Dynamics of shell with destructive heat-protective coating under running load

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Abstract: *The problem of dynamic deformation of a plate with a two-layer composite shell with a heat-shattering coating collapsing in time under the action of a running load is solved approximately. The problem is solved in a dynamic formulation, considering that the deformed state of the shell depends both on the spatial coordinates and on time. The problem is reduced to solving two differential equations of the shell in partial derivatives with respect to deflections and the stress function. These equations contain discontinuous ratios for unknowns, which are associated with the dynamic destruction of the heat-shielding coating. According to the Bubnov method, the problem is also reduced to a system of differential equations, but already in ordinary derivatives. The solution of these equations is obtained in closed form. In addition, the natural vibration frequencies of the structure and the critical velocities of the load are found depending on the degree of damage to the protective layer. Formulas for oscillation frequencies and critical speeds are obtained in closed form.*

Key Words*: two-layer composite shell, running load, collapsing heat-shielding coating, dynamic deformation, natural vibration frequencies, critical speeds of movement*

1. INTRODUCTION

Structural elements of space vehicle (SC) during operation are subjected to the action of a moving load and experience significant aerodynamic heating. To combat it, the surface of the space vehicle is covered with a composite protective layer, which for various reasons can be destroyed during operation. In the destruction zone, large thermal deformations occur, leading to the destruction of the bearing shell. That is what caused the death of the space vehicle US "Columbia". In the proposed work, only one of the facets of this most complex scientific and technical problem is investigated – the determination of the dynamic behavior and the intrinsic characteristics of the aircraft skin element under the action of a pressure wave. All of the above leads to the need to study non-stationary deformation of the casing of the space vehicle with a dynamically collapsing protective layer under the action of a moving load, as well as to find its own dynamic characteristics of the system. A review of works on the effect of such loads on the elements of thin-walled structures is given, for example, in [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32].

2. METHODOLOGY

The solution of the problem under consideration is based on the use of the equations of acclivous shell in a mixed form with an additional inertial layer that simulates a heat-shielding coating and the use of generalized functions. The resolving system of partial differential equations according to the Bubnov method can be reduced to a system of differential equations, but already in ordinary derivatives. Under certain conditions, this system splits into separate equations. motions of a damaged structure, for which, in turn, solutions are obtained in closed form. The lower part of the spectrum of the natural oscillations of the structure is found in accordance with the ideas of Euler, and the critical velocities of the movement of the load are found on the basis of the dynamic stability criterion. In accordance with it, the critical state of the shell corresponds to the equality to zero of one of the natural frequencies of its oscillations.

3. RESULTS AND DISCUSSIONS

Let's consider a thin rectangular cylindrical composite shell, rectangular in plan, consisting of two layers referred to the curvilinear orthogonal coordinate system *Oxyz* (Fig. 1).

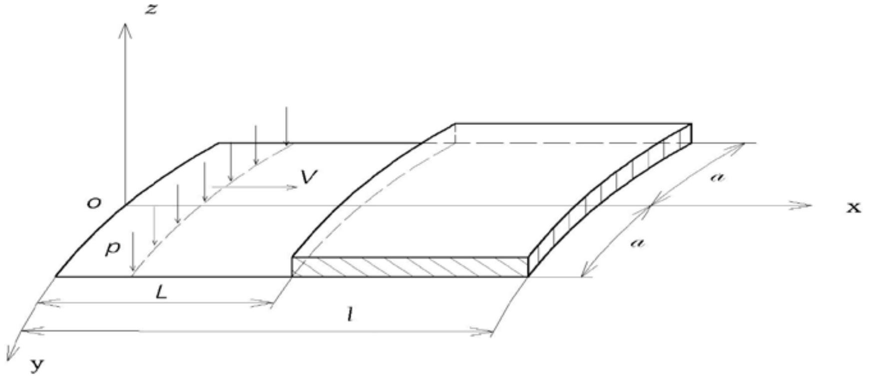

Fig. 1 – Acclivous shell with partially destroyed heat-resistant coating under the action of a moving load

Its inner layer is carrying and provides strength, and the outer – heat-shielding coating. An infinite uniformly distributed normal load of intensity p simulating a pressure wave moves along the shell in the direction of the x axis with constant velocity V. In fig. 1, it is conventionally depicted as running forces applied along the line parallel to the y axis. Under the action of this load, the heat-shielding layer is being destroyed. The front of its destruction is also parallel to the y- axis and with the speed V_0 shifts in the x direction. The entire protective layer will be destroyed in time $t_1 = l/V_0$. The surviving part of the coating in Figure 1 is shaded. The linear masses of both layers are commensurable, and the stiffness characteristics of the coating are small compared to the corresponding characteristics of the carrier layer. As a consequence, we consider the entire structure as an original two-layer composite shell, the outer coating of which is treated as an inertial layer, changing only the dynamic properties of the system as a whole. The solution of the problem is divided into two stages, at the first of which with $t < t_1$ there is a destruction of the protective layer, and at the second at $t > t_1$ it is already completely destroyed.

