Integral Method for Flow Induced Transverse Vibrations Analysis of Flexible Pipes

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Section 1. Basic Methods in Fluid Mechanics

Abstract: *This paper reviews some existing studies and numerical methods used for flow induced transverse vibrations analysis of flexible pipes. An integral method, based on the use of Green's functions, already used for different straight beam dynamic analysis is adapted for the proposed subject. This approximate method leads to a matrix formulation and to an eigenvalue problem for free vibration analysis. The presented approach is able to estimate also the critical fluid velocities. Effects of boundary conditions and of elastic foundation characteristics, Coriolis terms and of other parameters on dynamic behavior of a pipe, can be included. Some numerical examples are also presented for comparisons with results obtained by FEM or with other data from literature. They show good agreement.*

Key Words*: Pipeline, Fluid-Structure Interaction, Winkler Foundation, Green's Functions, FEM, Vibrations, Stability*

1. INTRODUCTION

Vibrations of piping systems are studied in many engineering fields. The steady flow at high velocities in thin walled pipes can produces vibrations with large amplitudes and instabilities. One of the first studies of flow induced vibrations started from a case of trans-Arabian pipeline [1]. In 1952 Housner [2] derived the corresponding equations of motion and predicted the instability according to the boundary conditions. Simply supported and cantilever pipes have been considered. Starting with the work [3], Paidoussis has extensively studied different aspects of flow induced pipeline instabilities. These has been presented in review [4] and in his book [5]. Another book also reviewing this type of studies is presented by Blevins [6]. From numerical point of view, as one can speak about a distributed parameter system, the classic Rayleigh-Ritz and Galerkin methods can be employed for such analysis. Studies as [7], [8] are based on the use of finite element method, while [9] applied the spectral element method. The equations involved for vibration analysis of pipelines can be also solved by Differential Transformation Method (DTM), as it is demonstrated in [10], while [11] uses a formulation based on absolute nodal coordinates (ANCF). The use of Green's functions for the study of different slender beam structures is presented in references as [12-14].

Such a method is employed for example in [15] for transverse (bending) vibration analysis of Euler-Bernoulli and Timoshenko beams.

In this paper, the pipes conveying fluid induced vibrations are analyzed by employing the integral formulation described in [13, 14] and using the Green's functions, determined in [13], for the case of the cantilevered beam.

The presented approach is a matrix one leading to eigenvalues and eigenvectors problem making possible the critical fluid velocity determination. A results comparison with available data from literature is then discussed.

2. PROBLEM FORMULATION

The analyzed cantilever pipeline configuration is presented in Fig. 1. The pipe is considered a uniform Euler beam and the fluid has a plug flow with constant velocity U. The corresponding equation of motion for bending vibration analysis are of the form, [5], [8]:

$$EI\frac{\partial^4 w}{\partial x^4} + MU^2\frac{\partial^2 w}{\partial x^2} + 2MU\frac{\partial^2 w}{\partial x \partial t} + (m+M)\frac{\partial^2 w}{\partial t^2} + Kw = 0 . \tag{1}$$

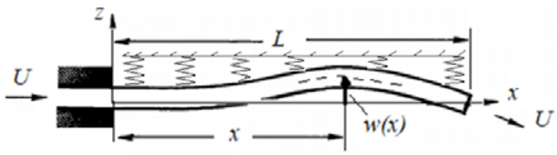

Fig. 1 – Fluid conveying cantilever pipe configuration

The employed notations in (1) are: E is Young modulus of elasticity of pipe material; I is moment of inertia of the cross-section; m is the mass of unit length for the pipe ($m = \rho A$, with ρ the mass density of the pipe material and A the cross-section area), M is the mass of unit length for the fluid ($M = \rho_f A_f$, with ρ_f the mass density of the fluid and A_f the cross-section area corresponding to the fluid) and t is the time.

As the presence of a foundation can stabilize the motion [7], the last term represents the Winkler foundation reaction proportional to the bending deflection w, where K is the distributed soil stiffness constant. The following nondimensional parameters are also used:

$$u = UL\sqrt{\frac{M}{EI}}; \quad \beta = \frac{M}{m}; \quad k = \frac{KL^4}{EI}. \tag{2}$$

In (2) the first parameter u is a flow velocity parameter, the second parameter β measures the ratio between the mass of the fluid and mass of the pipe and k is the Winkler foundation coefficient.

