Dynamics of a cylindrical shell with a collapsing elastic base under the action of a pressure wave

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Abstract: *In the dynamic and quasi-static statements, the issue of non-stationary deformation and stability of the solid propellant rocket engine (SPRE) was approximately solved. It is modeled by a thin, smooth cylindrical shell, inside of which, on a part of its length, there is an elastic base corresponding to a gradually burning powder charge. A pressure wave is moving along the outer surface of the body, simulated by the running load. The deformed state of the shell is considered axisymmetric and is determined on the basis of the moment theory of the shells. For diverse variants of mounting the ends of the shell in a closed form, expressions were obtained for the critical velocity of the load. Examples were considered.*

Key Words: *cylindrical shell, elastic foundation, Winkler hypothesis, moving load, dynamic and quasistatic equationtion of the issue, critical speeds.*

1. INTRODUCTION

The solid propellant rocket motor (SPRE) is structurally a thin cylindrical shell into which the powder charge is placed. When the engine runs, the powder burns out, creating an overpressure inside the shell that acts on the walls of the engine. At the same time, the zone of elevated pressure increases all the time as the charge burns out [1], [2], [3]. A pressure wave acts on the outer surface of the shell, simulated in the calculations by an axisymmetric moving radial load. Under its action, the shell may lose stability, but the inner pressure has a supporting effect on it. The proposed work is dedicated to the determination of the critical velocity of the moving load, taking into account the supporting action of the pressure of powder gases [4].

2. METHODOLOGY

The solution of the issue is based on the use of the partial differential equation of the moment axisymmetric deformation of a cylindrical shell when using the apparatus on general functions necessary to determine the boundaries of the charge burning zone. As a result, the resolving equation contains discontinuous coefficients. We considered two solutions to the issue. In both cases, the solution uses the Bubnov method, which reduces the issue either to a system of differential equations in ordinary derivatives (dynamic problem statement) or to a system of linear algebraic equations (quasistatic problem statement). The resulting systems are solved either numerically or analytically. An approximate equation for the minimum critical velocity of the load is obtained in a closed form for different variants of fastening the ends of the shell.

3. RESULTS AND DISCUSSIONS

Let's consider a thin circular cylindrical shell, inside of which a part of its length is elastic base which is being destroyed at a constant speed V_0 The inner pressure of intensity p acts on the free part of the shell .On the outer surface of the cylinder with a constant velocity V, *a* pressure wave is moving, simulated when solving a task by infinite linear radial load of intensity q [5], [6]. A similar problem, but taking into account temperature deformation, occurs when calculating the durability of the shell of a solid-propellant rocket engine [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], in which the role of an elastic base is played by a powder stick, and the inner pressure corresponds to the pressure of burnt powder gases. The time of complete destruction of the elastic base is $t_0 = L/V_0$. Figure 1 shows the state of the structure at an arbitrary time t $(0 < t < t_0)$. The part of the base that has been preserved by this time is shaded.

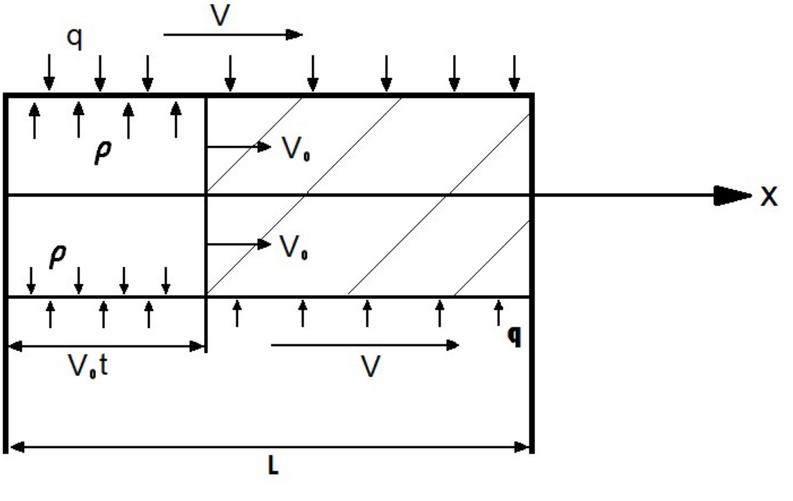

Fig. 1 – Cylindrical shell under the influence of applied loads

Under the action of the considered loads, the shell undergoes axisymmetric deformation. The main purpose of the work is to determine the critical speed of the moving load at which the loss of stability of the shell occurs.

