Numerical illustration of the global and local stability and instability of the constant spatially developing 2D gas flow

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***Abstract**: In this paper different types of stabilities (global, local) with respect to instantaneous perturbations and permanent source produced time harmonic perturbations are numerically illustrated in case of a constant spatially developing 2D gas flow. Some types of instabilities (global absolute, local convective) are also illustrated. For this purpose the 2D Euler equations linearized at the constant gas flow are used. It is illustrated for instance, that the constant gas flow is global absolutely unstable with respect to some instantaneous and some permanent source produced time harmonic perturbations. The locally convective instability is also illustrated with respect to some instantaneous and some permanent source produced time harmonic perturbations.*

***Key Words:** global/ local stability/ instability; spatially developing gas flow*

1. INTRODUCTION

In the 2D gas flow model, the nonlinear Euler equations governing the flow of an inviscid, compressible, non heat conducting, isentropic, perfect gas are [1]:

$$\begin{gathered}
\frac{\partial v_x}{\partial t} + v_x \cdot \frac{\partial v_x}{\partial x} + v_y \cdot \frac{\partial v_x}{\partial y} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0 \\
\frac{\partial v_y}{\partial t} + v_x \cdot \frac{\partial v_y}{\partial x} + v_y \cdot \frac{\partial v_y}{\partial y} + \frac{1}{\rho}\frac{\partial p}{\partial y} = 0 \qquad (1) \\
\frac{\partial \rho}{\partial t} + v_x \cdot \frac{\partial \rho}{\partial x} + v_y \cdot \frac{\partial \rho}{\partial y} + \rho \cdot \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) = 0
\end{gathered}$$

Here: t - time, v_x, v_y - velocity components along the Ox, Oy axis respectively;

p - pressure, ρ - density.
The pressure p, the density ρ and the absolute temperature T satisfy the state equation of the perfect gas

$$p = \rho \cdot R \cdot T \tag{2}$$

where the specific gas constant $R = c_p - c_v$; c_p and c_v being the specific heat capacities at constant pressure and constant volume, respectively.
If $v_x = U_0 = const > 0$; $v_y = 0$; $\rho = \rho_0 = const > 0$; $p = p_0 = const > 0$, then according to (2): $p_0 = \rho_0 \cdot R \cdot T_0$ and the associated isentropic sound speed c_0 verifies $c_0{}^2 = \frac{p_0}{\rho_0} = R \cdot T_0$. Linearizing (1) at $v_x = U_0$; $v_y = 0$; $p = p_0$; $\rho = \rho_0$ and using that the perturbations p', ρ' satisfy:

$$\left(\frac{\partial}{\partial t} + U_0 \cdot \frac{\partial}{\partial x}\right)\left(p' - c_0{}^2 \cdot \rho'\right) = 0 \tag{3}$$

the following system of linear Euler equations is obtained:

$$\begin{gathered}
\frac{\partial v'_x}{\partial t} + U_0 \cdot \frac{\partial v'_x}{\partial x} + \frac{1}{\rho_0} \cdot \frac{\partial p'}{\partial x} = 0 \\
\frac{\partial v'_y}{\partial t} + U_0 \cdot \frac{\partial v'_y}{\partial x} + \frac{1}{\rho_0} \cdot \frac{\partial p'}{\partial y} = 0 \\
\frac{\partial p'}{\partial t} + U_0 \cdot \frac{\partial p'}{\partial x} + \rho_0 \cdot c_0^2 \left(\frac{\partial v'_x}{\partial x} + \frac{\partial v'_y}{\partial y}\right) = 0
\end{gathered} \tag{4}$$

In [2] the Lyapunov stability, with respect to instantaneous perturbations and source produced permanent time harmonic perturbations, of the null solution of the equation (4) is investigated in a particular infinite dimensional phase space.
Following [2], for a given set $\mathcal{I} = \{I\}$ of initial values $I = (F, G, P)$ the solution of the initial value problem (4), (5)

$$v_x'(x, y, 0) = F;\; v_y'(x, y, 0) = G;\; p'(x, y, 0) = P \tag{5}$$

equal to zero for $t < 0$ is called ***instantaneous perturbation propagation problem.***
Beside the instantaneous perturbation propagation problem (4), (5) in [1] ***the source produced permanent time harmonic perturbation propagation problem*** is also considered. That is the solution of the equation

