Quasi-static stability of a ribbed shell interacting with moving load

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Abstract: *Loss of stability of thin-walled structural elements of aircraft is the most common type of destruction. In this work, the definition of the critical velocity of a pressure wave moving over the surface of an acclivous, ribbed cylindrical shell, which is the most common design element of all aviation and rocket systems, is considered. The article takes into account the discreteness of the location of stiffeners (stringers), in contrast to other works, where ribbing is taken into account only within the framework of a constructive orthotropic model. In the quasi-static formulation, the problem of the stability of a shallow shell with a discrete arrangement of stringers under the action of a moving radial load has been solved. The range of critical speeds of movement of the load is defined. An example is considered.*

Key Words: *flat shell, discrete location of the ribs, mobile radial load, quasi-static solution, critical speed, loss of stability.*

1. INTRODUCTION

Discretely supported thin shells are the main structural element of all aircraft without exception. During their operation, the aircraft lining under the action of loads of different nature can lose stability. To combat this phenomenon, thin cladding shells are supported by discretely standing force elements such as stringers, which leads to an increase in critical loads and ultimately to an extension of the service life of the aircraft. A peculiar and specific type of loading of an aircraft is the action of a pressure wave, which is usually simulated by moving loads. In the proposed work, unlike in others, the problem is solved by taking into account the discreteness of the location of the stringers and analyzing effect of these elements on the critical speed of the moving load for various forms of loss of stability.

2. METHODOLOGY

The proposed method of solution is based on the use of generalized functions and the quasi-static hypothesis of deformation of the structure, according to which the perturbed state of the shell depends only on the spatial coordinates and does not depend on time. The main idea of the solution is the harmonic combination of the Bubnov method, which reduces the solution of differential equations to solving linear algebraic ones, with the approach of Leonard Euler to finding the eigenvalues of the problem, identified in this case with critical velocities of the load. Thanks to this approach, it was possible to obtain formulas for critical velocities in a closed form.

3. RESULTS

Consider a gentle panel of a cylindrical shell, referred to a curved orthogonal coordinate system *Oxyz*, in which a row of elastic one-dimensional reinforcing elements is located parallel to the *x* axis. Figure 1 conditionally shows only one stringer.

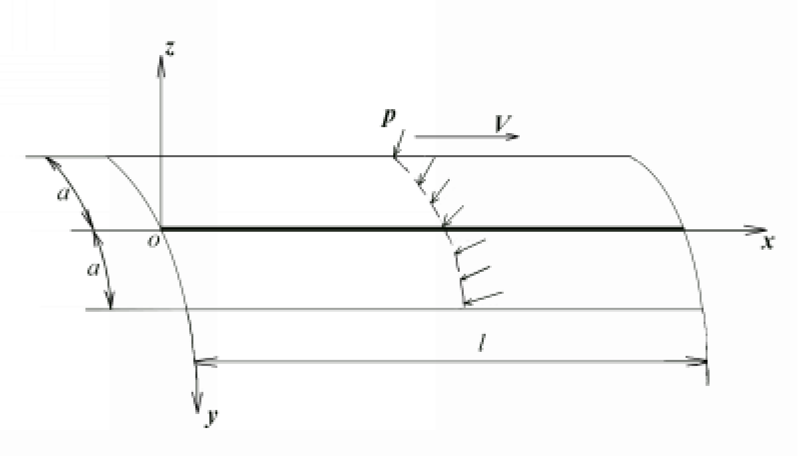

Figure 1 – Panel of discretely supported cylindrical shell under the action of a moving load

In this figure, the infinite uniform inertial load of intensity *p* moving at constant speed *V is* conventionally shown as linear forces distributed along a line perpendicular to the *x* axis. In solving the problem, we neglect the mass and inertia of motion of the actual elements of the structure. We consider its deformed state to be quasi-static, in which the curved surface of the shell *w(x, y)* does not depend on time. The moving load element for the time interval *t* will pass the path *x = Vt*, and the total gravitational and inertial load developed by it in the normal to the shell surface direction is defined by the formula [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28]:

$$p-\frac{p}{g}\frac{\partial^2 w}{\partial t^2}=p-\frac{p}{g}V^2\frac{\partial^2 w}{\partial x^2} \tag{1}$$

To solve the problem, we mentally separate the stringers from the shell and replace their effect with unknown contact reactions normal to its surface. $q_i(x)$ distributed along the line of contact of bodies in the middle surface of the panel.