3. INTEGRAL FORM OF THE EQUATIONS

One starts from the differential equation governing the bending behavior of a straight beam, loaded by the distributed force $p(x)$:

$$\frac{d^2}{dx^2}\left(EI\frac{d^2 w}{dx^2}\right) = p(x), \tag{3}$$

which can take the integral form, [13, 14]:

$$w(x) = \int_0^L G_w(x,\xi)\, p(\xi)\, d\xi \tag{4}$$

where the Green function for the clamped-free beam is, [13]:

$$G_w(x,\xi) = \int_0^{\min(x,\xi)} \frac{(x-\xi_1)(\xi-\xi_1)}{EI(\xi_1)} d\xi_1 . \tag{5}$$

It represents the bending deflection $w(x,\xi)$ at distance x due to a unity force applied at distance ξ (Fig. 2). For uniform clamped-free beam, relation (5) takes the form obtained also in [16].

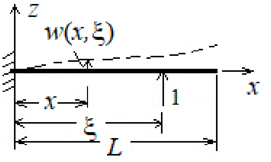

Fig. 2 – Green's functions for cantilever beam

The Green functions are computed by numerical integration using n sampling (collocation) points ξ_i with $f_i = f(\xi_i)$ with a relation of the form:

$$\int_0^L f(\xi) d\xi = \sum_{i=1}^{n} f_i \cdot W_i , \tag{6}$$

where W_i are weighting numbers.
Relation (4) gives the deflections $w(x)$ for known distributed force $p(x)$. It can be written in matrix form:

$$\{w\} = [G_w][W]\{p\} , \tag{7}$$

where one can notice the following ($n \times n$) matrices:
$[G_w]$ contains the calculated influence coefficients $G_w(\xi_i,\ \xi_j)$; $[W]$ is a weighting matrix depending on the integration method (Simpson, here).

The last term in (7) is a column vector containing the distributed load values in the n chosen collocation points.

The idea is to consider equation (1) of the form (3) with $p(x)$ given by:

$$p(x) = -MU^2 \frac{\partial^2 w}{\partial x^2} - 2MU \frac{\partial^2 w}{\partial x \partial t} - (m+M)\frac{\partial^2 w}{\partial t^2} - Kw . \tag{8}$$

It can take the integral form:

$$\begin{aligned} \{w\} &= -MU^2 [G_w][W][D_2]\{w\} - 2MU[G_w][W][D_1]\{\dot{w}\} \\ &-(m+M)[G_w][W]\{\ddot{w}\} - K[G_w][W]\{w\}. \end{aligned} \tag{9}$$

The above matrix relation can be written in the standard form:

$$[M_n]\{\ddot{w}\} + [C_n]\{\dot{w}\} + [K_n]\{w\} = \{0\} \tag{10}$$

where one can highlight the ($n \times n$) matrices namely mass matrix, an equivalent damping type matrix and stiffness matrix:

$$\begin{gathered}[M_n]=(m+M)[G_w][W]; \quad [C_n]=2MU[G_w][W][D_1];\\ [K_n]=[I_n]+K[G_w][W]+MU^2[G_w][W][D_2],\end{gathered} \tag{11}$$

where $[I_n]$ is the identity matrix of size n. In order to avoid the differentiation matrices $[D_1]$ and $[D_2]$ it is also possible to use for the bending deflection $w(x)$ an expression based on collocation functions according to the boundary conditions (clamped-free beam here):

$$w(x)=\sum_{k=1}^{p} C_k f_k(x), \tag{12}$$

where $f_k(x)$ are p known functions with C_k constant coefficients. In matrix form, in the case of the use of n collocation points, one can obtain the following three matrix relations:

$$\{w\}=[F]\{C\}; \quad \{w'\}=[F_1]\{C\}; \quad \{w''\}=[F_2]\{C\}, \tag{13}$$

where $[F]$, $[F_1]$, $[F_2]$, are $(n\times p)$ size matrices containing the values of $f_k(x)$ in collocation points and their corresponding first and second derivatives, respectively.
Using this formulation, relation (9) can be written as:

$$\begin{gathered}[F]\{C\}=-MU^2[G_w][W][F_2]\{C\}-2MU[G_w][W][F_1]\{\dot{C}\}\\ -(m+M)[G_w][W][F]\{\ddot{C}\}-K[G_w][W][F]\{C\}.\end{gathered} \tag{14}$$

