Numerical Simulations for Fuel Aircraft Management System

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Abstract: *The Fuel Management Systems for civil aircrafts have a direct impact on the airlines profitability, competitiveness and sustainability, because the fuel represents the highest cost for airlines, on average 32% of overall operating costs; extra costs can frequently be taken into account, due to the increase in time of approach and straight-in landing versus circle to land maneuver. The core philosophy of the Fuel Management Systems, targeted to aircraft safety, must be focused on the most important aspects regarding the efficient operation of an airline and its proper management expressed in terms of efficacy, efficiency and effectiveness; an actual concern of great interest is the adequate fuel management. In this paper, a thorough study was carried on based on the state of art detailed investigation, with numerical simulations for significant case studies which are relevant for the implementation of robust fuel management processes in order to increase the internal efficiency and reduce the overall costs. In this paper, fuel management solutions and specific mode of their implementation are presented. The benefits of Fuel Aircraft Management Systems are multiple related and refer to: minimizing the costs, reducing the workload, improving internal and external data sharing, reducing unnecessary communications, controlling fuel costs, making better decisions, improving cash flow and creating a more cohesive approach to the Fuel Aircraft Management System. The applications of Fuel Aircraft Management System are beneficial to the aviation industry, and particularly, this study brings a significant contribution to the topic, i.e. advances in Fuel Aircraft Management System.*

Key Words: *Aviation industry, automatic control, engine control, Fuel Aircraft Management System*

1. INTRODUCTION

There is a continuous demand for effectiveness of performances versus costs and resources, for all the players involved in the aviation industry, from design to manufacturing, operation, maintenance, repair and overhaul. In the case of commercial airliners, as well as other civil aircraft operators, since the fuel costs represent the highest costs for airlines, on average 32% of overall operating costs (with frequent occurrence of extra costs due to the increase of time of approach and straight-in landing versus circle to land maneuver), it is of utmost interest for airlines in terms of profitability, competitiveness and sustainability; within this context, the

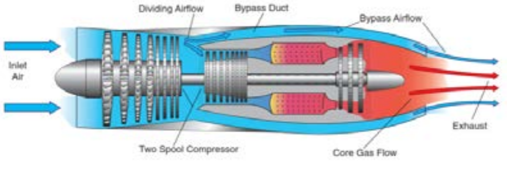
Fuel Management Systems represent an important key instrument for the aircraft safety, efficient airline operation and its proper management expressed in terms of efficacy, efficiency, effectiveness. Based on this motivation, the adequate fuel management, in conjunction with adequate aircraft engine control, Engine Condition Monitoring/ Engine Health Monitoring / Advanced Health Monitoring represent an actual concern of great interest. Aircraft Health Management Technology for jet engines includes a wide range of applications and standardized procedures, customized for each significant part or assembly which has been proven crucial for the safe operation of the aircraft, for all the jet engine running regimes and flight regimes supposed by the flight envelope and the aircraft flight missions. Referring to the jet engine the concept of Aircraft Health Management AHM turns to focus on Engine Health Management EHM, which further is based on Engine Condition Monitoring ECM. Aircraft Health Management AHM with application to jet engines can be expressed by Engine Health Management EHM and Engine Condition Monitoring ECM. The main goal and benefit of the Engine Condition Monitoring ECM is the cost reduction, from the most important key points: 1/ operational, 2/ financial, 3/ Maintenance, Repair and Overhaul MRO. By the means of the AHM and consequently EHM & ECM, a series of long-term significant benefits are obtained by each of the main players in the aviation industry, namely: the designers of aircrafts and jet engines, the manufacturers, as well as the factors involved in operation of civil, commercial and military airplanes. The key benefits with regards to the jet engines are related to reducing the engine life cycle costs, improving the fuel efficiency, increasing the engines durability and life cycle. Aircraft Health Management Technology includes multiple goals of aircraft propulsion control, diagnostics problems, prognostics realized, and their proper integration in control systems. Modern control for Aircraft Health Management Technology is based on improved control techniques and therefore provides improved aircraft propulsion system performances. The technology actually involved in AHM & EHM is high-end and in order to enable an improved level of performance that far exceeds the current one, propulsion systems must be compliant with the terms of reduction harmful emissions, maximization of fuel efficiency and minimization of noise, while improving system affordability and safety. In this paper, a thorough study was carried on based on the state of art detailed investigation, with numerical simulations for significant case studies which are relevant for the implementation of robust fuel management processes in order to increase the internal efficiency and reduce the overall costs. In this paper, fuel management solutions and specific mode of their implementation are presented. The problem of jet engine control can be solved if prior is set a mathematical model to describe with adequate accuracy the behaviour of the jet engine for all the operating regimes and all the flight regimes required by the flight mission profile; the numerical simulations carried on based on the jet engine mathematical model can provide accurate results on the performances of the studied jet engine (i.e. the thrust, specific thrust and specific fuel consumption) for the design and all the off-design regimes, for the steady state analysis. The dynamic analysis of the jet engine highlights the jet engine behaviour to transient regimes and/ or non-stationary. Adequate mathematical modelling can provide proper results for the dynamic analysis and jet engine control. Further, in conjunction with the Engine Condition Monitoring ECM and Engine Health Monitoring, the jet engine control problem (which includes algorithms, procedures and techniques) can be a solid foundation for Advanced Health Monitoring AHM. In this context, of particular interest are the researches regarding the mathematical modelling of the turbojet, turbofan and mixed flows turbofan engines, oriented and customized for the performance prediction at the design and off-design regimes, as well as the steady state analysis and the dynamic analysis, the jet engines and aircraft control, optimizations, in order to create and develop in-house software.