$$\begin{gathered}
\frac{\partial v'_y}{\partial t} + U_0 \cdot \frac{\partial v'_y}{\partial x} + \frac{1}{\rho_0} \cdot \frac{\partial p'}{\partial y} = h(t) \cdot G \cdot \sin \omega_f t \\
\frac{\partial v'_y}{\partial t} + U_0 \cdot \frac{\partial v'_y}{\partial x} + \frac{1}{\rho_0} \cdot \frac{\partial p'}{\partial y} = h(t) \cdot G \cdot \sin \omega_f t \\
\frac{\partial p'}{\partial t} + U_0 \cdot \frac{\partial p'}{\partial x} + \rho_0 \cdot c_0^2 \left(\frac{\partial v'_x}{\partial x} + \frac{\partial v'_y}{\partial y}\right) = h(t) \cdot P \cdot \sin \omega_f t
\end{gathered} \tag{6}$$

which is equal to zero for $t \le 0$.
In equations (6) $A=(F,G,P)$ is the amplitude of the source produced permanent time harmonic perturbation; ω_f is the angular frequency and $h(t)$ is the Heaviside function.
F,G,P appearing in equalities (5) and (6) are continuously differentiable real valued functions depending on the variables x, y.
For the Lyapunov stability of the null solution of (4) in [2] the following infinite dimensional topological function spaces are considered:
a) X - the topological function space ([3], [4]) of the set of systems $I=(F,G,P)$ (or $A=(F,G,P)$) of functions $F,G,P : R^2 \to R^1$ which are continuously differentiable, endowed with the usual algebraic operations and topology generated by the uniform convergence on R^2 [5].
A neighborhood of the origin O in X is a set V_0 of systems I from X having the property that there exists $\varepsilon \in R^1$, $\varepsilon > 0$, such that if for $I=(F,G,P) \in X$ inequalities $|F(x,y)| < \varepsilon$; $|G(x,y)| < \varepsilon$; $|P(x,y)| < \varepsilon$ hold for any $(x,y) \in R^2$, then $I \in V_0$.
The set V_0^{ε} defined by:

$$V_0^{\varepsilon} = \left\{ (F,G,P) \in X \,\vdots\, |F(x,y)| < \varepsilon \; \textit{and so on, for any} \, (x,y) \in R^2 \right\} \tag{7}$$

is a neighborhood of the origin O in X.
b) Y is the topological function space of the set of systems $f=(f_1, f_2, f_3)$ of functions $f_i : R^1 \to R^1$, $i=\overline{1,3}$ which are continuously differentiable endowed with the usual algebraic operations and topology generated by the uniform convergence on R^1.
A neighborhood of the origin O in Y is a set W_0 of systems f from Y having the property that there exists $\varepsilon \in R^1$, $\varepsilon > 0$, such that if for $f=(f_1, f_2, f_3) \in Y$ inequalities $|f_i(\xi)| < \varepsilon$, $i=\overline{1,3}$ hold for any $\xi \in R^1$, then $f \in W_0$.
The set W_0^{ε} defined by:

$$W_0^{\varepsilon} = \left\{ (f_1, f_2, f_3) \in Y \,\vdots\, |f_i(\xi)| < \varepsilon \quad \textit{for any} \, \xi \in R^1 \; \textit{and} \; i=\overline{1,3} \right\} \tag{8}$$

is a neighborhood of the origin O in Y.
c) Z is the topological function space of the set of systems $I=(F,G,P)$ of the form

$$\begin{gathered} F(x,y) = k_1 \cdot f_1(k_1 x + l_1 y) + k_2 \cdot f_2(k_2 x + l_2 y) + \frac{l_3}{k_3} \cdot f_3(k_3 x + l_3 y) \\ G(x,y) = l_1 \cdot f_1(k_1 x + l_1 y) + l_2 \cdot f_2(k_2 x + l_2 y) - f_3(k_3 x + l_3 y) \\ P(x,y) = -c_0 \rho_0 \sqrt{k_1^2 + l_1^2} \cdot f_1(k_1 x + l_1 y) + c_0 \rho_0 \sqrt{k_2^2 + l_2^2} \cdot f_2(k_2 x + l_2 y) \end{gathered} \tag{9}$$

obtained for a given set of constants k_i, l_i $i=\overline{1,3}$ with $k_3 \neq 0$ endowed with the usual algebraic operations and the natural relative topology generated by the topology of X [6].

