On the Conditional Probabilities in PRA

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Abstract: *Mathematical modeling of existing hazards and risks in a system such as aviation starts from identifying and evaluating significant numerical parameters by different procedures. The chosen criteria and the expression of the optimized model for risk analysis process will show the efficiency of the modality to evaluate risk and continuous management. An important result achieved will be to ensure compliance with the quality requirements and levels of safety.*
Quantitative description of risk, knowing the particularities of the system, the study of interdependencies, restrictions and limitations, create a complete picture of the analyzed elements. The mathematical model thus defined can be complex, difficult to analyze, but the results will present an accurate picture of the addressed problem, removing information and data that are insignificant or redundant.

Key Words: *Probabilistic risk assessment, conditional probabilities, Bayes theorem, UAV.*

1. INTRODUCTION

Risk analysis tools are modeling the sequence of events depending on scenarios that can be achieved. Each such scenario is assigned a fixed probability for production (or scenario can be described by a random variable). In this context, the greatest challenge is the integration of the characteristics that describe a versatile and complex assessment and modeling environment that can analyze various scenarios and probabilities.

Information collected through the identification and classification of risks will be available for the calculations. Actual methods are structured to calculate the frequency of occurrence of the risk, barriers efficiency and, in the second part of the study, to assess the accident severity. Random processes modeled using PRA techniques involve the use of probabilistic models. [8]

The concept of probability is related to the definition of risk. It must therefore be established the distinction between probability and frequency; the probability is founded on experiments based on real (and proven) facts. [9]

2. PROBABILISTIC RISK MODELLING

The representation in a general, standard form of the algorithms for risk analysis may involve ignoring some essential aspects; therefore the differences in terms of risk classes must be treated with special attention and particular elements must be highlighted. In this manner, firstly we can talk about effective methods of evaluation by identifying special properties and secondly, about an easier way to solve the problems.

An analyzed event can be represented by the occurrence of a failure after a certain time of operation and can have an independent way and probability of occurrence. Depending on a large number of variables, these can be more or less probable to happen in comparison to other events.

The probability is an indicator of the completion of a number of conditions according to a certain criterion. The probability involves assigning a number that indicates the possibility of achievement of each event that can be classified as:

- Certain (E) - it will happen in a mandatory manner,

$$P(E) = 1$$

- Impossible (Ø) – it will certainly not happen,

$$P(\Phi) = 0$$

- Or it can be a random event (A_n) that can or cannot happen.

In the probability theory, systematization targets the events compatibility and follows independent/ dependent events whose occurrence will (or will not) influence the probability of another event.

For two events that are considered compatible, the reunion probability will be calculated as follows:

$$P(A) = P\ (A_1 \cup A_2) = P(A_1) + P\ (A_2) - P\ (A_1 \cap A_2)$$

In the case of "n" events, the relation has the following form (the Poincaré relation):

$$P(A) = P\left(\bigcup_{i=1}^{n}(A_i)\right) =$$

$$= \sum_{i=1}^{n} P(A_i) - \sum_{j=2}^{n}\sum_{i=1}^{j-1} P\left(A_i \cap A_j\right) + \sum_{k=3}^{n}\sum_{j=2}^{k-1}\sum_{i=1}^{j-1} P\left(A_i \cap A_j \cap A_k\right) + (-1)^{n-1} P\left(\bigcap_{i=1}^{n}(A_i)\right)$$

For compatible dependent events, the intersection probability is calculated using the conditional probabilities $P\ (A_2|A_1)$. [2]

$$P\ (A_1 \cap A_2)\ =\ P(A_1)\ \cdot\ P\ (A_2|A_1) \text{ or}$$

$$P\ (A_2|A_1) = \frac{P\ (A_1 \cap A_2)}{P\ (A_1)}$$

For "n" events:

$$A = \bigcap_{i=1}^{n}(\ A_i)$$

And the attached probability:

$$P(A) = P\left(\bigcap_{i=1}^{n}(\ A_i)\right) = P(A_1) \cdot P\ (A_2|A_1) \cdot P\ (A_2|A_1 \cap A_2) \cdots P\ (A_n|\bigcap_{i=1}^{n-1}(\ A_i)$$

For independent events, the probability of an event is not influenced (of the fulfilment or unfulfilment) of another event. [2]

The probability of meeting incompatible events, has the general form:

$$P(A) = P\left(\bigcup_{i=1}^{n}(A_i)\right) = \sum_{i=1}^{n} P(A_i)$$

And the intersection: $A_1 \cap A_2 = \emptyset$

If the case of independent compatible events, the probability of the intersection is:

$$P\,(A_1 \cap A_2) = P(A_1) \cdot P\,(A_2)$$

For "*n*" events, the function will be:

$$A = \bigcap_{i=1}^{n}(\,A_i)$$

and the probability attached:

$$P(A) = P\left(\bigcap_{i=1}^{n}(\,A_i)\right) = \prod_{i=1}^{n} P(A_i)$$

If the following conditions are met, then we can say that 3 events $A_1,\ A_2,\ A_3$ are independent.