Multiplying (14) at left with the transpose of the matrix $[F]$ and making the following notations:

$$\begin{gathered}[FF]=[F]^T[F]; \quad [FF_1]=[F]^T[G_w][W][F_1];\\ [FF_0]=[F]^T[G_w][W][F]; \quad [FF_2]=[F]^T[G_w][W][F_2],\end{gathered} \tag{15}$$

the relation (14) becomes:

$$[FF]\{C\}=-MU^2[FF_2]\{C\}-2MU[FF_1]\{\dot{C}\}-(m+M)[FF_0]\{\ddot{C}\}-K[FF_0]\{C\}. \tag{16}$$

Rearranging one can obtain a standard form:

$$[M_p]\{\ddot{C}\}+[C_p]\{\dot{C}\}+[K_p]\{C\}=\{0\}, \tag{17}$$

where one can highlight the mass matrix, damping type matrix and stiffness matrix having now the $(p\times p)$ size:

$$\begin{gathered}[M_p]=(m+M)[FF_0]; \quad [C_p]=2MU[FF_1];\\ [K_p]=[FF]+K[FF_0]+MU^2[FF_2],\end{gathered} \tag{18}$$

The size of relation (17) is $p < n$, therefore this form will be used instead of (10). As it is concluded in [17], for the numerical calculations of the dynamic of rotor bearing systems (including the critical speeds estimation), it is convenient to write an equation like (17) in the state space form:

$$[A]\{\dot{q}\}+[B]\{q\}=\{0\}, \tag{19}$$

where the matrices and vector $\{q\}$ having the size $2p$ are given by:

$$[A]=\begin{bmatrix}[M_p] & [0]\\ [0_p] & [I_p]\end{bmatrix},\quad [B]=\begin{bmatrix}[C_p] & [K_p]\\ -[I_p] & [0_p]\end{bmatrix},\quad \{q\}=\begin{Bmatrix}\dot{C}\\ C\end{Bmatrix}. \tag{20}$$

For (19) trying solutions of the form:

$$\{q\}=\{q_0\}e^{\lambda t}, \tag{21}$$

one can obtain the following generalized eigenvalue problem:

$$\left(\lambda[A]+[B]\right)\{q_0\}=\{0\}, \tag{22}$$

which can take the standard form:

$$\left(-[A]^{-1}[B]\right)\{q_0\}=\lambda\{q_0\}. \tag{23}$$

The eigenvalues of (23) are the eigenvalues of the following unsymmetrical real matrix:

$$[D]=\begin{bmatrix}-[\mathrm{M}_p]^{-1}[C_p] & -[\mathrm{M}_p]^{-1}[K_p]\\ [I_p] & [0_p]\end{bmatrix}. \tag{24}$$

As consequence, the eigenvalues which are functions of the fluid velocity U occur in complex conjugate pairs:

$$\lambda_r=\alpha_r+i\omega_r,\quad \lambda_r^*=\alpha_r-i\omega_r\quad r=1,2,\ldots p. \tag{25}$$

The imaginary parts ω_r represent the natural frequencies of bending vibrations taking into account the influence of the fluid and of its velocity. The real parts α_r, when they are not zero values represent attenuation or growth constants. In this case, critical velocities U_{cr} are obtained when an imaginary part becomes positive.