A neighborhood of the origin O in Z is a set V_0' of systems $I=(F,G,P)$ from Z having the property that there exists $\varepsilon \in R^1$, $\varepsilon > 0$, such that if for $I=(F,G,P) \in Z$

inequalities $|F(x,y)| < \varepsilon$ and so on hold for any $(x,y) \in R^2$, then $I \in V_0'$. The set $V_0'^{\varepsilon}$ defined by:

$$V_0'^{\varepsilon} = \left\{(F,G,P) \in Z \vdots |F(x,y)| < \varepsilon \text{ and so on for any } (x,y) \in R^2\right\} \tag{10}$$

is a neighborhood of the origin O in Z.

For a given set of constants k_i, l_i $i = \overline{1,3}$ for which the relations:

$$k_1 k_2 k_3 \neq 0 \tag{11}$$

and

$$\Delta = \frac{c_0 \rho_0}{k_3}\left[\sqrt{k_1^2 + l_1^2}(k_2 k_3 + l_2 l_3) + \sqrt{k_2^2 + l_2^2}(k_1 k_3 + l_1 l_3)\right] \neq 0 \tag{12}$$

hold, the topological function space Z is the phase space for the perturbation propagation problems (4), (5) and (6), respectively and the Lyapunov stability is analyzed in this phase space.

In [2] it is shown that in a phase space Z the ***instantaneous perturbation propagation problem*** is well posed, the solution of the initial value problem (4), (5) is given by

$$\begin{aligned}
v_x'(x,y,t) = {} & k_1 \cdot h(t) \cdot f_1\left[k_1 x + l_1 y - \left(k_1 U_0 - c_0\sqrt{k_1^2 + l_1^2}\right)\cdot t\right] + \\
& k_2 \cdot h(t) \cdot f_2\left[k_2 x + l_2 y - \left(k_2 U_0 + c_0\sqrt{k_2^2 + l_2^2}\right)\cdot t\right] + \\
& \frac{l_3}{k_3} \cdot h(t) \cdot f_3\left[k_3 x + l_3 y - k_3 U_0 t\right] \\
v_y'(x,y,t) = {} & l_1 \cdot h(t) \cdot f_1\left[k_1 x + l_1 y - \left(k_1 U_0 - c_0\sqrt{k_1^2 + l_1^2}\right)\cdot t\right] + \\
& l_2 \cdot h(t) \cdot f_2\left[k_2 x + l_2 y - \left(k_2 U_0 + c_0\sqrt{k_2^2 + l_2^2}\right)\cdot t\right] - \\
& h(t) \cdot f_3\left[k_3 x + l_3 y - k_3 U_0 t\right]
\end{aligned} \tag{13}$$

$$\begin{aligned}
p'(x,y,t) = {} & -c_0 \rho_0 \sqrt{k_1^2 + l_1^2} \cdot h(t) \cdot f_1\left[k_1 x + l_1 y - \left(k_1 U_0 - c_0\sqrt{k_1^2 + l_1^2}\right)\cdot t\right] + \\
& c_0 \rho_0 \sqrt{k_2^2 + l_2^2} \cdot h(t) \cdot f_2\left[k_2 x + l_2 y - \left(k_2 U_0 + c_0\sqrt{k_2^2 + l_2^2}\right)\cdot t\right]
\end{aligned}$$

where the initial perturbation $I = (F,G,P)$ is given by (9) and the null solution of (4) is stable in Lyapunov sense**.**