$$\begin{cases} P\,(A_1 \cap A_2) = P\,(A_1)P\,(A_2) \\ P\,(A_1 \cap A_3) = P\,(A_1)P\,(A_3) \\ P\,(A_2 \cap A_3) = P\,(A_2)P\,(A_3) \\ P\,(A_1 \cap A_2 \cap A_3) = P\,(A_1)P\,(A_2)P\,(A_3) \end{cases}$$

Structural probability is based on correct outcomes of experiments, it requires no additional measurements and depends on the type and characteristics of the analyzed problem.

Starting from the study of single events, indirect methods used in probability theory examine complex events using already known probabilities of the events that required a direct analysis.

Firstly, the choice of calculation methods needs to be representative and adapted for the analyzed process, and secondly, the optimization of the results is valuable (establishing the optimal decisions), after setting the admissibility domain and directions, and also the restrictions, if any.

3. ANALYSIS OF MARGINAL AND CONDITIONAL PROBABILITIES THROUGH BAYES THEOREM

Probabilistic Risk Assessment models are founded on the principle of decomposion (the process is evaluated in detail by its subsystems) and the principle of Bayesian probabilistic analysis based on the concept of conditional probability, which states that the probability is proportional to the probability following prior and current data; therefore, the calculation of the following probability is done by allocating data / new values to previous information. [1]

Building Bayesian analysis starts from the definition of conditional probability:

$$P(A_1|A_2) = \frac{\frac{m}{n}}{\frac{p}{n}} = \frac{P\ (A_1 \cap A_2)}{P\ (A_2)}$$

m, p - random events
n - elementary equally probable events

$$P(A_1|A_2) = \frac{P\ (A_1 \cap A_2)}{P\ (A_2)} = \frac{P\ (A_1, A_2)}{P\ (A_2)}$$

$$P\ (A_2) \neq 0$$

The a priori probability (A_2) is being evaluated prior to any calculation or experiment.

In order to outline the general form of the multiplication rule for "n" conditional events, we will consider:

$A_1, A_2, A_3 \ \dots \ , A_n$ with the probabilities: $P\ (A_1), P\ (A_2), P\ (A_3), \dots, P\ (A_n)$

$$\text{Considering: } \begin{cases} P\ (A_1) \neq 0 \\ P\ (A_1 \cap A_2) \neq 0 \\ \vdots \\ P\ (A_1 \cap A_2 \cap A_3 \cap \dots \cap A_{n-1}) \neq 0 \end{cases}$$

Then, the general form will be:

$$P\ (A_1 \cap A_2 \cap A_3 \cap \dots \cap A_n) = P\ (A_1) \cdot P\ (A_2|A_1) \cdot P\ (A_3|A_1 \cap A_2) \cdot \dots \cdot P\ (A_n|A_1 \cap A_2 \cap \dots \cap A_{n-1})$$

Bayes' theorem allows calculating the probability of various scenarios, quantifying both the likelihood and consequences of an event [7]:

$$P(A_1|A_2) = \frac{P\ (A_2|A_1) \cdot P\ (A_1)}{P\ (A_2)}$$

After outlining the shape of the relationship above, for the two events A_1 and A_2 , the first conclusion we can note is that the probability A_1 to occur if the event A_2 has already occurred is distinct from the probability of A_2 to produce conditioned by A_1, being necessary the imposition of other different elements, such as marginal probabilities.

This theorem is a powerful mathematical tool, with applications in diverse areas, using a process designated for improving the precision of results, depending on the accuracy of existing data (previous distribution).

Bayes' formula can be determined based on the following types of probabilities:

$P\ (A_i)$ – marginal probabilities
$P(A|A_i)$ – conditional probabilities
$P(A_i|A)$ – conditional probabilities

The events A_i are mutually exclusive (any two events from the set cannot happen simultaneously) and exhaustive (the events from the set describe all the states of the system).

A_i – the causes of occuring the event A (unknown)

May $(A_i)_{i \in I} \subset \Sigma$ be a complete system of events, $\forall\ i \in I$

The general form of Bayes' formula is:

$$P(A_i|A) = \frac{P\ (A|A_i) \cdot P\ (A_i)}{\sum_{i \in I} P\ (A|A_i) \cdot P\ (A_i)}$$

Bayes' theorem outlines the hypothesis according to which the prior probability of an event provides a computing base for determining the probability of an analyzed event. [10]

Modern technology interaction with natural phenomena and factors, influences the degree of risk, whose awareness, understanding and interpretation are essential for decisions to be taken to restore the previous situation and risk treatment.