Since the rigidity of the shell in the tangential directions is much greater than in the radial, then the tangential components of the forces of interaction in the direction of the *x* and *y* axes are further neglected. To describe the dynamic deformed state of the shell, we use its mixed-form equations for the radial displacement (deflection) *w* and stress functions *F* based on hypotheses of the technical theory of shells [29]. In the considered case of quasi-static deformation of the shell, taking into account the action of contact reactions $q_i(x)$ they take the form

$$\frac{D}{h}\nabla^2\nabla^2 w - \frac{1}{R}\frac{\partial^2 F}{\partial x^2} = \frac{p}{h} - \frac{p}{gh}V^2\frac{\partial^2 w}{\partial x^2} - \sum_{i=1}^{C} q_i\delta(y - y_i) \tag{2}$$

$$\frac{1}{E}\nabla^2\nabla^2 F + \frac{1}{R}\frac{\partial^2 w}{\partial x^2} = 0 \tag{3}$$

where ∇^2 is the Laplace operator, D and h are the cylindrical rigidity and shell thickness, respectively, g is the gravitational acceleration, C is the number of stringers, and the delta is the Dirac functions $\delta(y - y_i)$ that set the coordinates of their location y_i along the y axis. Each of the stringers is referred to the rectangular coordinate system *0xz* (Figure 1). The equation of its equilibrium in the projection on the z axis has the form:

$$EJ_i\frac{d^4 z_i}{dx^4} = q_i \ (i = 1,2, \dots\ C\ ) \tag{4}$$

where EJ_i and z_i are their flexural rigidity and deflection of stringers, respectively. On each of the contact lines, the condition of equal deflection of both bodies is satisfied. $w(x, y_i) = z_i$, but at the same time we believe that the deformed state of the shell caused by neighboring contact reactions do not interfere with each other. Then substituting contact reactions q_i from (4) to the equation of bending of the shell (1) we obtain the resolving system of equations

$$\frac{D}{h}\nabla^2\nabla^2 w - \frac{1}{R}\frac{\partial^2 F}{\partial x^2} + \frac{p}{gh}V^2\frac{\partial^2 w}{\partial x^2} + \sum_{i}^{C}\delta(y - y_i)(EJ_i\frac{\partial^4 w}{\partial x^4}) = \frac{p}{h}, \tag{5}$$

$$\frac{1}{E}\nabla^2\nabla^2 F + \frac{1}{R}\frac{\partial^2 w}{\partial x^2} = 0 \tag{6}$$

Equations (5, 6) are a system of two partial differential equations with coefficients that are discontinuous in the y coordinate, which is due to the presence in the equation (5) of the Dirac delta functions defining the position of stringers on the shell surface. In addition to these equations, the speed of movement of the load V is included as a parameter. For their approximate solution, we use the Bubnov method, in accordance with which we represent the deflection of the shell w and the stress function F as expansions

$$w = \sum_{m}^{K}\sum_{n}^{L} w_{mn}\phi_{mn}(x, y)\ , \quad F = \sum_{m}^{K}\sum_{n}^{L} F_{mn}\psi_{mn}(x, y) \tag{7}$$

where w_{mn} and F_{mn} – unknown coefficients, $\phi_{mn}(x, y)$ and $\psi_{mn}(x, y)$ – orthogonal forms of natural oscillations of a smooth shell panel. Applying the procedure of the Bubnov method to equations (4), we obtain a system of related linear algebraic equations of order $2 \times K \times L$ relative to unknown coefficients w_{mn} and F_{mn} in expansions (7). However, algebraic equations corresponding to the second of relations (5, 6), due to the orthogonality of the coordinate functions in expansions (7), decompose into separate independent equations for each pair of values m and n. Let's express F_{mn} through w_{mn}. Taking this into account, after a series of transformations, the first equations (5, 6) in the matrix form of the record take the form

$$\big[[K] - V^2[M]\big]\{W\} = \{P\} \tag{8}$$

where

$$K = [k_{mn}^{kl}], \quad M = [m_{mn}^{kl}], \quad W = \{w_{kl}\}, \quad P = \{p_{kl}\}. \tag{9}$$

The dimension of the stiffness matrices *K*, masses *M* and the vectors *W* and *P* is determined by the number of terms of the series stored in the decomposition (9). The elements of the matrices *K*, *M* and the vector *F* have the form

$$k_{mn}^{kl} = \frac{D}{h}\int_S \nabla^2\nabla^2\varphi_{mn}\cdot\varphi_{kl}dS + \frac{E}{R^2}\frac{\int_S \frac{\partial^2\varphi_{mn}}{\partial x^2}\psi_{kl}dS \cdot \int_S \frac{\partial^2\psi_{mn}}{\partial x^2}\varphi_{kl}dS}{\int_S \nabla^2\nabla^2\psi_{mn}\cdot\psi_{mn}dS} + \sum_{i=1}^{C} EJ_i\int_S \delta(y-y_i)\frac{\partial^4\varphi_{mn}}{\partial x^4}\varphi_{kl}dS, \tag{10}$$