It is also shown that ***the source produced permanent time harmonic perturbation propagation problem*** (6) is well posed in a phase space Z, the solution is given by:

$$v_x'(x,y,t) = h(t)\cdot k_1\cdot\int_0^t f_1\left[k_1x + l_1y - \left(k_1U_0 - c_0\sqrt{k_1^{\,2}+l_1^{\,2}}\right)\cdot(t-\tau)\right]\sin\omega_f\tau\,d\tau +$$
$$h(t)\cdot k_2\cdot\int_0^t f_2\left[k_2x + l_2y - \left(k_2U_0 + c_0\sqrt{k_2^{\,2}+l_2^{\,2}}\right)\cdot(t-\tau)\right]\sin\omega_f\tau\,d\tau +$$
$$h(t)\cdot\frac{l_3}{k_3}\cdot\int_0^t f_3\left[k_3x + l_3y - k_3U_0(t-\tau)\right]\sin\omega_f\tau\,d\tau +$$

$$v_y'(x,y,t) = h(t)\cdot l_1\cdot\int_0^t f_1\left[k_1x + l_1y - \left(k_1U_0 - c_0\sqrt{k_1^{\,2}+l_1^{\,2}}\right)\cdot(t-\tau)\right]\sin\omega_f\tau\,d\tau +$$
$$h(t)\cdot l_2\cdot\int_0^t f_2\left[k_2x + l_2y - \left(k_2U_0 + c_0\sqrt{k_2^{\,2}+l_2^{\,2}}\right)\cdot(t-\tau)\right]\sin\omega_f\tau\,d\tau - \qquad (14)$$
$$h(t)\cdot\int_0^t f_3\left[k_3x + l_3y - k_3U_0(t-\tau)\right]\sin\omega_f\tau\,d\tau$$

$$p'(x,y,t) = -h(t)c_0\rho_0\sqrt{k_1^{\,2}+l_1^{\,2}}\cdot$$
$$\int_0^t f_1\left[k_1x + l_1y - \left(k_1U_0 - \sqrt{k_1^{\,2}+l_1^{\,2}}\right)(t-\tau)\right]\sin\omega_f\tau\,d\tau +$$
$$h(t)c_0\rho_0\sqrt{k_2^{\,2}+l_2^{\,2}}\cdot$$
$$\int_0^t f_2\left[k_2x + l_2y - \left(k_2U_0 + c_0\sqrt{k_2^{\,2}+l_2^{\,2}}\right)(t-\tau)\right]\sin\omega_f\tau\,d\tau$$

where the amplitude $A=(F,G,P)$ is given by (9) and the null solution of (4) is unstable in Lyapunov sense.

In [7] the phase space is the same. What is different is the type of stability. The stability considered there is moreover the type of stability considered in the papers [8]-[17].

In the present paper we provide numerical illustration for the global and local stability and instability results of the constant spatially developing 2D gas flow reported in [7].

2. NUMERICAL ILLUSTRATION OF SOME GLOBAL AND LOCAL STABILITIES; ABSOLUTE AND CONVECTIVE INSTABILITIES WITH RESPECT TO INSTANTANEOUS PERTURBATION IN 2D

In [7] several types of stabilities and instabilities of the null solution of (4) with respect to instantaneous perturbation are presented.

They are different from those introduced by Lyapunov [18] from which the concept of hydrodynamic stability [19] was derived.

One of the differences is that the stability and instability are considered here with respect to a given perturbation (not with respect to a set of perturbations) and the magnitude of the perturbation as well that of the perturbation propagation is compared to a prior given fixed value, called ***tolerance.***

Definition 2.1 For the instantaneous perturbation $I = (F, G, P)$ from *X* the magnitude of the perturbation at the point (x, y) is the maximum of $|F(x, y)|, |G(x, y)|, |P(x, y)|$ and it is denoted by

$$|I(x, y)| = \max\{|F(x, y)|, |G(x, y)|, |P(x, y)|\}.$$

The magnitude of the propagation at a point $(x, y) \in R^2$ at the moment of time $t \geq 0$ (i.e. that of the solution $S'(x, y, t) = (v_x'(x, y, t), v_y'(x, y, t), p'(x, y, t))$ of the initial value problem (4), (5), equal to zero for $t < 0$) is the maximum of $|v_x'(x, y, t)|, |v_y'(x, y, t)|, |p'(x, y, t)|$ and is denoted by

$$|S'(x, y, t)| = \max\{|v_x'(x, y, t)|, |v_y'(x, y, t)|, |p'(x, y, t)|\}.$$

Definition 2.2 The null solution of (4) is ***globally stable*** with respect to the instantaneous perturbation $I = (F, G, P)$ from *Z* and the prior given tolerance $\varepsilon > 0$ if the magnitude of its propagation (i.e. that of the solution of the initial value problem (4), (5), equal to zero for $t < 0$ at any point $(x, y) \in R^2$ and at every moment of time $t \geq 0$ is less than the tolerance $\varepsilon > 0$.

