About an original method of optimizing the design of axial compressor cascades operating in the stability domain

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Abstract: The purpose of this paper is to present the development of an original optimization method, designed in order to obtain improved performances due to the fact that all the operating regimes of the axial compressor AC cascades are found within the stability domain. The optimization method is based on new correlations between the cascade’s geometric and aerodynamic parameters and the universal map of cascades. A new criterion was introduced in purpose to assess the stability range of the running regimes in case of the AC cascades, by using the Zweifel number and the universal map of cascades. The optimization method elaborated by the author finds its practical validation in the wide-chord fan and compressor blades constructions that are already being built for modern turbofans, by constructors, as the General Electric, Pratt & Whitney and Rolls-Royce. The motivation of the constructors is to achieve the dynamic stability, but from the author’s point of view, the static stability was also included.

Key Words: axial compressor cascades, optimization, stability, parameters correlations, Zweifel number, compressor maps, operating regimes.

1. INTRODUCTION

Axial compression systems are widely used in many aerodynamic applications and as jet engine parts mainly. Actual concerns of the designers of the modern jet engines refer to the fulfillment of economical and ecological demands, with the continuous preoccupation for performance improvement, i.e. high efficiency as well as higher power output, based on both growing mass flows and increasing specific work. Jet engines have to operate with sufficient stall margin in different climate conditions, at rotating speed variations due to frequency deviation in the power supply system, and at part-load conditions. The actual trends in the design of the modern jet engines are aimed towards size and weight reduction mainly, and wherever possible, the reducing of the technical construction’s complexity and costs bring down. Strong research efforts are focused at present to the designing of highly efficient products.

That is why the design of the aviation axial compressors is focused on the optimization of the construction, being driven by performance improvement.

The author presented in her Ph.D. Thesis, [1], a thorough study dedicated to the axial flow compressor AC, original contributions that have been introduced to complete the study and the design of the AC’s, which focused on this topic are:

(1) - A new correlation between the cascade’s geometric and aerodynamic parameters, [1, 3];
(2) - A new criterion purposed to assess the stability range of the running regimes in case of the AC cascades, by using Zweifel number and the cascades universal map, [1];

(3) - An original optimization method for designing the cascade such that to run within the stability domain.

This paper focuses on the development of an original optimization method, designed in order to obtain improved performances due to the fact that all the operating regimes of the AC cascades are found within the stability domain. The study was carried on for the first-stage transonic cascade of a NASA axial compressor, [22-24]. Comparative studies have been developed for variate radial designs of the blade, by using the Fully Radial Equilibrium Theory FRET and the radial design laws: Solid Body SB, Constant Vortex CV, Constant Reaction Degree CRD, fully details are given in ref. [1].

2. A NEW CORRELATION BETWEEN THE CASCADE’S GEOMETRIC AND AERODYNAMIC PARAMETERS

The development of the correlations between geometric and aerodynamic parameters with Zweifel number starts with the equation (1) which has been deduced in 2000 by Song S. J. & Cho S. Ho [33]; eqn (1) becomes (2) when being written with the usual parameter notations.

The influence of the \textit{geometric parameters} is expressed by the means of the \textit{blade span} $H$, \textit{pitch} $t$, \textit{chord} $b$ and/or \textit{relative pitch} ($t/b$).

The geometry of the fluid path can be introduced by using the \textit{radii at blade hub} $R_B$ and \textit{blade tip} $R_V$. Then, the \textit{blade span} $H$ can be replaced by the difference between the the \textit{radii at blade tip} $R_V$ and \textit{blade hub} $R_B$.

\[ ZW = 2 \left( \frac{c}{b} \right) \cos^2 \left( \beta_2 \right) \left( \tan \left( \beta_1 \right) - \tan \left( \beta_2 \right) \right) \]  

\[ ZW = 2 \left( \frac{t}{b} \right) \sin^2 \left( \beta_2 \right) \left( \tan^{-1} \left( \beta_1 \right) - \tan^{-1} \left( \beta_2 \right) \right) \]  

The influence of the \textit{aerodynamic parameters} is expressed by the \textit{flow angles} $\beta_1$ and $\beta_2$ in relative frame.