Of greatest interest is the solution to the first part of the problem, so we'll dwell on it. We solve the problem in a dynamic formulation, assuming the deformed state of the plate depends not only on the spatial coordinates *x* and *y*, but also on time *t*. The element of the moving load for a period of time *t* will pass the way $x = Vt$ and the vertical acceleration it develops will be the full derivative

$$\frac{d^2w}{dt^2} = \frac{\partial^2 w}{\partial t^2} + 2V\frac{\partial^2 w}{\partial x \partial t} + V^2\frac{\partial^2 w}{\partial x^2} \tag{1}$$

The second term in formula (1) contains a mixed derivative corresponding to Coriolis acceleration and is usually neglected in solving practical problems [1], [2], [3]. To describe the dynamic deformed state of the plate under consideration, we use the equations of the theory of acclivous shells in a mixed form with respect to the deflection of the shell *w* and the stress function *F* [4]. For the case in question, they take the form

$$\frac{D}{h}\nabla^4 w - \frac{1}{R}\frac{\partial^2 F}{\partial x^2} + \frac{p}{gh}V^2\frac{\partial^2 w}{\partial x^2} + \frac{p}{gh}\frac{\partial^2 w}{\partial t^2} + p\left[1 + \frac{m_1}{m}\Psi\right]\frac{\partial^2 w}{\partial t^2} = \frac{p}{h} \tag{2}$$

$$\frac{1}{E}\nabla^4 F + \frac{1}{R}\frac{\partial^2 w}{\partial x^2} = 0 \tag{3}$$

Here $D = Eh^3/12(1 - m^2)$ cylindrical rigidity of the bearing plate, R and h are the radius and thickness of the bearing shell, *E* and *m*- modulus of elasticity and Poisson's ratio of its material, m_1 and *m* are the linear masses of the protective layer and the carrier shell, respectively, g is the gravitational acceleration. Rupture function Y defines the area of damage to the thermal protection and is found by the ratio

$$\Psi = \begin{cases} 0 & x \in L \\ 1 & x \notin L \end{cases}. \tag{4}$$

For an approximate solution of equations (2-3), we use the Bubnov method, in accordance with which we represent the deflection of the shell *w* and the stress function *F* in the form of expansions

$$w = \sum_{m}^{K}\sum_{n}^{C} w_{mn}(t)\varphi_{mn}(x,y), \qquad F = \sum_{m}^{K}\sum_{n}^{C} F_{mn}(t)\phi_{mn}(x,y) \tag{5}$$

where $w_{mn}(t)$ and $F_{mn}(t)$ - unknown functions of time $\varphi_{mn}(x,y)$ and $\phi_{mn}(x,y)$ - set coordinate functions. From equation (3) we express $F_{mn}(t)$ through $w_{mn}(t)$ and substitute this expression in equation (2). Applying to it also the procedure of the Bubnov method, we obtain the system $K \times C$ second order differential equations in ordinary derivatives with respect to $w_{mn}(t)$. In the matrix form of the record, it has the form

$$M\ddot{W} + KW = P \tag{6}$$

Here *M* and *K* are square matrices of mass and rigidity of the plate, respectively, *W* and *P* are columns of unknown functions and loads. The points above the desired functions *W* here and below denote their time derivatives.

$$K = \lfloor k_{mn}^{cl} \rfloor, \qquad M = \lfloor m_{mn}^{cl} \rfloor, \qquad W = \{w_{cl}(t)\}, \qquad P = \{p_{cl}\} \tag{7}$$

