\in(\varepsilon_j,\varepsilon_{j+1})\\[2ex] 0, & \text{otherwise.}\end{cases}$$

It is easy to check that $S_L(\varepsilon_j)=T^j$.

We can now make precise the above interpolation by fixing our data as follows: $T^0=-35$, $T^1=-25$, $T^2=-17$, $T^3=0$, $T^4=10$, $T^5=13$, $T^6=23$ while the corresponding ε are -10.2; -0.2; -0.1; 0; 0.1; 0.3; 10.2.

We remark that all the previous interpolation functions $S(\varepsilon)$, $S_{sh}(\varepsilon)$ and $S_L(\varepsilon)$ could be inverted, i.e., we can also have $\varepsilon(S)$, $\varepsilon(S_{sh})$ and $\varepsilon(S_L)$ respectively. Of course this could be done in the case of $\varepsilon(S)$ for instance, by a similar spline interpolation approximation associated with the following data, i.e.

$\varepsilon^0(-35)=-0.4$, $\varepsilon^1(-25)=-0.2$, $\varepsilon^2(-17)=-0.1$, $\varepsilon^3(0)=0$, $\varepsilon^4(10)=0.1$, $\varepsilon^5(13)=0.3$, $\varepsilon^6(23)=0.5$.

Taking now into account the structure (geometry) of the considered pseudoaneurysm in the vicinity of the junction between the deformed genuine vessel and the deformed prosthesis, we will be interested only in the points 3, 4, 5, 6 associated with nonnegative deformations. As the maximum value for ε - which is also the maximum thickness of the pseudoaneurysm curve, is $\varepsilon^6 = 1$ we may state that the corresponding stress at this point should be (by using the linear interpolation)

$$S_L(1) = T^0 S_L^0(1) + T^1 S_L^1(1) + T^2 S_L^2(1) + T^3 S_L^3(1) + T^4 S_L^4(1) + T^5 S_L^5(1) + T^6 S_L^6(1) \equiv$$

$$\equiv T^6 S_L^6(1) = 23 \cdot \frac{1-10.2}{0.2-10.2} = 20.7.$$

This value of the stress will be compared with the value of the WSS evaluated at the same junction point for assessing the possibility of a “jerk” (“rupture”). If $WSS\big|_{junctionpoint} > T$, it is possible that a rupture accident shows up.

4. CONCLUSIONS

To compare the values of the shear stress with the values of the wall shear stress in several points of the vessel wall we use the values obtained by numerical calculations with COMSOL 4.3.

The evolution of the WSS exerted by the blood versus the shear stress acting by the wall at the same points of the vessel wall is presented in figure 6.

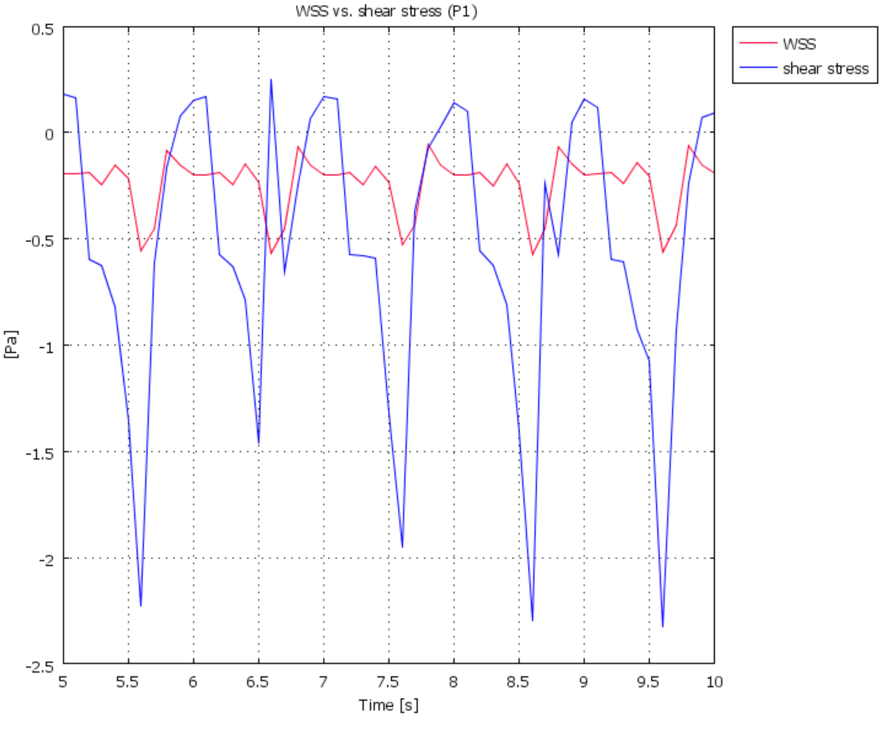

Figure 6. Variation of WSS versus shear stress at point P1 (see figure 5)

Figure 6 shows that the values of the shear stress and the WSS are of the same order of magnitude at point P1. A possible “rupture” may appear if the absolute value of the WSS overpasses the absolute value of the shear stress. At the moment t = 6.6s, for instance, the absolute value of the WSS is 0.545 Pa, while the absolute value of the shear stress is 0.230 Pa. So a possible “rupture” could take place in this point, at this moment. However, most of the time the absolute value of the shear stress remains over of the absolute value of the WSS.

This multidisciplinary research has been done by a group formed by physicians, mathematicians and specialists in continuum mechanics.

Vitalie Vacaras was responsible with the medical investigations and interpretations. Balazs Albert has elaborated the continuum mechanics approach and also the mathematical calculations.

Balazs Albert, on the one hand, and Vitalie Vacaras, on the other hand, contributed equally to the present paper.

The whole research has been performed under the leadership of Professor Titus Petrila, the coordinator of the research group on blood flow modeling.

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