Acoustic modes [19] - [22] can be seen as propagations of some particular instantaneous perturbations.

In 2D the acoustic modes are propagation of the form $v_x'(x, y, t) = A\exp i(\omega t - kx - ly)$, $v_y'(x, y, t) = B\exp i(\omega t - kx - ly)$, $p'(x, y, t) = C\exp i(\omega t - kx - ly)$ with real or complex frequency $\omega = \omega_1 + i\omega_2$ and wave numbers $k = k_1 + ik_2$, $l = l_1 + il_2$.

For $kl \neq 0$, and ω satisfying $\omega - kU_0 = 0$ real acoustic mode is of the form:

$$\begin{aligned}
v_x'(x, y, t) &= B \cdot e^{[k_2(x - U_0 t) + l_2 y]} \\
&\left[\tfrac{l_2 k_1 - l_1 k_2}{k_1^2 + k_2^2} \sin(k_1(x - U_0 t) + l_1 y) - \tfrac{l_1 k_1 + l_2 k_2}{k_1^2 + k_2^2} \cos(k_1(x - U_0 t) + l_1 y)\right] \\
v_y'(x, y, t) &= B \cdot e^{[k_2(x - U_0 t) + l_2 y]} \cos(k_1(x - U_0 t) + l_1 y) \\
p'(x, y, t) &= 0
\end{aligned} \tag{15}$$

and corresponds to the instantaneous perturbation

$$\begin{aligned}
F(x, y) &= B \cdot e^{[k_2 x + l_2 y]}\left[\tfrac{l_2 k_1 - l_1 k_2}{k_1^2 + k_2^2} \sin(k_1 x + l_1 y) - \tfrac{l_1 k_1 + l_2 k_2}{k_1^2 + k_2^2} \cos(k_1 x + l_1 y)\right] \\
G(x, y) &= B \cdot e^{[k_2 x + l_2 y]} \cos(k_1 x + l_1 y) \\
P(x, y) &= 0
\end{aligned} \tag{16}$$

or of the form

$$v_x'(x,y,t) = B \cdot e^{[k_2(x-U_0t)+l_2y]}$$
$$\left[\frac{l_1k_1+l_2k_2}{k_1^2+k_2^2}\sin(k_1(x-U_0t)+l_1y) - \frac{l_2k_1-l_1k_2}{k_1^2+k_2^2}\cos(k_1(x-U_0t)+l_1y)\right] \tag{17}$$
$$v_y'(x,y,t) = -B \cdot e^{[k_2(x-U_0t)+l_2y]}\sin(k_1(x-U_0t)+l_1y)$$
$$p'(x,y,t) = 0$$

and corresponds to the instantaneous perturbation

$$F(x,y) = B \cdot e^{[k_2x+l_2y]}\left[\frac{l_1k_1+l_2k_2}{k_1^2+k_2^2}\sin(k_1x+l_1y) - \frac{l_2k_1-l_1k_2}{k_1^2+k_2^2}\cos(k_1x+l_1y)\right]$$
$$G(x,y) = -B \cdot e^{[k_2x+l_2y]}\sin(k_1x+l_1y) \tag{18}$$
$$P(x,y) = 0$$

For $kl \neq 0,$ and ω satisfying $(\omega - kU_0)^2 = c_0{}^2(k^2 + l^2)$ real acoustic mode is of the form:

$$v_x'(x,y,t) = \frac{C}{\rho_0} \cdot e^{[k_2x+l_2y-\omega_2t]}$$
$$\left[\frac{-k_1(\omega_2-k_2)+k_2(\omega_1-k_1)}{(\omega_1-k_1)^2+(\omega_2-k_2)^2}\sin(k_1x+l_1y-\omega_1t) + \frac{k_1(\omega_1-k_1)+k_2(\omega_2-k_2)}{(\omega_1-k_1)^2+(\omega_2-k_2)^2}\cos(k_1x+l_1y-\omega_1t)\right]$$
$$v_y'(x,y,t) = \frac{C}{\rho_0} \cdot e^{[k_2x+l_2y-\omega_2t]} \tag{19}$$
$$\left[\frac{-l_1(\omega_2-l_2)+l_2(\omega_1-l_1)}{(\omega_1-l_1)^2+(\omega_2-l_2)^2}\sin(k_1x+l_1y-\omega_1t) + \frac{l_1(\omega_1-l_1)+l_2(\omega_2-l_2)}{(\omega_1-l_1)^2+(\omega_2-l_2)^2}\cos(k_1x+l_1y-\omega_1t)\right]$$
$$p'(x,y,t) = C \cdot e^{[k_2x+l_2y-\omega_2t]}\cos(k_1x+l_1y-\omega_1t)$$