3.1 Dynamic equation of the problem

In this equationtion of the issue, the deflection of the shell *w* depends on both the longitudinal coordinate x and the time t. During time t, the pressure wave element will travel a distance along the cylinder $x = Vt$ then the speed and acceleration of the radial displacements of the walls of the shell *w* will be full derivatives and can be determined by the relations

$$\frac{dw}{dt} = \frac{\partial w}{\partial t} + V\frac{\partial w}{\partial x}, \qquad \frac{d^2w}{dt^2} = \frac{\partial^2 w}{\partial t^2} + 2V\frac{\partial^2 w}{\partial t \partial x} + V^2\frac{\partial^2 w}{\partial x^2} \tag{1}$$

In the equation for accelerations, the second term corresponds to Coriolis acceleration and is usually neglected in solving practical issues. Pressure on the shell in the zone of the elastic base determined by the coordinate $V_0t \leq x \leq L$ we find on the basis of the Winkler hypothesis, considering it to be dependent on the deflection of the shell *w* and the stiffness coefficient of the base *b*. The scope of this load is set using the Heaviside function $H(x - V_0t)$. The zone of action of the inner pressure p is determined using the difference of the Heaviside functions given at the points $x = 0$ and $x = V_0t$. In view of the preceding, the axisymmetric deformation of a cylindrical shell is investigated on the basis of the general theory of shells. In this case, the resolving equation of the problem takes the form

$$\frac{\partial^4 w}{\partial x^4} + \frac{q}{gD}\frac{\partial^2 w}{\partial t^2} + w\left(4\beta^4 + \frac{b}{D}H(x - V_0t)\right) + \frac{q}{gD}V^2\frac{\partial^2 w}{\partial x^2} = \frac{p}{D}\left(H(x-0) - H(x - V_0t)\right) \tag{2}$$

where $D = Eh^3/12(1-\mu^2)$, $\beta^4 = 3(1 - m\mu^2)/R^2h^2$, *R* and *h* are the radius and thickness of the shell, *E* and *m* – modulus of elasticity and Poisson's ratio of its material, respectively. Equation (2) is a partial differential equation containing discontinuous coefficients with Heaviside functions defining the boundary of the zone of the elastic base. In addition, it contains as a parameter the movement velocity of the external load *V*. This equation is solved approximately by the Bubnov method, according to which we represent the shell deflections as an expansion

$$w = \sum_{i=1}^{N} w_i(t)\varphi_i(x), \tag{3}$$

where $w_i(t)$– unknown functions of time (generalized coordinates), $\varphi_i(x)$– given coordinate functions satisfying the boundary conditions at the ends of the shell. Applying the procedure of the Bubnov method to equation (2), we reduce the problem to a coupled system of second-order differential equations, but in ordinary derivatives. In the matrix entry form, it has the form

$$M\ddot{W} + KW = C, \tag{4}$$

Matrices of masses *M*, rigidity *K* and vectors *W* and *C* included in (4) are

$$M = [m_{ij}],\ K = [k_{ij}],\ \ddot{W} = \{\ddot{w}_j\},\ W = \{w_j\},\ C = \{c_j\} \tag{5}$$

Hereinafter, points above the function *w* denote its derivatives with respect to time. The dimension of the matrices and vectors is determined by the number of members of the series stored in the expansion (3), and their elements are equal.

$$k_{ij} = \left\{\int_0^L \varphi_i^{IV}\varphi_j dx + 4\beta^4\int_0^L \varphi_i\,\varphi_j dx + \frac{b}{D}\int_0^L H(x - V_0t)\,\varphi_i\varphi_j dx + \frac{qV^2}{gD}\int_0^L \varphi_i''\,\varphi_j dx\right\}$$
$$m_{ij} = \frac{q}{gD}\int_0^l \phi_i\phi_j dx \quad c_j = \frac{p}{D}\int_0^L\left(H(x-0)H(z - V_0t)\phi_j dx\right) \tag{6}$$

Coefficient k_{ij} as the parameter contains the square of the speed of the load V^2. The solution of the system of differential equations (4) under the given initial conditions is performed only numerically.

If at some values of the velocity of motion of the load, the deflections of the shell begin to increase sharply, then this means that we are entering the near-resonant mode of motion.

Analytical equations for these critical velocities can be obtained only if one or two terms of the series are preserved in the decomposition (3).

3.2 Quasistatic equation of the problem

In this simpler version of the solution of the problem under consideration, we assume that the deflection of the shell *w* depends only on the longitudinal coordinate *x*. Then the inertial load on the cylinder with the ratio $x = Vt$ is defined as $\frac{q}{g}\frac{\partial^2 w}{\partial x^2}$, and the solving equation of the problem becomes simpler

$$\frac{\partial^4 w}{\partial x^4} + w\left(4\beta^4 + \frac{b}{D}H(x - V_0 t)\right) + \frac{q}{gD}V^2\frac{\partial^2 w}{\partial x^2} = \frac{p}{D}(H(x-0) - H(x - V_0 t)) \tag{7}$$