The elements of the matrices *K*, *M* and the vector *P* have the form

$$k_{mn}^{cl} = \frac{D}{h}\int_S \nabla^4 \varphi_{mn} \cdot \varphi_{cl} dS + \frac{p}{gh}\int_S \varphi_{mn}'' \varphi_{cl} dS + \frac{E}{R}\frac{\int_S \varphi_{mn}'' \phi_{cl} dS \cdot \int_S \phi_{mn}'' \varphi_{cl} dS}{\int_S \nabla^4 \phi_{mn} \cdot \phi_{cl} dS} \tag{8}$$

$$m_{mn}^{cl} = \left(\frac{p}{gh} + \rho\left(1 + \frac{m_1}{m}\Psi\right)\right)\int_S \varphi_{mn} \cdot \varphi_{cl} dS\,;\; p_{cl} = \frac{p}{h}\int_S \varphi_{cl} dS \tag{9}$$

Hereinafter, the strokes above the approximating functions denote their derivatives with respect to the *x* coordinate. In equation (6) as a parameter includes the speed of movement of the load. Under given initial conditions, this system can only be solved numerically. If at some values of the velocity *V*, the deflection of the shell begins to increase indefinitely, then this means that we are approaching its critical value, at which the loss of stability of the shell occurs.

If we additionally assume that the elastic and inertial interaction between different oscillation modes are small, then system (6) splits into separate unrelated equations for each pair of *m* and *n* values.

$$\ddot{w}_{mn} - \omega_{mn}^2 w_{mn} = \frac{p_{mn}}{m_{mn}^{mn}}\ (m\ =\ 1{,}2,\ldots.K, n\ =\ 1{,}2,\ldots.C\ ), \tag{10}$$

where $w_{mn}^2 = k_{mn}^{mn}(V)\ /m_{mn}^{mn}$ - the square of the natural frequency of oscillation of the shell in shape $m \times n$ Ratios $k_{mn}^{mn}(V)$ and m_{mn}^{mn} are found by formulas (8-9) with *m* = *c* and *n* = *l*.

$$\omega_{mn}^2 = \frac{\frac{D}{h}\int_S \nabla^4 \varphi_{mn}^2 dS + \frac{p}{gh}V^2\int_S \varphi_{mn}'' \phi_{mn} dS + \frac{E}{R^2}\frac{\int_S \varphi_{mn}'' \phi_{mn} dS \cdot \int_S \phi_{mn}'' \varphi_{mn} dS}{\int_S \nabla^4 \phi_{mn} \cdot \phi_{mn} dS}}{\left(\frac{p}{gh} + \rho\left(\int_S \varphi_{mn}^2 dS + \frac{m_1}{m}\int_S \varphi_{mn}^2 \Psi dS\right)\right)} \tag{11}$$

In accordance with the dynamic stability criterion, the critical values of this velocity are in the form $m \times n$ are found from the condition $w_{mn}^2 = 0$

$$V_{KP}^2 = \frac{\frac{D}{h}\int_S \nabla^4 \varphi_{mn}^2 dS + \frac{E}{R^2}\frac{\int_S \varphi_{mn}'' \phi_{mn} dS \cdot \int_S \phi_{mn}'' \varphi_{mn} dS}{\int_S \nabla^4 \phi_{mn} \cdot \phi_{mn} dS}}{\frac{p}{gh}\int_S \phi_{mn}'' \varphi_{mn} dS} \tag{12}$$

The deflections of the shell find the sum of the solutions of equations (10) of the form

$$\begin{aligned} w &= \sum_m^K \sum_n^C w_{mn}(t)\varphi_{mn}(x,y) \\ &= \sum_m^K \sum_n^C \left(\frac{p_{mn}}{k_{mn}}(1 - cos\,\omega_{mn}\,t) + C_{1mn}\,sin\,\omega_{mn}\,t + C_{2mn}\,cos\,\omega_{mn}\,t)\right)\varphi_{mn} \end{aligned} \tag{13}$$

Here are the ratios k_{mn} and p_{mn} found by the formulas (8-9), and w_{mn} - according to (11). Constants of integration C_{1mn} and C_{2mn} are found from the initial conditions at *t = 0.*

As examples, we consider an approximate definition in the single-part approximation of the natural frequencies of the oscillations of the shell and the critical velocities of the movement of the load and using formulas (11) and (12), respectively. The approximating functions in expansions (5) are taken as

$$\varphi_{mn} = \phi_{mn} = \sin \mu_m x \cdot \cos \lambda_n y \tag{14}$$

where $m_m = \frac{mp}{l}, l_m = np/2a$. Then the squares of the natural frequencies and critical velocities can be found by the following formulas

$$\omega_{mn}^2 = \frac{\frac{D}{h}(\mu_m^2 + \lambda_n^2)^2 - \frac{p}{gh}V^2\mu_m^2 + \frac{E}{R^2}\frac{\mu_m^4}{(\mu_m^2+\lambda_n^2)^2}}{\frac{p}{gh} + p\left(1 + \frac{m_1}{m}\int_S \varphi_{mn}^2 \Psi dS\right)} \tag{15}$$

$$V_{KP}^2 = \frac{gh\left(\frac{D}{h}(\mu_m^2 + \lambda_n^2)^2 + \frac{E}{R^2}\frac{\mu_m^4}{(\mu_m^2+\lambda_n^2)^2}\right)}{p\mu_m^2} \tag{16}$$