For the steady-state analysis of a jet engine, which refers to the prediction and simulation of the jet engine operation, that is performance calculation at design and off-design regimes, the input data (as engine and fluid parameters) must be known. Some parameters are given in catalogues or technical specifications of the manufacturers (e.g. Pratt & Whitney, Jane's All the World's Aircraft), while others are not revealed to public access. For most of the cases, some engine input parameters are given here, such as the airflow, pressure ratio, by-pass ratio and the values of the performances (as the thrust and fuel specific consumption) at Sea Level Static SLS take off T/O and cruise conditions. For other specific situations of missing information on engine input data, some engine parameters must be identified or determined, in order to complete the mathematical model of the jet engine.

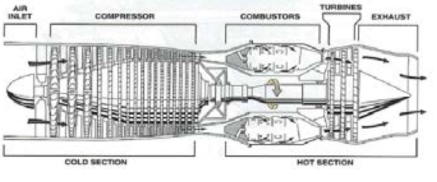
2. JET ENGINES TYPES AND STUDY CASE

The types of jet engines are: the turbojet, the turbofan engine, the turboprop and the mixed flows turbofan engine. This study focuses on the categories of jet engines used as propulsion systems for powering commercial airliners, civil airplanes and military aircraft (combat, fighter or multirole aircraft) that are as presented in Fig. 1 – Fig. 5: the turbojet, the mixed flows turbofan (equiv. low by-pass turbofan), the twin spool or triple spool turbofan (equiv. high by-pass turbofan). The commercial airliners can be powered by pairs of 2, 4 or 6 high by pass turbofan engines (which are also referred as un-mixed flows turbofan engines); e.g. the A380 aircraft powered by 4 Rolls Royce Trent 900 turbofan engines. In current use are also other types of turbofan engines, such as CF6-50/80 C2, V2500 IAE, PW 2000, CFM 56-3/58/7, CF 34-8/10, GE 90-110B/115B that can be mentioned [1-8].

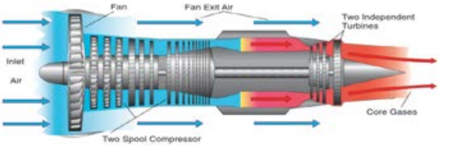
The turbofan engines are designed such that to provide lower fuel consumption, while for the military applications, the turbojet and the mixed flows turbofan engines are designed such that to provide high levels of thrust [12].



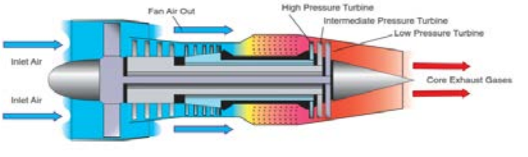
(a) Mixed Flows Turbofan



(b) Turbojet



(c) Twin spool Turbofan



(d) Triple spool Turbofan

Fig. 1 – Schematic diagrams of turbojet and turbofan engines [3]

The reheated mixed flows turbofan engines F100-PW220 and F100-PW229 manufactured by Pratt & Whitney can be mentioned as examples of turbofan engines, in current use. The J-85 GE-17A Turbojet Engine and the J-79 GE Reheated Turbojet Engine, can be considered as suitable examples [12-15].

Due to the reduced cross section, the turbojet engines and the mixed flows turbofan engines are preferred as propulsion systems for the military aircraft (combat/ fighter/ multirole). The thrust of both turbojet and mixed flows turbofan engines can be significantly increased, for short periods of time, by the means of the reheat engine cycle; such engines are equipped with an afterburner/ afterburn unit. The low by-pass turbofans, as the military F100/ F110/ EJ200 show the capability of better adjustment to fuel consumption levels (and with lower values of specific fuel consumption) with respect to the turbojet family [21,26].

The CASE STUDY is represented by the reheated mixed flows turbofan engine F100-PW220 manufactured by Pratt & Whitney, illustrated in Fig. 2 [8].

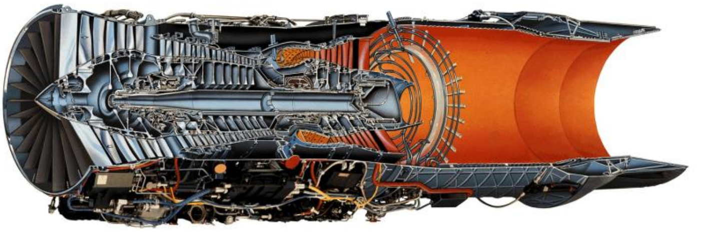


Fig. 2 – The CASE STUDY: the F100 PW 220 Reheated Mixed Flows Turbofan Engine, [8] powering the F16 Fighting Falcon

3. PROBLEM STATEMENT: TURBOJET AND TURBOFAN ENGINES CONTROL FOR CIVIL AND MILITARY APPLICATIONS

The turbofan engines for civil applications are dedicates to power commercial airliners, e.g. A 380 powered by four Rolls Royce Trent 900 turbofan engines [7]; the schematic diagram of turbofan engines is illustrated in Fig. 1c (twin spool construction) and Fig. 1d (triple spool construction). Since the exhaust nozzle area is fixed, then the control is performed by the means of a single variable, that is the fuel flow rate; consequently, the control vector consists of one element [9-11]. The turbofan engines for military applications are destined to power combat aircrafts (fighter and/ or multirole aircraft); the construction of such engines, namely reheated mixed flows turbofan, is radically different from the previous types; the schematic diagram of a mixed flows turbofan engine MFTE is depicted in Fig. 1a. Since the profile mission of a combat aircraft often requires a sudden increase of available thrust, the MFTE is equipped with an afterburner, which provides a significant augmentation of the engine's thrust, by the means of a reheat cycle [12-15], [21, 26]. In the case of MFTE and the reheated MFTE respectively, for a better adaptability of the engine regimes to the aircraft flight regimes, and a safer engine operation (due to the fact that the work regimes line is shifted farther from the surge line), the geometry of the exhaust nozzle is variable. Fig. 2 shows a cut view of the F100 PW 220 Reheated Mixed Flows Turbofan Engine, [8], which is powering the F16 Fighting Falcon, aircraft, in current use in the Romanian Air Force. In the case of MFTE, the control is performed by the means of two variables: the fuel flow rate and the exit nozzle area; consequently, the control vector consists of two elements. The jet engine control [9-11], [18] can be described by either non-linear, linearized or linear models; in this study, a linearized mathematical model was used, expressed by the state (1) and output (2) equations, where the matrices **A**, **B**, **C** and **D** (of appropriate order) are computed for each representative operating point (of the jet engine), in accordance with the flight path and flight map.