The flow angles can further be replaced by the \textit{flow coefficient} $\bar{C}_a$ (3), the \textit{load coefficient} $\bar{I}_a$ (4) and the \textit{stage reaction degree} $\rho_c$ (5), after doing some computations (21)-(30) and using the properties of the trigonometric functions.

The \textit{reaction degree} can be expressed for \textit{compressible flow} (5.a) or \textit{uncompressible flow} (5.b), as the ratio of the variations of the enthalpy $h$ (or the pressure $p$ respectively), inside the rotor vs. the stage.

The variation of either enthalpy or pressure determines the increasing of the fluid force which acts upon the cascades of the stage.

The reaction degree reflects the way that the AC stage compresses the air; when ranging from 0.5 up to 1, the reaction degree indicates a \textit{reactive stage}, meaning that the compression is done mainly inside the moving blades, while the fixed blades are guiding the flow. The \textit{fully reactive stage} (rotor: 100% compression, stator: fluid deviation) corresponds to the reaction degree equal to 1, while the \textit{fully active stage} (stator: 100% compression, rotor: fluid deviation) features the reaction degree equal to 0.
Optimization issues in design of axial compressor cascades

\[ \overline{C_a} = \frac{C_a}{u} \]  

\[ \tilde{l}_u = \frac{l^*_\text{stage}}{u^2} \]  

\[ \rho_c = \frac{h_2 - h_1}{h_3 - h_1} \]  

\[ \rho_c = \frac{h_2 - h_1}{h_3 - h_1} \]

The author proposes an original definition, with the set of the most significant geometric parameters, that is the **Geometric Factor** \( FG \), expressed by eqns. (6)÷(10).

\[ FG = 2 \left( \frac{r/b}{H/b} \right) \]  

\[ FG = \frac{t}{H} \]  

\[ FG = 2 \left( \frac{\tilde{t}}{H/b} \right) \]  

\[ FG = 2 \frac{\tilde{t}}{(R_v - R_B) b} \]  

\[ FG = 2 \cdot \frac{\tilde{t}}{R_v / b} \cdot \left[ 1 - \frac{R_B}{R_v} \right]^{-1} \]

Also, the author proposes another original definition, the **Aerodynamic Factor** \( FA \), expressed by eqns. (11)÷(15), intended to point out the influence of the aerodynamics of flow through axial compressor cascade and stage; these equations have been developed by using the velocity diagram of the rotor blade cascade, Fig. 1, and its consequent relations (21-30).

\[ FA = \sin^2(\beta_2) \cdot (\cotg(\beta_1) - \cotg(\beta_2)) \]  

\[ FA = \frac{\tilde{l}_u \cdot \overline{C_a}}{(\overline{C_a})^2 + \left( \rho_c - \frac{\tilde{l}_u}{2} \right)^2} \]

\[ FA = \frac{\tilde{l}_u}{\overline{C_a}} \cdot \frac{1}{1 + \left[ \frac{1}{\overline{C_a}} \cdot \left( \rho_c - \frac{\tilde{l}_u}{2} \right) \right]^2} \]

**INCAS BULLETIN, Volume 5, Issue 4/2013**
\[ FA = \frac{\tilde{u}_l}{C_a} \left[ 1 + \left[ \frac{\rho_c}{C_a} - \frac{1}{2} \cdot \frac{\tilde{u}_l}{C_a} \right]^2 \right]^{-1} \]  

(14)

\[ FA = \left[ \frac{C_a}{\tilde{u}_l} + \frac{\rho_c}{C_a} \cdot \left( \frac{\rho_c}{\tilde{u}_l} - 1 \right) + \frac{1}{4} \cdot \frac{\tilde{u}_l}{C_a} \right]^{-1} \]  