It also has discontinuous coefficients at an unknown deflection *w* associated with the boundary of the zone of destruction of the elastic layer. To solve it, we also use the Bubnov method, according to which we represent the deflection of the shell *w* as an expansion

$$w = \sum_{i=1}^{N} w_i \varphi_i(x), \tag{8}$$

where w_i – unknown coefficients, and $\varphi_i(x)$ – given coordinate functions satisfying the boundary conditions at the ends of the shell. Substituting the expansion (8) into equation (7) and applying to the latter the procedure of the Bubnov method, we obtain a connected system of *N* linear algebraic equations for the unknown coefficients w_i (*i = 1,2, ..N*). In the matrix entry form, it has the form

$$KW = C, \tag{9}$$

The stiffness matrix *K* and vectors *W* and *C* included in (9) are

$$K = \lfloor k_{ij}(V) \rfloor, W = \{w_j\}, C = \{c_j\} \tag{10}$$

Their dimension is determined by the number of members of the series stored in the expansion (8), and the elements are determined by equations (6). Coefficient $k_{ij}(V)$ as a parameter contains the speed of movement of the load. The solution of system (9) in high approximations is possible only numerically. If at some values of velocity *V*, the deflection of the shell begins to increase sharply, this also means that we are approaching near the resonant velocity. Equations in closed form for critical speeds V_{KP} can also be obtained only in one or two-term approximations.

3.3 Example

Practically important minimum critical speed corresponds, as a rule, to the simplest form of loss of shell stability.

Therefore, we solve the problem, for example, in the dynamic equationtion in the single-term approximation at $w = w_1(t)\varphi_1(x)$. Then instead of the system of equations (4) we get only one equation of the form

$$\ddot{w}_1 + \omega_1^2 w_1 = c_1/m_{11} \tag{11}$$

where $\omega_1^2 = k_{11}/m_{11}$ – is the square of the frequency of natural oscillations of the structure, its coefficients are calculated by the equations (6) with $i = j = 1$. The solution of equation (11) determining the shape of the curved surface of the shell is

$$w_1 = A\, sin\omega_1 t + B\, cos\, \omega_1 t + \frac{c_1}{k_{11}} \tag{12}$$

The integration constants A and B included in this solution are from the initial conditions.

Based on the dynamic stability criterion, the square of the critical velocity V_{KP}^2 at an arbitrary moment of time t can be found from the condition that the natural frequency of oscillation is zero $\omega_1^2 = 0$ which is converted to form $k_{11} = 0$

$$V_{KP}^2 = -\frac{gD}{q\int_0^L \varphi_1''\varphi_1 dx}\left[\int_0^L \varphi_1^{IV}\varphi_1 dx + 4\beta^4\int_0^L \varphi_1^2 dx + \frac{b}{D}\int_0^L H(x - V_0 t)\varphi_1^2 dx\right] \tag{13}$$

The minus sign in front of equation (14) is canceled as a result of specific calculations. Let's suppose that both ends of the shell at $x = 0$ and $x = L$ be freely supported. In this case, we accept $\varphi_1 = \sin(\pi x/L)$. Then, on the basis of equation (13), we obtain the square of the critical velocity in the form

$$V_{KP}^2 = \frac{2gDL}{q\pi^2}\left[\frac{\pi^4}{2L^3} + 2\beta^4 L + \frac{b}{D}\left\{\frac{L - V_0 t}{2} + \frac{L}{4\pi} sin\frac{2\pi V_0 t}{L}\right\}\right] \tag{14}$$

If we assume that the ends of the shell are rigidly clamped, then the approximating function can be taken in the form $\varphi_1 = 1 - \cos(2\pi x/L)$. Then the square of the critical velocity is

$$V_{KP}^2 = \frac{gDL}{q2\pi^2}\left[\frac{8\pi^4}{L^3} + 6\beta^4 L + \frac{b}{D}\left\{\frac{3(L - V_0 t)}{2} - \frac{L}{8\pi} sin\frac{2\pi V_0 t}{L}\right\}\right] \tag{15}$$

In both equations, the critical velocity nonlinearly depends on the magnitude of the destruction of the inner elastic layer, characterized by the parameter $V_0 t$. In addition, equations (14) and (15) have the same structure and if we compare them by terms we will see that in the case of rigid pinching of the shell ends, the critical speed is higher than in the case of their free support.

4. CONCLUSIONS

The problem of dynamic deformation and stability of a cylindrical shell under the action of an axisymmetric linear load moving on its outer side is solved. The shell on the part of its length has an inner elastic base conforming to Winkler's hypothesis. An inner pressure increases the critical loads on the part of the cylinder that is free from the elastic base. The problem is solved in both dynamic and quasistatic approximations. Equations were obtained for the minimum critical velocity of the moving load under various settings for fixing the ends of the shell. The supporting effect of inner pressure on critical loads was analyzed. It is shown that with free support of the ends of the shell, the loss of stability occurs at a lower speed of movement of the load than in the case of their rigid fixation.

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