and corresponds to the instantaneous perturbation

$$F(x,y) = \frac{C}{\rho_0} \cdot e^{[k_2x+l_2y]}\left[\frac{-k_1(\omega_2-k_2)+k_2(\omega_1-k_1)}{(\omega_1-k_1)^2+(\omega_2-k_2)^2}\sin(k_1x+l_1y) + \frac{k_1(\omega_1-k_1)+k_2(\omega_2-k_2)}{(\omega_1-k_1)^2+(\omega_2-k_2)^2}\cos(k_1x+l_1y)\right]$$
$$G(x,y) = \frac{C}{\rho_0} \cdot e^{[k_2x+l_2y]}\left[\frac{-l_1(\omega_2-l_2)+l_2(\omega_1-l_1)}{(\omega_1-l_1)^2+(\omega_2-l_2)^2}\sin(k_1x+l_1y) + \frac{l_1(\omega_1-l_1)+l_2(\omega_2-l_2)}{(\omega_1-l_1)^2+(\omega_2-l_2)^2}\cos(k_1x+l_1y)\right] \tag{20}$$
$$P(x,y) = C \cdot e^{[k_2x+l_2y]}\cos(k_1x+l_1y)$$

or of the form

$$v_x'(x,y,t) = \frac{C}{\rho_0} \cdot e^{[k_2x+l_2y-\omega_2t]}$$
$$\left[\frac{-k_1(\omega_1-k_1)+k_2(\omega_2-k_2)}{(\omega_1-k_1)^2+(\omega_2-k_2)^2}\sin(k_1x+l_1y-\omega_1t) + \frac{-k_1(\omega_2-k_2)+k_2(\omega_1-k_1)}{(\omega_1-k_1)^2+(\omega_2-k_2)^2}\cos(k_1x+l_1y-\omega_1t)\right]$$
$$v_y'(x,y,t) = \frac{C}{\rho_0} \cdot e^{[k_2x+l_2y-\omega_2t]} \tag{21}$$
$$\left[-\frac{l_1(\omega_1-l_1)+l_2(\omega_2-l_2)}{(\omega_1-l_1)^2+(\omega_2-l_2)^2}\sin(k_1x+l_1y-\omega_1t) + \frac{-l_1(\omega_2-l_2)+l_2(\omega_1-l_1)}{(\omega_1-l_1)^2+(\omega_2-l_2)^2}\cos(k_1x+l_1y-\omega_1t)\right]$$
$$p'(x,y,t) = -C \cdot e^{[k_2x+l_2y-\omega_2t]}\sin(k_1x+l_1y-\omega_1t)$$

and corresponds to the instantaneous perturbation

$$F(x,y) = \frac{C}{\rho_0} \cdot e^{[k_2 x + l_2 y]} \left[\frac{-k_1(\omega_1 - k_1) + k_2(\omega_2 - k_2)}{(\omega_1 - k_1)^2 + (\omega_2 - k_2)^2} \sin(k_1 x + l_1 y) + \frac{-k_1(\omega_2 - k_2) + k_2(\omega_1 - k_1)}{(\omega_1 - k_1)^2 + (\omega_2 - k_2)^2} \cos(k_1 x + l_1 y) \right]$$

$$G(x,y) = \frac{C}{\rho_0} \cdot e^{[k_2 x + l_2 y]} \left[-\frac{l_1(\omega_1 - l_1) + l_2(\omega_2 - l_2)}{(\omega_1 - l_1)^2 + (\omega_2 - l_2)^2} \sin(k_1 x + l_1 y) + \frac{-l_1(\omega_2 - l_2) + l_2(\omega_1 - l_1)}{(\omega_1 - l_1)^2 + (\omega_2 - l_2)^2} \cos(k_1 x + l_1 y) \right] \quad (22)$$

$$P(x,y) = -C \cdot e^{[k_2 x + l_2 y]} \sin(k_1 x + l_1 y)$$

In [7] it was shown that if the null solution of (4) is globally stable with respect to a mode-type instantaneous perturbation $I = (F, G, P)$ and tolerance $\varepsilon > 0$, then necessarily $k_2 = l_2 = 0$.