(15)

The universal map of cascades (34) defined by its coordinates \( m \) (16) and \( n \) (17), allows to obtain new expressions for the Aerodynamic Factor \( FA \), i.e. the eqns. (18)-(19).

\[ m = -\frac{\rho_c}{\tilde{u}_l} \]  

(16)

\[ n = \frac{C_a}{\tilde{u}_l} \]  

(17)

\[ FA = \frac{n}{n^2 + (m + 0.5)^2} \]  

(18)

\[ FA = \frac{n}{n^2 + m \cdot (m + 1) + 0.25} \]  

(19)

It comes out that Zweifel number (1-2) can be expressed directly (20) as the product between the Aerodynamic Factor \( FA \) (11)-(17) and the Geometric Factor \( FG \) (6)-(10), that is the correlation (20) that highlights more directly and following a much more simpler way, the global influence of the most significant aerodynamic and geometric parameters of both axial compressor’s cascade and stage.

\[ Zw = FA \cdot FG \]  

(20)

### 3. THE VELOCITY DIAGRAM IN ROTOR BLADE CASCADE

The aerodynamics of a rotor blade cascade is expressed by equations (21)-(30), based on the following parameters:

- \( l_{\text{stage}}^{*} \) - the specific work on compression of the stage, (21).
- \( u \) - rotational speed as transport velocity, (25)
- \( C = (C_a, C_u) \) - absolute velocity, given by its axial and tangential components
- \( W = (W_a, W_u) \) - relative velocity, given by its axial and tangential components
- \( \Delta C_u = \Delta W_u \) - flow deviation, as the variation of the tangential components of the velocity (either in absolute frame or relative frame) between exit and inlet stations.

The correlation between the specific work on compression and the kynematics of flow (i.e. velocities and flow angles, as shown in Fig. 1), is expressed by equations (21, 24):

\[ l_{\text{stage}}^{*} = u \cdot \Delta C_a = u \cdot \Delta W_u = u \cdot (W_{1u} - W_{2u}) \]  

(21)

\[ W_{1u} = C_a \cdot \text{ctg} \beta_1 \]  

(22)
\[ W_{2u} = C_u \cdot \cot \beta_2 \] (23)

\[ I_{stage}^* = u \cdot C_a \cdot (\cot \beta_1 - \cot \beta_2) \] (24)

\[ u = \omega \cdot R \] (25)

Taking into account the rotational speed (25), and after replacing the flow angles with the flow coefficient \( \bar{C}_a \) (3), the load coefficient \( \bar{I}_u \) (4) and the stage reaction degree \( \rho_c \) (5), one obtains the relations (26-30):

\[ \bar{I}_u = \frac{2 \cdot \rho_c}{\bar{C}_a} = 2 \cdot \cot \beta_m \] (26)

\[ \cot \beta_1 + \cot \beta_2 = \frac{1}{\bar{C}_a} \left( \rho_c + \frac{1}{2} \cdot \bar{I}_u \right) \] (27)

\[ \cot \beta_1 = \frac{1}{\bar{C}_a} \left( \rho_c + \frac{1}{2} \cdot \bar{I}_u \right) \] (28)

\[ \cot \beta_2 = \frac{1}{\bar{C}_a} \left( \rho_c - \frac{1}{2} \cdot \bar{I}_u \right) \] (29)

\[ \sin^2 \beta_2 = \frac{1}{1 + \cot^2 \beta_2} = \frac{1}{1 + \left[ \frac{1}{\bar{C}_a} \left( \rho_c - \frac{1}{2} \bar{I}_u \right) \right]^2} = \frac{\left( \bar{C}_a \right)^2}{\left( \bar{C}_a \right)^2 + \left( \rho_c - \frac{1}{2} \bar{I}_u \right)^2} \] (30)

![Fig. 1 – The velocity diagram of the rotor blade cascade](image)