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \tag{1}$$

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u} \tag{2}$$

In the following, the mathematical models [15-17], [19], [22, 23] used for turbojet and turbofan engines control will be presented from the standpoint of increasing the technological complexity features of appropriate jet engines. The basic jet engine architecture and type is the single-spool turbojet engine, Fig. 1.b, and the state vector (3), output vector (4) and control vector (5) are defined in Table 1.

Table 1 – Parameters defining the state vector *x*, output vector *y* and control vector *u* in case of a single spool turbojet engine

Vector	Parameters	
state vector $\boldsymbol{x}$	x_1	compressor speed, N [rpm]
	x_2	combustor internal pressure, [bar]
	x_3	combustion chamber internal energy, [kJ]
output vector $\boldsymbol{y}$	y_1	Engine thrust, [kN]
	y_2	Turbine inlet temperature T3T, [K]
	y_3	compressor surge margin
control vector $\boldsymbol{u}$	u_1	Combustor fuel flow rate, [kg/s]

$$\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \tag{3}$$

$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \tag{4}$$

$$\boldsymbol{u} = (u_1) \tag{5}$$

The next one from the standpoint of increasing the technological complexity features, is the twin-spool turbojet engine, for which the state vector (6), output vector (7) and control vector (8) are defined in Table 2.

Table 2 – Parameters defining the state vector *x*, output vector *y* and control vector *u* in case of twin spool turbojet engine, twin-spool turbofan engine

Vector	Parameters	
state vector $\boldsymbol{x}$	x_1	HPC speed, N2 [rpm]
	x_2	LPC speed, N1 [rpm]
	x_3	combustor internal pressure, [bar]
	x_4	combustion chamber internal energy, [kJ]
output vector $\boldsymbol{y}$	y_1	Engine thrust, [kN]
	y_2	Turbine inlet temperature T3T, [K]
	y_3	HPC surge margin
	y_4	LPC surge margin
control vector $\boldsymbol{u}$	u_1	Combustor fuel flow rate, [kg/s]

$$\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \tag{6}$$

$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} \tag{7}$$

$$\boldsymbol{u} = (u_1) \tag{8}$$

In the case of the reheated twin-spool turbojet engine, the state vector (6), output vector (7) are similar to those explained in Table 2, while the control vector (9) is detailed in Table 3.

Table 3 – Parameters defining the state vector *x*, output vector *y* and control vector *u* in case of reheated twin spool turbojet engine, reheated mixed flows turbofan engine

Vector	Parameters	
state vector ***x***	x_1	HPC speed, N2 [rpm]
	x_2	LPC speed, N1 [rpm]
	x_3	combustor internal pressure, [bar]
	x_4	combustion chamber internal energy, [kJ]
output vector ***y***	y_1	Engine thrust, [kN]
	y_2	Turbine inlet temperature T3T, [K]
	y_3	HPC surge margin
	y_4	LPC surge margin
control vector ***u***	u_1	Combustor fuel flow rate, [kg/s]
	u_2	Exhaust nozzle area, [m^2]

$$\boldsymbol{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \tag{9}$$

In the case of the twin-spool turbofan engine, Fig. 1.c, the state vector (6), output vector (7) and control vector (8) have the component elements introduced in Table 2. In Table 4 are defined the state vector (10), output vector (11) and the control vector (8) in case of the triple-spool turbofan engine (Fig. 1.d).

Table 4 – Parameters defining the state vector *x*, output vector *y* and control vector *u* in case of a triple spool turbofan engine

Vector	Parameters	
state vector ***x***	x_1	HPC speed, N3 [rpm]
	x_2	LPC speed, N2 [rpm]
	x_3	LPC/ Fan speed, N1 [rpm]
	x_4	combustor internal pressure, [bar]
	x_5	combustion chamber internal energy, [kJ]
output vector ***y***	y_1	Engine thrust, [kN]
	y_2	Turbine inlet temperature T3T, [K]
	y_3	HPC surge margin
	y_4	LPC surge margin
	y_5	LPC/ Fan surge margin
control vector ***u***	u_1	Combustor fuel flow rate, [kg/s]

$$\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \tag{10}$$

$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{pmatrix} \tag{11}$$

In the case of the mixed flows turbofan engine, Fig. 1.a, the state vector (6), output vector (7) and control vector (8) are as defined in Table 2. The most complex model of the jet engine control is that used for the reheated mixed flows turbofan engine, Fig. 2; the state vector *x* (10), output vector *y* (7) and control vector *u* (9) are explained in Table 5.

Table 5 – Parameters defining the state vector *x*, output vector *y* and control vector *u* in case of the reheated mixed flows (twin spool) turbofan engine

Vector	Parameters	
state vector ***x***	x_1	HPC speed, N2 [rpm]
	x_2	LPC speed, N1 [rpm]
	x_3	combustor internal pressure, [bar]
	x_4	afterburner pressure, [bar]
	x_5	combustion chamber internal energy, [kJ]
output vector ***y***	y_1	Engine thrust, [kN]
	y_2	Turbine inlet temperature T3T, [K]
	y_3	HPC surge margin
	y_4	LPC surge margin
control vector ***u***	u_1	Combustor fuel flow rate, [kg/s]
	u_2	Exhaust nozzle area, [m^2]

The proposed model for turbojet and turbofan engine control, being defined by equations (1) and (2) is flexible, fit for general use and easy to be implemented in accordance with the type and architecture of the jet engine, i.e. single-spool, twin-spool or triple-spool, basic or reheated construction, turbojet engine, turbofan engine or mixed flows turbofan engine. The specificity of the jet engine is introduced within the mathematical model by the component elements of the state vector *x*, output vector *y* and control vector *u*, as detailed above.