$$m_{mn}^{kl} = \frac{p}{g}\left|\int_S \frac{\partial^2\varphi_{mn}}{\partial x^2}\varphi_{kl}dS\right| \tag{11}$$

$$p_{kl} = \frac{p}{hg}\int_S \varphi_{kl}dS \tag{12}$$

The stiffness matrix *K* has a block diagonal character, since the Dirac delta functions are located only on the *y* axis. In addition, in each of its blocks, the first two terms in the stiffness coefficient k_{mn}^{kl} and all the mass coefficients are diagonal. For a given load velocity *V*, the solution of the coupled system of equations (8, 9) is carried out using conventional methods. If at some values of the velocity *V*, the deflection of the shell begins to increase indefinitely, then this means that we are approaching its critical value, at which a loss of stability of the ribbed panel occurs. If we additionally assume that the gravitational component of the load *p* is small compared to the inertial one, then the lower part of the spectrum of critical velocities can be found from the condition of equality to zero of the determinant of the system of equations (8, 9)

$$\det|K - V^2M| = 0 \tag{13}$$

Example. Let the shell be supported by one stringer along the *y* axis $(y_1 = 0)$ (Figure 1). In the first approximation, we use the simplified approach based on the one-term approximation in series (7) for arbitrary wave numbers *m* and *n*. We believe that all the edges of the shell panel have free support on flexible inextensible ribs, and they can deform about the *y* axis (Figure 1) either symmetrically or anti-symmetrically. In the case of symmetric deformation, the approximating function in expansions (7) is taken in the form (14a), and in the case of antisymmetric, in the form (14b)

$$\varphi_{mn} = \psi_{mn} = \sin\mu_m x \cos\lambda_n y \tag{14a}$$

$$\varphi_{mn} = \psi_{mn} = \sin\mu_m x \sin\lambda_n y \tag{14b}$$

where $\mu_m = m\pi/l$, $\lambda_n = n\pi/ka$, (*k = 2* or *k = 1* for symmetric or antisymmetric deformation of the shell, respectively).In both cases, the square of the critical velocity of the movement of the form $m \times n$ is determined by formula (15) obtained in accordance with condition (13).

$$V_{KP}^2 = \frac{g}{p\mu_m^2}\left[D(\mu_m^2+\lambda_n^2)^2 + \frac{Eh}{R^2}\frac{\mu_m^4}{(\mu_m^2+\lambda_n^2)^2} + \frac{2}{a}EJ\mu_m^4\right] \tag{15}$$

The first term from formula (15) is corresponding to the critical velocity for a smooth plate completely coincides with the velocity value from the book [30]. From the point of view of practical applications, the minimum critical speed is most important, which is probably realized with the simplest form of loss of stability, $m = n = 1$ in formula (15).The shell has the following dimensionless parameters: $l/R = 2,\ a/R = 0.5,\ \nu = 0.3$. Figure 2 shows the dependence of the dimensionless critical velocity of the load. $V_{KP}^{2*} = V_{KP}^{2} p/ghE$ on the relative thickness of the shell R/h with dimensionless moment of inertia stringer $J/h^4 = 10^3$. Curve 1 corresponds to symmetric deformations of the panel, and curve 2 – antisymmetric ones. The dashed line shows dependency V_{KP}^{2*} for a smooth shell without a stringer in case of symmetric deformation of the panel. Curves 1 and 2 confirm the fact that critical loads with an antisymmetric form of loss of stability are lower than with symmetric ones, and the presence of even one stringer dramatically increases the critical speed of the load.

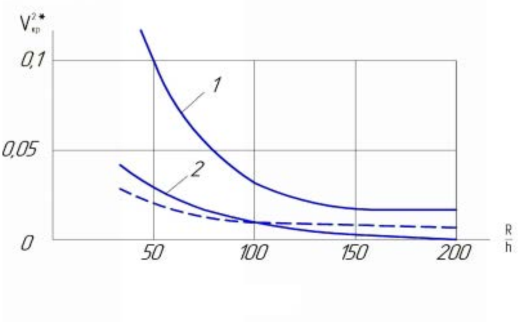

Figure 2 – The dependence of the critical velocity on the shell thickness

4. CONCLUSIONS

The work proposes an original method for determining the critical velocity of a pressure wave moving over the surface of a acclivous panel of an aircraft discretely supported by a stringer system. Formulas for determining the critical velocities for different (symmetric and antisymmetric) forms of loss of stability were obtained, and parametric studies were carried out. The obtained formulas make it possible to find the critical loads, both for the lowest and for the highest forms of loss of stability. The influence of the number and bending stiffness of stringers on the critical velocities was analyzed. It was shown that with decreasing shell thickness, critical speeds drop sharply, and the antisymmetric form of buckling is more dangerous than symmetric one.

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