**4. STUDY CASES**

The study cases (1)÷(4) are represented by four radial designs, [1], of a transonic rotor blades cascade; there have been considered the radial design laws in accordance with: 1) the **Fully Radial Equilibrium Theory** FRET, 2) the **Solid Body** SB model, 3) the **Constant Vortex** CV model and 4) the **Constant Reaction Degree** CRD model; a thorough study is exposed in [1]. The radial variation of the tangential velocity \( C_u \) is expressed for each of the radial design laws: 1) FRET, 2) SB, 3) CV and 4) CRD, accordingly as in Table 1.
The transonic rotor blades cascades originates from the first stage transonic rotor blades of a seven-staged NASA axial compressor, [22-24]; the geometry and the flow parameters at mean streamline (i.e. velocities, flow angles, Mach numbers in absolute and in relative frame respectively, pressure, temperature, enthalpy and entropy, as well as the stagnation parameters (pressure, temperature, enthalpy and work) are given in refs. [22-24].

The blade geometry has been determined for NACA 65 series airfoils cascades, subjected to achieve the velocity diagram as shown in fig. 1-b. For all the considered radial design laws (i.e. FRET, SB, CV, CRD), the parameters of flow at inlet have been preserved. A summary of cascade design parameters is specified in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Blade span-wise Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airfoils of NACA65 series</td>
<td>B</td>
</tr>
<tr>
<td>Chord b [mm]</td>
<td>88</td>
</tr>
<tr>
<td>Pitch t [mm]</td>
<td>39</td>
</tr>
<tr>
<td>Relative pitch</td>
<td>0,439</td>
</tr>
<tr>
<td>Camber θ [°]</td>
<td>37,838</td>
</tr>
<tr>
<td>Stagger βf [°]</td>
<td>60,605</td>
</tr>
<tr>
<td>Reynolds’ number</td>
<td>&gt;1.10·10^6</td>
</tr>
<tr>
<td>Inlet flow angle β1 [°]</td>
<td>43,692</td>
</tr>
<tr>
<td>Exit flow angle β2 [°]</td>
<td>68,917</td>
</tr>
<tr>
<td>Deviation Δβ [°]</td>
<td>25,225</td>
</tr>
<tr>
<td>Diffusion factor Dk</td>
<td>0,3088</td>
</tr>
<tr>
<td>Inlet Mach Mc_1</td>
<td>0,532</td>
</tr>
<tr>
<td>Exit Mach Mc_2</td>
<td>0,6954</td>
</tr>
<tr>
<td>Inlet Mach Mc_1</td>
<td>0,769</td>
</tr>
<tr>
<td>Exit Mach Mc_2</td>
<td>0,5857</td>
</tr>
<tr>
<td>Radius R [mm] at inlet</td>
<td>318</td>
</tr>
<tr>
<td>Radius R [mm] at exit</td>
<td>378</td>
</tr>
<tr>
<td>Radius ratio R/Rv</td>
<td>0,50–0,60</td>
</tr>
</tbody>
</table>

N.B. Mach number of absolute flow is referred by MC, while Mach number in relative frame is referred by Mw; index _1 is applied for the cascade’s inlet section and index _2 stands for the cascade’s exit section.

1 Reynolds number was computed with the relative velocity at cascade inlet and chord.
5. COMPRESSOR MAPS

The way the compressor works is expressed by its maps, which are: (a) stage map (also referred as the simplified map of the stage), (b) cascades map (that is the main map of the design regimes) and (c) the universal map of cascades.

a) **The simplified map of the stage**, fig. 1: shows the relation (31) between the flow coefficient \( \overline{C}_a \) (3) and the load coefficient \( \overline{I}_u \) (4).

b) **The main map of the design regimes**, fig. 2: links the angular deviation \( \Delta \beta_n \) with the flow angle at the design regime and the relative pitch \( \tilde{i} \), as in eqn. (32) or equivalent (33).