4. APPLICABLE THEORY

The numerical solution provided by the jet engines control equations (1) and (2) is significantly different for each jet engine type, since it is based on the Operational Map of the studied engine, which is generated from the Universal Map of the Compressor System, Fig. 3 and Turbine Universal Map. For each assembly of the Compressor System, e.g. the fan, low-pressure compressor LPC and high-pressure compressor HPC, there is determined the Compressor Universal Map and eventually, is composed the Universal Map of the Compressor System; likewise, for the Turbine Universal Map. In Fig. 3 is shown the Axial Compressor Universal Map, and likewise is the Fan Universal Map, which are expressed by non-dimensional parameters, as follows: *x*-wise is the corrected flow $\dot{M}_a \frac{\sqrt{T_1^*}}{p_1^*}$ and *y*-wise is the pressure ratio π_C^*. In red contours is represented the axial compressor surge line; in the field of this map are shown the lines of compressor constant efficiency, which range from 0.80 to 0.89 and in bolded black contours are shown the work lines of constant engine speed regimes $\bar{n}$, ranging from ground idling 0.45, then 0.5 up to 0.8, low cruise 0.85, cruise 0.9, high cruise 0.95, design 1.0 and maximum emergency 1.05.

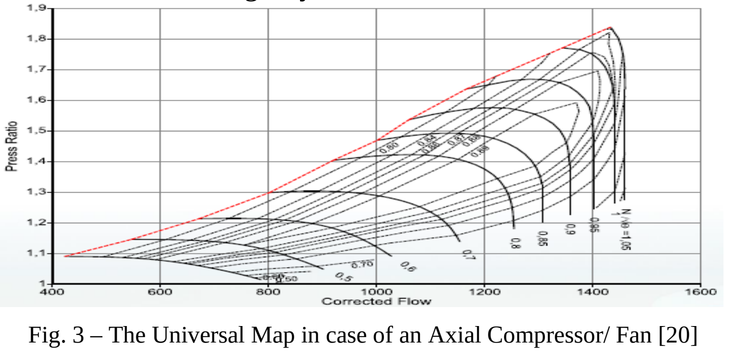


Fig. 3 – The Universal Map in case of an Axial Compressor/ Fan [20]

The importance of the Compressor Universal Map, as represented in Fig. 3, consists in enabling jet engine control, by maintaining the work line regimes within the safe domain, and out of surge. Either by proper design or by shifting the work-line regimes farther from the surge line, by specific means, as the bleed valve control, is enabled the safer operation of the compressor assembly and hence the safer operation of the jet engine. From the Universal Maps of the Low-Pressure and High-Pressure Compressor is determined the Compressor Surge Margins CSM, which are necessary for completing the output vector **y** (4), (7), (11).

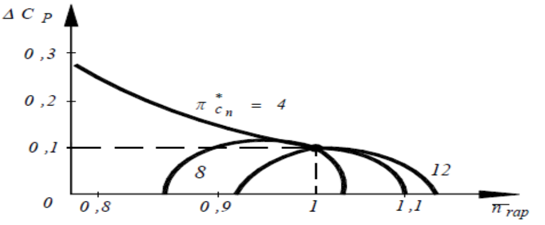


Fig. 4 – Variation of ΔC_P - the Compressor Surge Margin CSM versus the engine regime $\bar{n}$, for constant pressure ratios π_c^*, [24-25]

The Compressor Surge Margin CSM is defined as the coefficient of the jet engine's safe running ΔC_P (12), where the index SM refers to stall (as surge margin) and index OL refers to the operating line; the parameter p (13) is defined as follows:

$$\Delta C_{SM} = \frac{p_{SM}}{p_{OL}} - 1 \tag{12}$$

$$p = \frac{\pi_c^*}{\dot{M}_a \frac{\sqrt{T_1^*}}{p_1^*}} = \frac{\pi_c^*}{q(\lambda_1)} \tag{13}$$

$$\Delta C_P = 0 \tag{14}$$

In case that CSM vanishes, i.e. $\Delta C_P = 0$ (14), then the system is submitted to either surge S or choke C conditions. Both the compressor pressure ratio π_C^* and the speed n have strong influence on the CSM; in Fig. 4 is depicted the variation of the CSM versus the engine speed regime $\bar{n}$ for constant pressure ratios π_C^*. It easily comes out that the surge margin is influenced by the compressor pressure ratio and the operational speed regime $\bar{n}$.

$$\Delta C_P = f(\pi_{cn}^*, \bar{n}_{rap}) \tag{15}$$

$$\Delta C_P = f(\bar{n})_{\pi_{cn}^* = ct} \tag{16}$$

Fig. 4 points out that the compressor can overcross the surge margin (i.e. inside the instability domain) more rapidly with the increasing of the pressure ratio π_{cn}^*. That explains the necessity to determine the function (17) for each operating line.

$$\Delta C_P = f(\bar{n}_{rap}) \tag{17}$$

In order to establish the characteristics of a control system, the basic control characteristics and appropriate control functions for each system component must be determined or set. In the case of the jet engines, the control system can be: (a)- two position, (b)- open-loop scheduled, (c)- proportional, (d)- derivative, (e)- integrating, (f)- proportional and integral, (g)- proportional and derivative, (h)- proportional, derivative and integral. Modern control systems used on jet engines are multiple closed-loop scheduled.