Numerical illustration of this type of stability is given by the example:

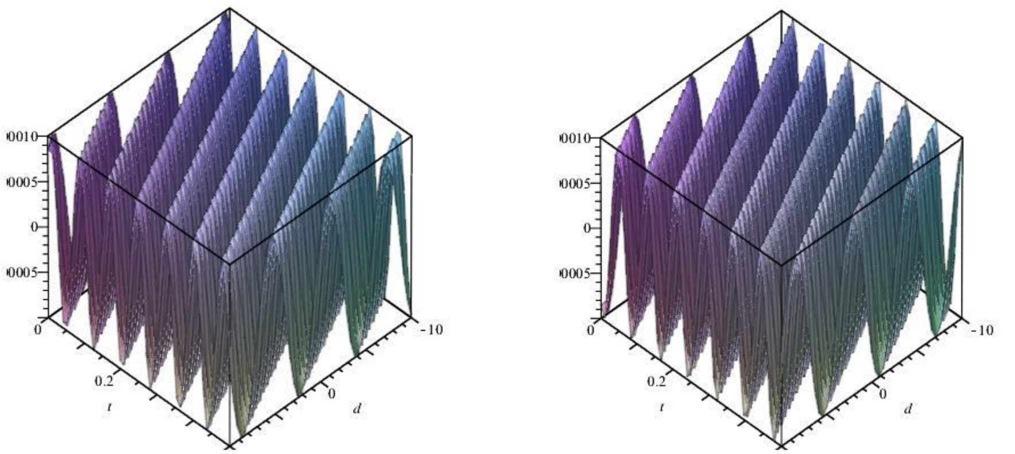
Example 2.2 is the illustration of the global stability of the null solution of equation (4) with respect to the mode type instantaneous perturbation (16)

$$F(x,y) = B \cdot e^{[k_2 x + l_2 y]} \left[\frac{l_2 k_1 - l_1 k_2}{k_1^2 + k_2^2} \sin(k_1 x + l_1 y) - \frac{l_1 k_1 + l_2 k_2}{k_1^2 + k_2^2} \cos(k_1 x + l_1 y) \right)$$

$$G(x,y) = B \cdot e^{[k_2 x + l_2 y]} \cos(k_1 x + l_1 y)$$

$$P(x,y) = 0$$

and the prior given tolerance $\varepsilon > 0$ for: $U_0 = 80\ m/s$; $\rho_0 = 1.20\ kg/m^3$; $c_0 = 345\ m/s$; $\varepsilon = 0.1$ and $|B| = 0.0001$, $k_2 = l_2 = 0$, $k_1 = l_1 = 1$.



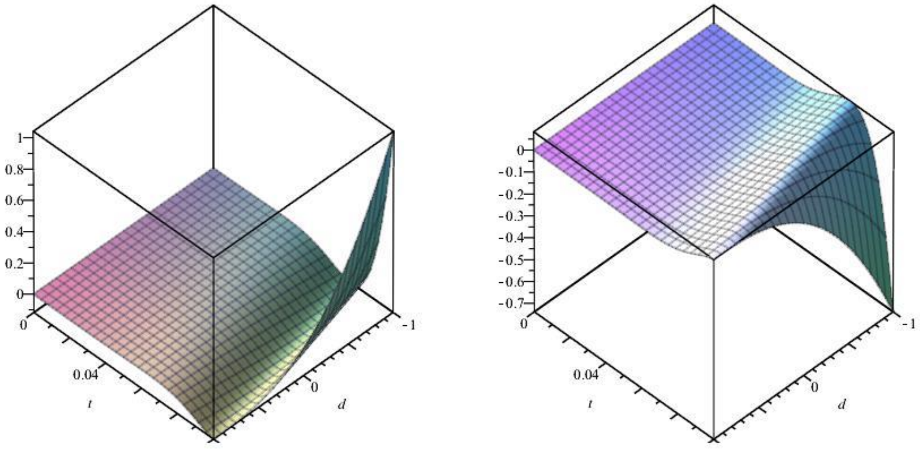
a. evolution of v'_x b. evolution of v_y'

Figure 1: Global stability with respect to the instantaneous perturbation at an arbitrary point of the line $x + y = d$ during the first $0.5\,s$.