c) **The universal map of cascades**, fig. 3: eqn. (34) features the variation of the coefficients \( m \) (18) vs. \( n \) (19).

\[
\overline{C}_a = f(\overline{I}_u) \tag{31}
\]

\[
\Delta \beta_c = f(\beta_{2c}, \tilde{i}) \tag{32}
\]

\[
\begin{cases}
\frac{\rho_c}{\overline{C}_a} = f_1(\tilde{i}) \\
\overline{I}_u = f_2(\tilde{i}) \\
m = f(n) \\
m = -\frac{\rho_c}{\overline{C}_a} 
\end{cases} \tag{33}
\]

Fig. 1 – The simplified map of the stage

Fig. 2 – The main map of the design regimes

Fig. 3 – The universal map of cascades

Fig. 4 – The correlation between the UMC and Zweifel number
6. RESULTS AND CONCLUSIONS

6.1 A new criterion to assess the stability domain by using the correlation between universal map of cascades and Zweifel number

From fig. 3 one can detect the occurrence of flow instabilities over blade span, for the radial design laws: FRET, CV and CRD, but in case of SB radial design the flow is stable.

From fig. 4 or its equivalent fig. 5 that aspect can be detected far much easier, and in addition, by using the correlations with Zweifel number, one can emphasize a critical threshold $Z_w^{\text{critical}}$, which separates the stability domain (i.e. running at stable regimes) $Z_w > Z_w^{\text{critical}}$ from the area of the unstable regimes $Z_w \leq Z_w^{\text{critical}}$. Fully details about the interpretation of the results are given in ref. [1]. On account of the above presented, one can note the following:

- the blade radially designed with SB runs with fully blade span within the stability domain;
- the blades radially designed with FRET and CV work with about 75% blade span (B-BM-M-MV) within the stability domain, while for the remaining 25% blade span (V-MV) occur instabilities of flow.
- the blade radially designed with CRD runs with about 40% blade span (B-BM$\rightarrow$M) within the stability domain, and with the remaining 60% blade span (V-MV$\rightarrow$M) runs inside the instability domain.

6.2 A new optimization method for designing the cascade to run within the stability domain

From the correlations between the universal map of cascades and Zweifel number comes out that the boundary of the stability domain is the critical value of the threshold $Z_w^{\text{critical}}$; the stability domain is found where the condition $Z_w > Z_w^{\text{critical}}$ is satisfied, while the area of the instability regimes is detected whenever the condition $Z_w \leq Z_w^{\text{critical}}$ is fulfilled.
The values of the threshold \( Zw_{\text{critic}} \) are different for each radial design, but these can be easily specified from fig. 4 or fig. 5. For instance, in case of radial design with FRET and CV, the condition for running outside the stability domain is \( Zw \leq 0.05 \div 0.06 \), which corresponds to the blade sections MV-V, whilst in case of radial design with CRD, the condition for running outside the stability domain is \( 0.05 \leq Zw \leq 0.10 \).

It comes out that the increasing of Zweifel number above the critical threshold represents an option to shift outwards the instability domain into the stability domain. Therefore, the condition that the blade should run within the stability domain also represent a potential mean in purpose to performance improvement.

The directions for the increasing Zweifel number (20), are to augmenting either the Geometric Factor (6)+ (10) and/or the Aerodynamic Factor (11)+ (17).

The options for augmenting the Geometric Factor FG (6)+ (10) are expressed by:

1. Increase relative pitch and/or decrease the ratio of blade span vs. chord, e.g. larger chords,
2. Increase pitch and/or reduce blade span,
3. Preserve relative pitch such that the velocity diagram does not change, but either increase chord or reduce the difference of radii between blade tip and blade hub, i.e. reduce blade span.

A closer look at these options, points out that the reducing blade span supposes the decreasing of the flow path cross-section, and therefore less air flow at inlet. Any modification of the relative pitch goes to the changing of the number of blades.