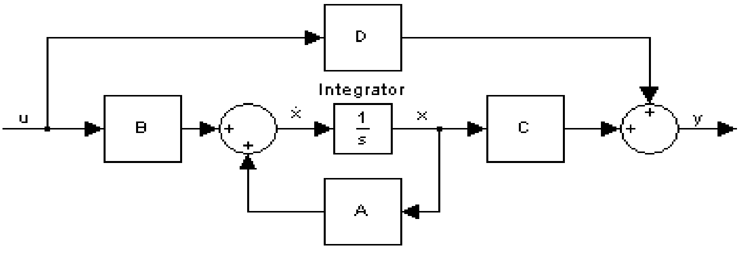


Fig. 5 – Block diagram for the uncontrolled engine [11]

An illustration of a block diagram representing the uncontrolled engine (i.e. open loop schedule) is given in Fig. 5; the uncontrolled system is not stable, on account of the system answer for open-loop schedule. Unlike the open-loop schedule, the closed-loop schedule, Fig. 6, contributes to stabilizing the system; Fig. 8 and 9 present the system answer for closed-loop schedule. The optimal control was obtained subsequent to the minimization of the performance index functions (18), which can be further expressed as (19) or (20), such that two different approaches were built; the linear quadratic approach was used in order to solve numerically the optimization problem:

$$\boldsymbol{J}=\int_0^{t_f}\left(\boldsymbol{Cx}(t)+\boldsymbol{Du}(t)\right)^2dt \tag{18}$$

$$\boldsymbol{J}=\int_0^{t_f}\left(\boldsymbol{Cx}(t)+\boldsymbol{Du}(t)\right)^T\left(\boldsymbol{Cx}(t)+\boldsymbol{Du}(t)\right)dt \tag{19}$$

$$\boldsymbol{J}=\int_0^{t_f}\left(\boldsymbol{x}^T(t)\boldsymbol{Qx}(t)+\boldsymbol{u}^T(t)\boldsymbol{L}^T\boldsymbol{x}(t)+\boldsymbol{x}^T(t)\boldsymbol{Lu}(t)+\boldsymbol{u}^T(t)\boldsymbol{Ru}(t)\right)dt \tag{20}$$

The states regulator problem (i.e. the states feedback regulator), where the performance index is expressed by relation (21), while the control is given by relation (23).

$$\boldsymbol{J}=\int_0^{t_f}\left(\boldsymbol{x}^T(t)\boldsymbol{Qx}(t)+\boldsymbol{u}^T(t)\boldsymbol{Ru}(t)\right)dt \tag{21}$$

$$\boldsymbol{Q}=\begin{bmatrix}300&0&0&0&0\\0&500&0&0&0\\0&0&100&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix};\ \boldsymbol{R}=10^8\cdot\begin{bmatrix}2&0\\0&50\end{bmatrix} \tag{22}$$

$$\boldsymbol{u}(t)=-\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{Sx}(t)=\boldsymbol{K}_x\boldsymbol{x}(t) \tag{23}$$

where **S** represents the solution of the Riccati linear quadratic equation associated to the optimization problem. The closed loop states feedback gain is represented by the parameter $\boldsymbol{K}_x$. The outputs regulator problem (i.e. the outputs feedback regulator) where the performance index is expressed by relation (24), while the appropriate control being expressed by relation (26).

$$\boldsymbol{J}=\int_0^{t_f}\left(\boldsymbol{y}^T(t)\boldsymbol{Qy}(t)+\boldsymbol{u}^T(t)\boldsymbol{Ru}(t)\right)dt \tag{24}$$

$$\boldsymbol{Q}=\begin{bmatrix}100000&0&0&0\\0&200000&0&0\\0&0&100&0\\0&0&0&100\end{bmatrix}\boldsymbol{R}=10^9\cdot\begin{bmatrix}9&0\\0&1\end{bmatrix} \tag{25}$$

$$\boldsymbol{u}(t) = -\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{S}\boldsymbol{x}(t) = \boldsymbol{K}_y\boldsymbol{x}(t) \tag{26}$$

where **S** represents the solution of the Riccati linear quadratic equation associated to the optimization problem. The closed loop states feedback gain is represented by the parameter $\boldsymbol{K}_y$.

5. NUMERICAL SIMULATIONS AND RESULTS

The study conducted within this paper aims to optimize the control of the reheated mixed flows turbofan engine F100, expressed in terms of jet engine performances (namely the thrust and the specific fuel consumption) versus the control vector; the influence of the flight conditions is expressed by the appropriate matrices A, B, C and D of the mathematical model.

Numerical simulations have been carried on, for various control laws, such as: 1)- step function, 2)- ramp, 3) a combination of limited ramp followed by a step function, and 4)- impulse.

The engine thermodynamic parameters are listed in Table 6.

Table 6 – Thermodynamic Parameters of the mixed flows turbofan engine

Parameters	Units	Values
Engine reference	F 100 Pratt & Whitney	
Overall Pressure Ratio	----	$\pi_c^* = 32$
Fan Pressure Ratio	----	$\pi_v^* = 4.38$
Optimum fan specific work	[kJ/kg]	$l_v^* = 172.400$
Bypass Ratio BPR	----	K=0.36
Overall Airflow Rate	[kg/s]	$\dot{M}_a = 105.371$
Turbine inlet temperature T3T	[K]	1623
Maximum Thrust F [kN] - military thrust	[kN]	77.5

The operating conditions of the jet engine (i.e. the F100 turbofan as the study case) are described by the flight envelope FE; therefore, the most significant operating points are, [11]:

1. Take off at sea level, standard day conditions;
2. Climb at 4000 ft (1.22 km), Mach number 0.3;
3. Subsonic cruise at 35000 ft (10.67 km), Mach number 0.6;
4. Approach at 1500 ft (457 m), Mach number 0.15;
5. Sea level, standard take off, military rating;
6. Supersonic flight at 35000 ft, Mach number 1.3.