Through $V = \Psi = 0$ formula (15) completely coincides with the corresponding expression from the reference book [33]. From the point of view of practical applications, the main oscillation frequency is the most significant, which corresponds to the simplest shell oscillation mode, which is realized at $m = n = 1$ in approximation (14), and, consequently, the minimum critical speed. Let's consider a shell characterized by the following dimensionless parameters: $R/h = 100$, $l/R = 5$, $l/a = 4$, $m_1/m = 1$. Fig. 2 shows the dependence of the square of the dimensionless frequency of oscillations. $\omega_*^2 = \omega^2\rho R^2/E$ from the dimensionless length of the destroyed layer of thermal protection L/l. Moreover, curve 1 corresponds to the case $V^2 = 0$ and curve 2 is obtained at a dimensionless rate of loading $V^{2^*} = 0{,}1 \times V_{KP}^{2^*}$ $\left(V_{KP}^{2^*} = V_{KP}^2 p/Egh\right)$. Calculations are carried out with conditionally accepted dimensionless load $p_* = p/ghp = 1$.

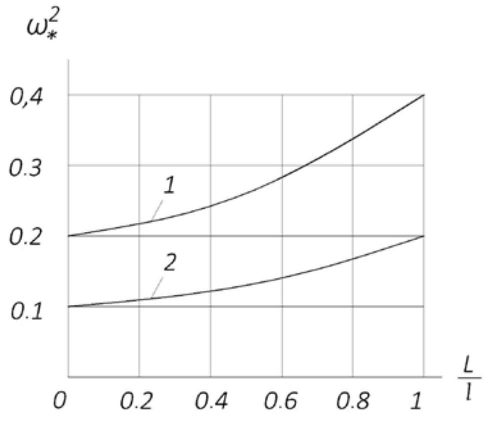

Fig. 2 – Dependence of the fundamental oscillation frequency of the shell on the size of the zone of destruction of the heat-shielding coating

Figure 3 shows the dependence of the square of the dimensionless critical velocity $V_{KP}^{2^*} = V_{KP}^2 p/Egh$ from the dimensionless length of the plate l/R while maintaining a constant ratio l/a=4.

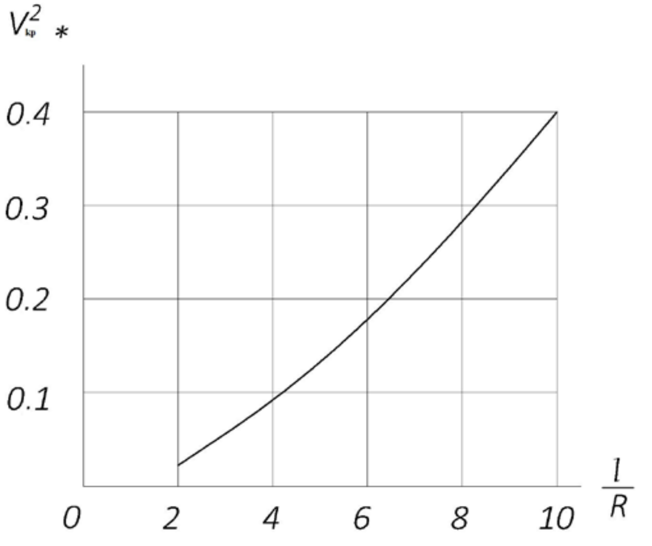

Fig. 3 – Dependence of critical velocity on shell size

4. CONCLUSIONS

The article proposes an original approach to solving the problem of dynamic behavior and finding the own dynamic characteristics of space vehicle trim elements with a dynamically destroyed heat shield. The method of solving the problem is based on the harmonic combination of the Bubnov method. and dynamic stability criteria for thin-walled structures. Formulas for the natural frequencies of oscillations and critical velocities of the motion of the load were obtained in closed form.

A number of parametric studies have been carried out, which allows to find the effect of various design parameters on the fundamental frequency of natural oscillations and critical velocities of the load of a discretely damaged shell.

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