4. NUMERICAL EXAMPLES

As a first example one considers an application described in [18]. The clamped-free stainless-steel tube has L = 10 m, EI = 3056937 Nm^2 and m = 24.498 kg/m. These data are taken in fact from [19] which is the first edition of the monograph [6]. The mass ratio β between 0.1 and 1.2 has been employed (for β = 1.18 the pipe conveys water). For $w(x)$ one can use the family of the mode shapes for transverse vibrations of uniform beams taken from [20], based on Krylov-Duncan functions:

$$S(x)=\frac{ch\,x+\cos x}{2},\quad T(x)=\frac{sh\,x+\sin x}{2} \tag{26}$$

and

$$U(x)=\frac{ch\,x-\cos x}{2},\quad V(x)=\frac{sh\,x-\sin x}{2}. \tag{27}$$

In the case of clamped-free beam, the collocation functions are of the form:

$$f_{bk}(x)=T(\beta_k)\cdot U\left(\beta_k\frac{x}{L}\right)-S(\beta_k)\cdot V\left(\beta_k\frac{x}{L}\right) \tag{28}$$

where β_k are those given by:

$$\beta_1=1.875;\ \beta_2=4.694;\ \beta_3=7.855;\ \beta_4=10.996;\ \beta_i=\frac{2i-1}{2}\pi \quad i\geq 5. \tag{29}$$

The presented method was used with n = 200 sampling (collocation) points and p = 10 collocation functions. For a given value of the mass parameter β the speed parameter u was changed until a real part of an eigenvalue becomes positive which is the indication of the instability. The eigenvalues were obtained using relations (23) and (24). Figure 3 presents the graphic of the critical flow speed parameter. The curve is practically the same as those obtained in [18].

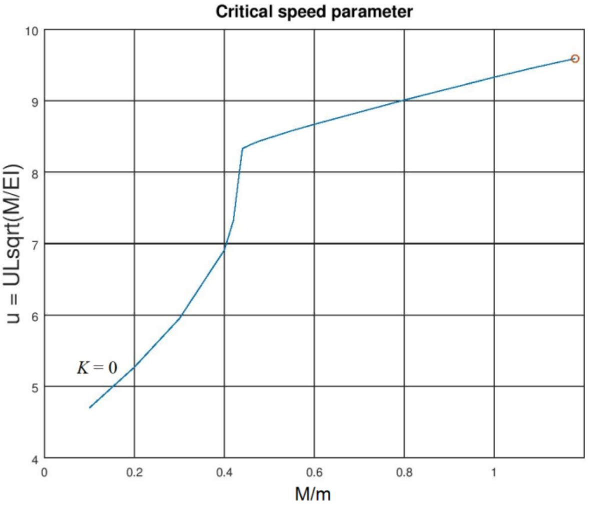

Fig. 3 – Critical flow speed parameter versus the mass ratio parameter

In order to check the results, the natural frequencies of bending vibrations for the L = 10 m pipe filled with non-flowing fluid (water with ρ_f = 1000 kg/m^3) were also computed with the presented formulation and using the finite element code ANSYS with several different models (according to [22], [23]). In order to obtain the stiffness data of the pipe and the parameter β = 1.18, the following values have been also considered: D_i = 191.85 mm (internal diameter), D_e = 201.94 mm (external diameter), ρ = 7850 kg/m^3 (mass density of the pipe material) and E = 202 GPa (Young's modulus). The first three natural frequencies in [Hz] were listed in the table below. They are in good agreement from engineering point of view.

Table 1 – Results comparison for the clamped-free pipe filled with water

Method	Frequency [Hz]		
	f_1	f_2	f_3
n = 200, p = 10 relations (23), (24)	1.3388	8.3902	23.492
ANSYS, Beam 188, 1000 elements	1.3379	8.3542	23.260
ANSYS, Pipe 288, 1000 elements	1.3388	8.3459	23.228

Another set of values is also taken from [18]. These are L = 10 m, EI = 265432 Nm^2 and m = 9.15 kg/m. The mass ratio β was taken between 0.1 and 1 (for β = 0.86 the pipe conveys water). For this case two constant, non-zero values of K, distributed soil stiffness coefficient were also employed: K_1 = 5000 N/m^2 and K_2= 10000 N/m^2. The results are shown in figure 4. Figure 5 presents the corresponding critical flow speeds. Increasing the soil stiffness constant, the critical flow speeds (flutter speeds) are also increasing.