The matrices **A**, **B**, **C** and **D** which appear in the state (1) and output (2) equations are calculated for each operating regime [11]; the values of the matrix coefficients **A**, **B**, **C** and **D** are given below.

$$\boldsymbol{A} = \begin{bmatrix} -5.08413 & -5.15966 & 2004.85 & 908.862 & 18.6713 \\ 0.282966 & -1.72003 & 508.627 & 332.326 & 15.3771 \\ 0.329032 & 0.861007 & -302.32 & -137.55 & 2.56234 \\ -0.0045155 & -0.0085174 & 4.65386 & -52.2517 & 0.0082936 \\ 3.07623 & -3.15127 & -2968.77 & -1841.25 & -91.5133 \end{bmatrix} \tag{27}$$

$$\boldsymbol{C} = \begin{bmatrix} 4.013e-04 & 0.0 & 0.0 & 7752.54 & -0.160553 \\ 0.0 & 0.0 & 0.0 & 0.0 & 4.93115 \\ -7.69e-05 & -4.582e-04 & 0.150167 & 0.0 & 0.0 \\ 0.0 & 6.73e-04 & -0.0502946 & 0.0 & 0.0 \end{bmatrix} \tag{28}$$

$$\boldsymbol{B} = \begin{bmatrix} 0.0 & 10648.8 \\ 0.0 & 84.4355 \\ 34.632799 & 2311.96 \\ 0.0 & 25.8363 \\ 34082.8 & 2311.96 \end{bmatrix} \tag{29}$$

$$\boldsymbol{D} = \begin{bmatrix} 0.0 & 845.73 \\ 0.0 & 0.0 \\ 0.0 & 0.768648 \\ 0.0 & -1.59633 \end{bmatrix} \tag{30}$$

The numerical accuracy in input data is very significant, since rounding or truncating errors in matrix **A** can affect the eigenvalues, and hence, may lead to inappropriate conclusions with regard to the stability assessment.

A stability analysis can be performed based on matrix **A**. The stability of the jet engine subjected to small perturbations of the running regimes is assessed by applying the Routh-Hurwitz criterion to the eigenvalues of the appropriate matrix **A** which contains information about the flight conditions.

The eigenvalues and eigenvectors of matrix **A** can be accurately computed by using the Danilewsky method of similar transformations; in accordance to this method, the matrix **A** is similarly transformed to the Frobenius form (31) and the coefficients of the characteristic polynomial are obtained from the first row of the Frobenius matrix (31).

$$\boldsymbol{Frobenius} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 & p_5 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -5.684 \cdot 10^{-14} & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{31}$$

The roots of the characteristic polynomial (32) are the eigenvalues (33) of the matrix **A**.

$$\boldsymbol{\lambda}^5 - p_1 \cdot \boldsymbol{\lambda}^4 - p_2 \cdot \boldsymbol{\lambda}^3 - p_3 \cdot \boldsymbol{\lambda}^2 - p_4 \cdot \boldsymbol{\lambda} - p_5 = 0 \tag{32}$$

$$\boldsymbol{\lambda} = \begin{pmatrix} -61.27292419 + i \cdot 80.82192156 \\ -61.27292419 - i \cdot 80.82192156 \\ -58.09302721 \\ 1.06346914 \\ -2.99375355 \end{pmatrix} \tag{33}$$

The stability analysis of a system (i.e. of the jet engine) in case of small perturbations of the running regimes is assessed by the eigenvalues (33) of matrix **A**, in conjunction with the Routh-Hurwitz criterion, which states that "If all the real eigenvalues are negative or if the complex eigenvalues have the real part negative, then the system is said to be stable." The presence of one or more positive real eigenvalues, or complex eigenvalues with the real part positive, indicates that the behaviour of the system subjected to small perturbations is unstable.

For the study case, i.e. the flight conditions being considered: subsonic cruise at 35000 ft (10.67 km), Mach number 0.6, there are four eigenvalues of the matrix **A** located within the stability domain, and another eigenvalue is out of range the stability domain.

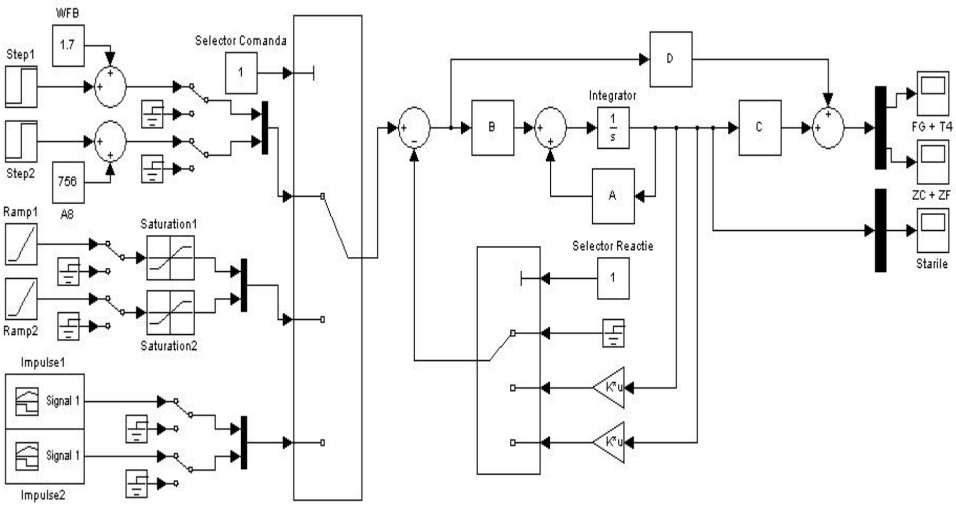


Fig. 6 - Block diagram tailored to control the low by-pass turbofan engine F 100

The physical interpretation refers means that the engine intended as a dynamic system is not stable for the particular flight condition: subsonic cruise at 35000 ft (10.67 km), Mach number 0.6. For this reason, the jet engine is continuously monitored and controlled for all the operating conditions and flight conditions, being equipped with Digital Engine Control Monitoring Units DECMU (i.e. health engine monitoring and control system).