The augmentation of the chord enables the blade to running regimes within the stability domain; that fact is highlighted by using the radial design SB, and which is more, it is proven by constructions that are already being built for modern turbofans, which are: wide-chord fan blade (GP7200, GenX, GE90, Trent 900, Trent 1000, PW4000 turbofans) and wide-chord forward swept stage 1 (GP7000 turbofan) of constructors like the General Electric, Pratt & Whitney and Rolls-Royce. The motivation of the constructors is to achieve the dynamic stability, but from the author’s point of view, the static stability was included.

The options for augmenting the Aerodynamic Factor FA (11)+ (17) refer to modifying the load coefficient (4), flow coefficient (3) and the reaction degree (5).

By using the correlations with Zweifel number, if the load coefficient is augmented, then Zweifel increases, e.g. for values of the load coefficient larger than \( \bar{I}_u \geq 0.3 \div 0.45 \) the Zweifel number exceeds the critical threshold, for any radial design of the blade. One must take into account the radial design law when the modification of the flow coefficient is intended, since for the FRET, CV and CRD the flow coefficient should be increased, but for the SB it should be decreased, in order to obtain the augmentation of Zweifel number.

The reaction degree should be reduced such that Zweifel increases, for the radial designs FRET, CV and SB;for the CRD design, it is convenient to use the correlation between Zweifel number and the coefficient \( m \) (18), which has to be increased in order to obtain the augmentation of Zweifel number.

Fully details about the interpretation of the results based on the correlations with Zweifel number are given in ref. [1].

It is important to underline that the augmentation of the Aerodynamic Factor FA (11)+ (17) and the Geometric Factor FG (6)+ (10) is applied locally, in specific areas on blade span, where there is detected the occurrence of the instability condition. The corrections must include the blade spanwise distributions of the Aerodynamic Factor FA (11)+ (17) and the Geometric Factor FG (6)+ (10).
7. CONCLUDING REMARKS

This paper intents to introduce and develop an original optimization method, targeted to performance improvement by designing the axial flow compressor AC cascades to run inside the stability range.

New correlations between the cascade’s geometric and aerodynamic parameters have been introduced, as well as other new correlations between the maps of stage and cascades with Zweifel number. A new criterion was introduced for the purpose of assessing the stability range of the running regimes in case of the AC cascades, by using Zweifel number and the universal map of cascades.

The use of Zweifel number as a new criterion to assess the stability domain proves to be advantageous and it is justified by the following aspects:

- For any radial design of the blade, a critical threshold $Z_{\text{w, critical}}$ can be emphasized by using Zweifel number; the boundary of the stability domain is just this critical threshold.
- The stability domain corresponds to the condition: $Z_w > Z_{\text{w, critical}}$.
- The instabilities area corresponds to the condition: $Z_w \leq Z_{\text{w, critical}}$.
- The new criterion allows to appreciate if instabilities could occur and where are the areas with flow instabilities of flow located on blade span sections.

The improvement of the construction in case of the blades detected with flow instabilities, consists in modifying the radial velocity distribution and if still necessary, reconsidering certain design steps.

The original optimization method is based on new correlations between the cascade’s geometric and aerodynamic parameters and the universal map of cascades.

The principle of the method consists in increasing the Zweifel number so as to overtake the critical threshold $Z_{\text{w, critical}}$, by using the options exposed above. As a consequence, the instability area is shifted with the stability domain, so that the purpose of blade cascades running within the stability domain is achieved.

Under these circumstances, this new method represents a new potential mean in purpose to performance improvement; an other advantage of the optimization method refers to the easiness to be put into practice, when applied for improving the blade design.

The optimization method elaborated by the author finds its practical validation in the wide-chord fan and compressor blades constructions that are already being built for modern turbofans, by constructors like the General Electric, Pratt & Whitney and Rolls-Royce. The motivation of the constructors is to achieve the dynamic stability, but from the author’s point of view, the static stability was included.

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