4. APPLICATIONS ON THE THEOREM OF HYPOTESES

The anticipation and recognition of errors (latent or current problems) of a system is essential to developing a functional strategy for risk treatment; then, risk hierarchy and the way to prioritize resources and actions are considered.

Probabilistic risk assessment aims the identification and assessment of specific risks in complex technological systems, so PRA techniques are structured on improving safety and system performance. [4]

Risk assessment is the most difficult part of the process of risk management, as involves determining the severity of the consequences of a hazard; the lack of information on the probability represents a critical shortcoming. Expected values of risk and the imposed limits are calculated by complex and rigorous methods based on quantification of the involved factors.

Application 1

Starting from the study of conditional probabilities, an adapted mathematical model for the study of probabilistic risk of takeoff in bad weather conditions (analyzing a decision of flight in adverse conditions) would be:

Considering the moment of takeoff of an experimental airplane remotely piloted (an UAV) with advanced technological features as light weighted and performant avionics, capable of flying at high altitudes, we can analyze two events (meteorological factors): icing (A_1) and side wind (A_2), and assume that their probabilities are:

The probability of takeoff on side wind conditions:

$$P(A_2\ ) = 10^{-\mathbf{2}}$$

And the probability that at takeoff, icing may be conditioned by the side wind is (therefore, the probability that in winter, icing can occur in side wind conditions):

$$P\ (A_1|A_2) = 3 \cdot 10^{-\mathbf{2}}$$

Then, choosing randomly a winter day for the UAV's takeoff, the probability of both weather conditions (side wind and frost) to be realized, is:

$$P\ (A_1 \cap A_2) = \ P\ (A_2) \cdot P(A_1|A_2)$$

Therefore, the conditional probability $P(A_1|A_2)$ offers the opportunity to formulate the probability for the intersection of the events marked with A_1 and A_2, icing and side wind, respectively.

$$P\ (A_1 \cap A_2) = \ 10^{-\mathbf{2}} * 3 \cdot 10^{-\mathbf{2}} = 3 \cdot 10^{-\mathbf{4}}$$

Application 2

In the following lines, as well as for the first example, a study on the remotely piloted planes will be considered. The UAV uses efficient photovoltaic cells and batteries, with high yield, disposed on the upper surface of the wing.

It is known that batteries for photovoltaic systems have a limited life of service, and their efficiency decreases depending on the cycles of loading / unloading and temperature (although this example targets a UAV for flight at high altitudes, which has a temperature control system).

The destructive action of this factor is well known; in addition, extra use and aging causes gradual exhaustion. The result is the need to replace the batteries.

Studies realized in time on the batteries exposed to overheating, have shown that in 10^6 cases, defects were found in a number of 6.948 batteries.

In this context, the absolute risk of failure of a battery in the exposed conditions (i.e. overheating) can be determined using the formula:

$$P(A_2|A_1) = \frac{P\ (A_1 \cap A_2)}{P\ (A_1)} = \frac{P\ (A_1, A_2)}{P\ (A_1)}$$

The following events will be note as:

A_1- overheating and

A_2 - battery failure

$$P(A_2|A_1) = \frac{P\ (A_1 \cap A_2)}{P\ (A_1)} = \frac{6.948}{1000000} = 6.948 \cdot 10^{-6}$$

Therefore, the absolute risk of failure of a battery in the given conditions has the value: $6.948 \cdot 10^{-6}$. Calculation of total probability and risk control through redundancy, namely by multiplying safety systems allowed the opening, understanding and efficiency of important key issues, thus putting into question a proactive approach to risk management.

From the perspective of systems complexity, the risk assessment will never be complete and this objective can not be achieved 100%, but existing probabilistic methods can improve the current rate.

5. CONCLUSIONS

Risk assessment approach is supported by rigorous mathematical models of quantitative analysis, but mostly by a qualitative approach of the specific variables, through existing data and analytical aspects and their relationship.

Methods built on connections between the constituent elements (variables, constants, parameters, symbols) aim to outline a clear image on a phenomenon / process or a class of processes.

Special attention should be directed on generalization produced by the methods (even if validated) on a broad category of systems, moreover in the case of the variety of issues encountered in aviation.

Minimizing risks, and therefore the consequences and losses, involves reducing the vulnerability of systems as a whole, especially regarding the human factor. The intervention on the ability to adapt to factors involved in system operation, especially on those that show functioning variations as against the design phase, can be a response to certain external conditions.

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