The block diagram of the controlled F 100 turbofan engine (by using a closed-loop schedule) is presented in Fig. 6. The results of the stability analysis performed for the controlled F 100 turbofan engine are summarized in Fig. 7 ÷ Fig. 17, as follows.

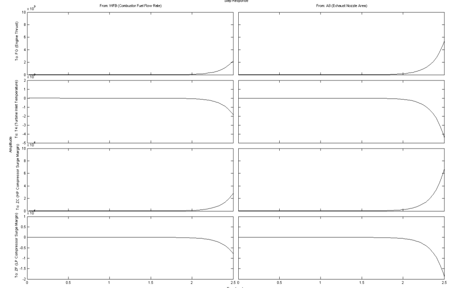


Fig. 7 – System answer for open loop schedule with step controls (unstable)

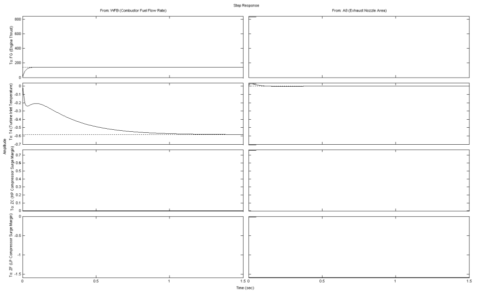


Fig. 8 – System answer for optimal states schedule with step controls

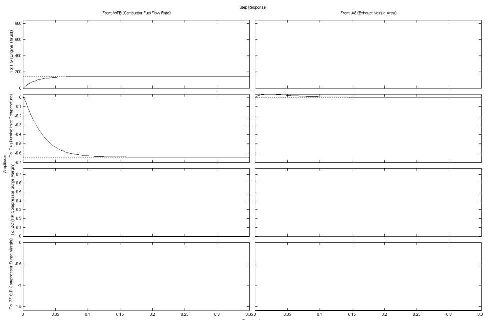


Fig 9 – System answer for optimal outputs schedule with step controls

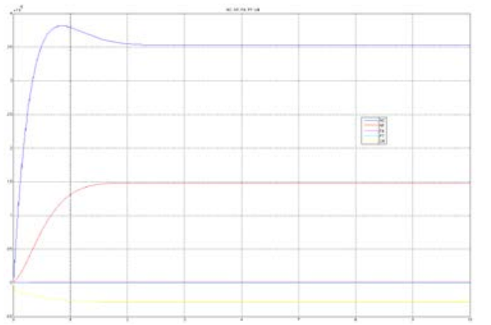


Fig 10 – Evolution of system states for Optimal Control

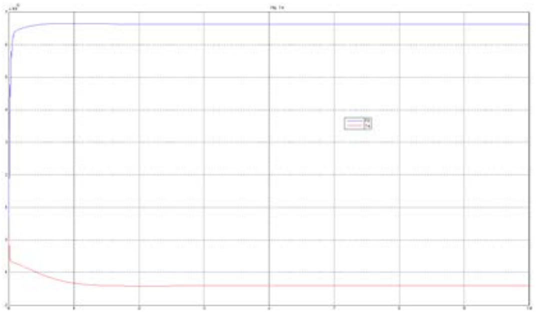


Fig 11 – Evolution of Thrust and Turbine Inlet Temperature for Optimal Control

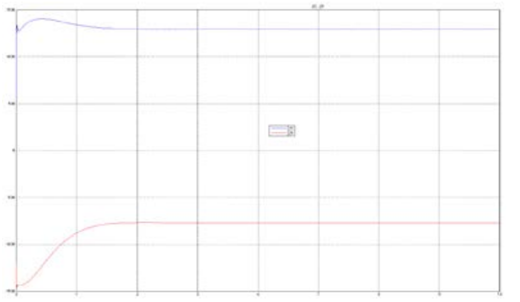


Fig 12 – Evolution of Surge Limit for Optimal Control

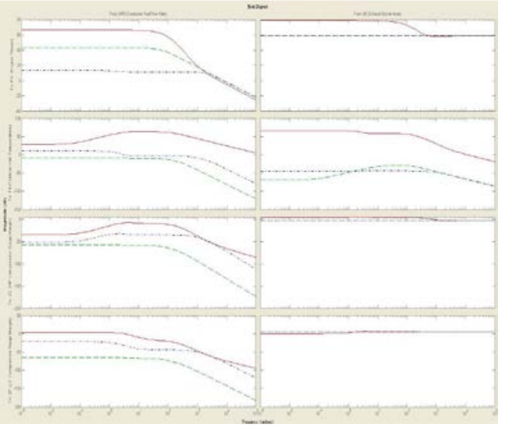


Fig. 13 – Bode Diagrams

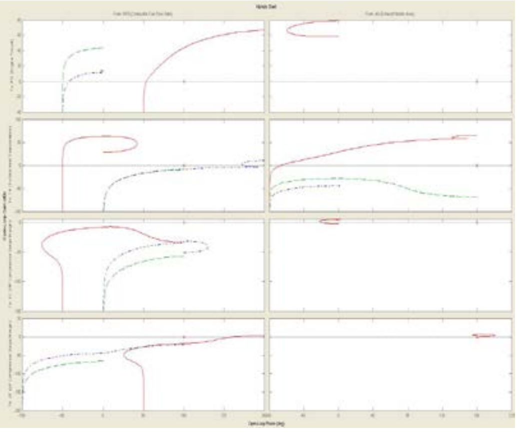


Fig. 14 – Nichols Diagrams

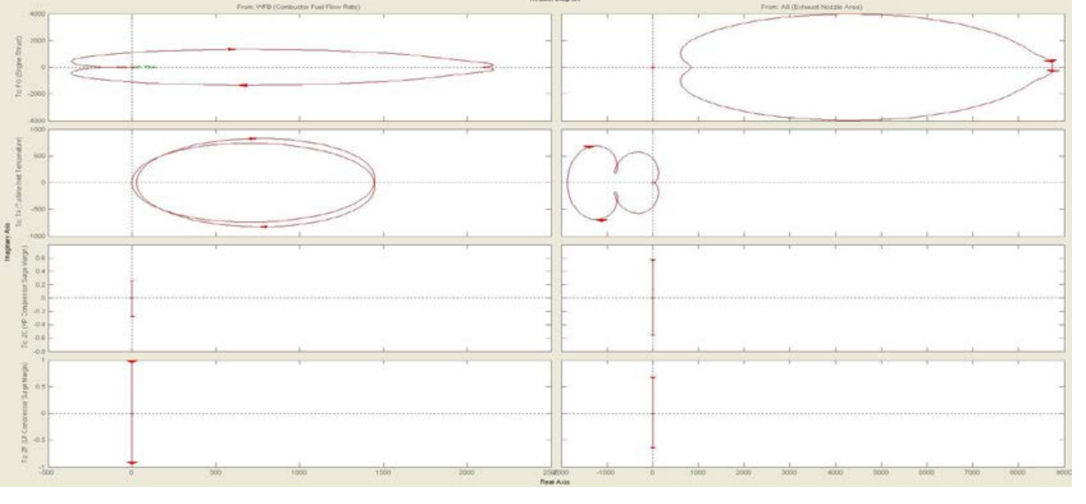


Fig. 15 – Nyquist Diagrams

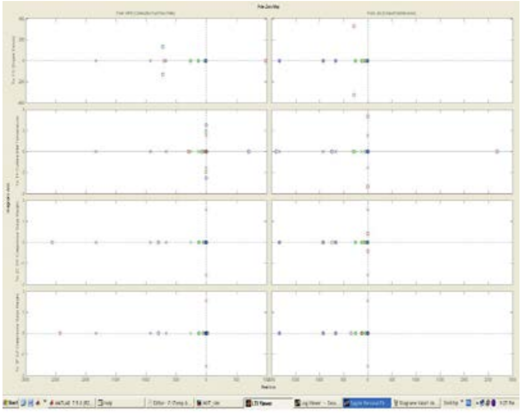


Fig. 16 – Map of zeros and poles

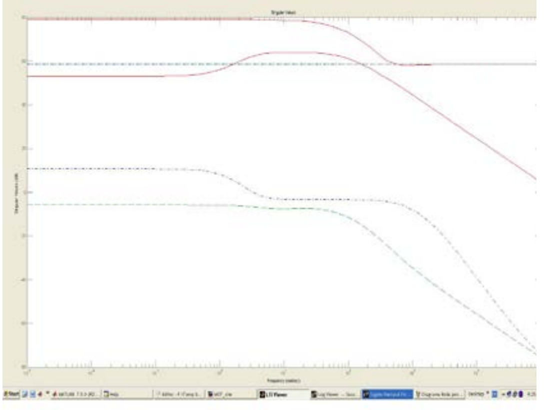


Fig. 17 – Map of eigenvalues

6. CONCLUSIONS

In this paper, a thorough study was carried based on the state of art detailed investigation, with numerical simulations for the low by-pass turbofan engine F 100 considered as case study. The conclusions drawn from this study are relevant to implement robust fuel management processes in order to increase the internal efficiencies and to reduce the fuel consumption and consequently, the operating costs and overall expenses.

Emphasizing the key-role of the jet engine as the propulsion system, its operating specific aspects and major influence on the aircraft-engine integration, from the global Fuel Aircraft Management System FAMS the focus is shifted to Fuel Engine Management System FEMS, based on a thorough analysis of the jet engine's dynamic behaviour; fuel management for specified operating conditions and flight regimes is necessary for maintaining the stability of the jet engine intended as a system. The paper presents the dynamic analysis of a low by-pass turbofan engine, which demonstrates improved features, such as: reduced fuel consumption levels, adaptability to all operating regimes and flight conditions and safer engine operation, due to the continuously engine monitoring and control, for the entire the flight mission profile. These reasons promote the low by-pas turbofan engines to be more efficient and environmentally friendly.