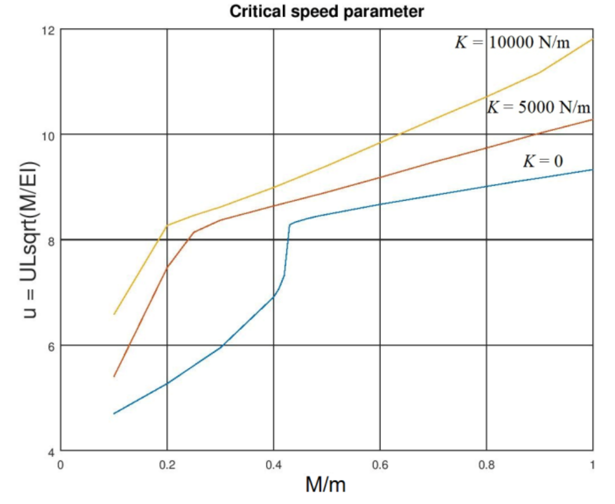

Fig. 4 – Critical flow speed parameter for three different K values

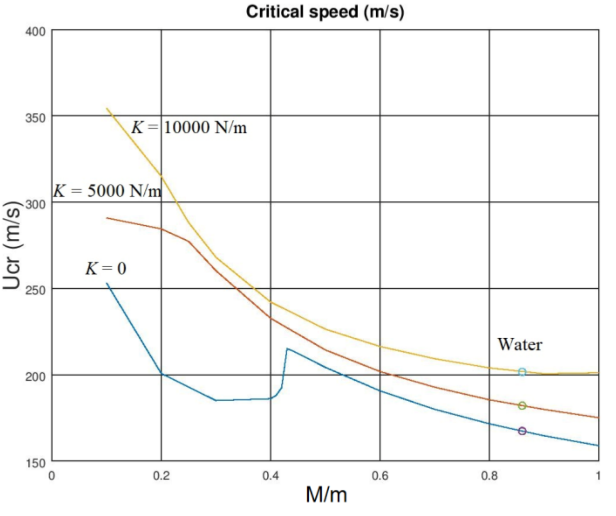

Fig. 5 – Critical flow speed versus the mass ratio parameter

The results obtained in figures 4 and 5 reproduced with good agreement the results presented in figure 1 in [18] and figure 3 in [21], respectively. The marked points in figures 3 and 5 correspond to pipe conveying water. The support of the pipe described by the Winkler model can pay a benefic role by increasing the critical speeds.

5. CONCLUSIONS

This work addresses the critical flow speed calculation for the case of a clamped-free pipe conveying fluid and supported by a constant stiffness elastic foundation. There are a lot of numerical methods used for this purpose, starting from classic Rayleigh-Ritz and Galerkin methods up to FEM and recent method as DTM and ANCF. The approach described in the present paper is an *integral method* based on the use of Green's functions. This method, also used for aeroelastic analysis of clamped-free wing ([13]), is adapted here for this type of calculations. It takes a matrix form using *integration* and *differentiation matrices*. In order to avoid the differentiation matrices and to reduce the dimension of the eigenvalue problem one can also use *collocation functions* according to the actual clamped-free boundary conditions. Due to the presence of the Coriolis term in the solved differential equation, it is convenient to write the equation in state space form. In this manner the final form of the eigenvalue problem is two times bigger. For instance, in the presented examples n = 200 sampling points and p = 10 collocation functions were used, so the dimension of the eigenvalue problem was finally reduced from $2n$ to $2p$. The comparison was performed with results obtained by Galerkin method. Good agreement was achieved for two pipe conveying fluid configurations analyzed in [18] and [21]. For the first one, the natural frequencies were also computed by FEM using the FE code ANSYS, in the case of the pipe filled with non-flowing water. For the second configuration, two cases of the pipe restrained by constant stiffness elastic foundation have been also analyzed obtaining a good agreement. It was the main purpose of the presented paper to check the validity of the presented simple integral approach, used to obtain the critical values of fluid speed. Further developments of the presented method can include the study of pipes having other boundary conditions, the analyze of other stiffness foundation configurations and the inclusion of different new features as foundation external damping, gravity effects etc.

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