The mathematical model for the dynamic analysis of the low by-pass turbofan engine was represented by a linearized model of the jet engine, described by the output and state equations. It comes out that in spite of the fact that the operating line in case of the turbofans is farther from the surge line (i.e. safer running regimes with respect to the turbojets), the dynamic behaviour of the turbofans can be maintained stable provided that there is at least one health engine monitoring and control system.

The answer of the jet engine as a system is shown in Fig. 7 (for open loop schedule with step controls, which is unstable), in Fig. 8 (for optimal states schedule with step controls) and Fig. 9 (for optimal outputs schedule with step controls). The evolution of parameters was monitored for optimal control and illustrated in Fig. 10 (evolution of system states for Optimal Control), Fig 11 (Thrust and Turbine Inlet Temperature) and Fig 12 (Surge Limit). Information about the phase magnitude are provided by Bode Diagrams (Fig. 13) and Nichols Diagrams (Fig. 14) which allow to assess the stability and robustness of a linear system and can be used to obtain the closed loop transfer function directly from the open loop transfer. Nyquist

Diagrams that provide information about the stability are presented in Fig. 15. Fig. 16 shows the Map of zeros and poles, and Fig. 17 presents the Map of eigenvalues.

The study is completed by the investigation on fuel management solutions and specific mode of their implementation. The benefits of Fuel Management Systems integrated into Fuel Aircraft Management Systems are multiple related and refer to: minimizing the costs, reducing the workload, continuously improving internal and external data sharing, reducing the unnecessary communications, controlling fuel costs, making better decisions, improving cash flow and creating a more cohesive approach to Fuel Aircraft Management System. The applications of Fuel Aircraft Management System are beneficial to the aviation industry, and particularly, this study brings a significant contribution to the topic, i.e. advances in Fuel Aircraft Management System.

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