Definition 2.3. The null solution of (4) is ***globally absolutely unstable*** with respect to the instantaneous perturbation $I = (F, G, P)$ if for any point $(x,y) \in R^2$ any real numbers $M > 0, N > 0$ (M and N big) there exists a moment of time $t > N$ such that the magnitude of its propagation (i.e. that of the solution of the initial value problem (4), (5), equal to zero for $t < 0$) at the point (x,y) and at moment of time t is greater than M.

In [7] it was shown that if $k_1 \neq 0, k_2 < 0$, and $B \neq 0$, then the null solution of (4) is ***globally absolutely unstable*** with respect to the instantaneous perturbation $I = (F, G, P)$, where F, G, P are given by (16) or (18).

Numerical illustration of this type of global absolute instability is given by the example:
Example 2.3 is the illustration of the global absolute instability of the null solution of equation (4) with respect to the mode type instantaneous perturbation (16), for:
$U_0 = 80\ m/s$; $\rho_0 = 1.20\ kg/m^3$; $c_0 = 345\ m/s$; and
$|B| = 0.0001, k_1 = 1, k_2 = -1, l_1 = 1, l_2 = -1.$



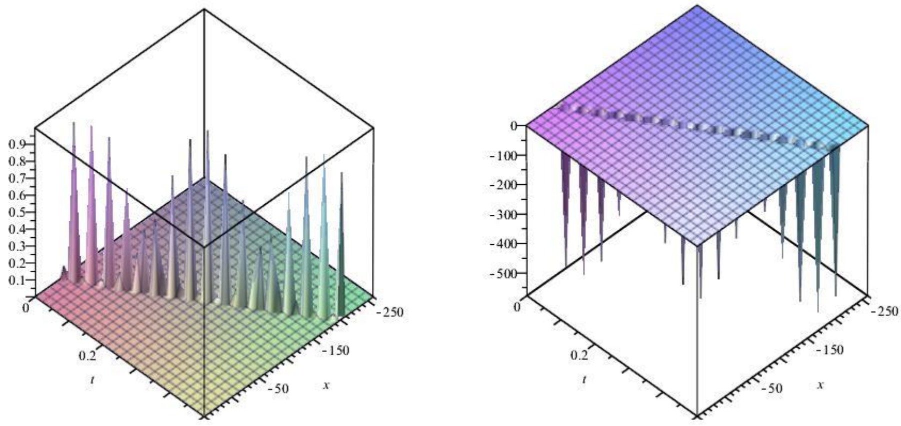
a. evolution of v'_x b. evolution of v_y'

Figure 2: Global absolute instability with respect to the instantaneous perturbation at the points of the lines $d = x + y$, d in the range [-1,1] during the first $0.1\,s$.

Definition 2.4. The null solution of (4) is ***locally stable*** on $\Omega \subset R^2$, with respect to the instantaneous perturbation $I = (F, G, P)$ from Z and the prior given tolerance $\varepsilon > 0$ if the magnitude of its propagation (i.e. the solution of the initial value problem (4), (5) which is equal to zero for $t < 0$) at any point $x \in \Omega$ and moment of time $t > 0$ is less than the tolerance $\varepsilon > 0$.
In [7] several statements concerning local stability of the null solution with respect to instantaneous perturbations which propagate as acoustic modes were given.

Numerical illustration of a local stability is given by the example:
Example 2.4 is the illustration of the local stability of the null solution of equation (4) with respect to the instantaneous perturbation $I = (F, G, P)$ from a phase space Z ($k_1 = 1, l_1 = 1, f_1(\xi) = e^{-\xi^2}, f_2(\xi) = f_3(\xi) = 0$) of amplitude $F(x,y) = G(x,y) = e^{-(x+y)^2}$; $P(x,y) = -c_0 \cdot \rho_0 \sqrt{2} \cdot e^{-(x+y)^2}$, and tolerance $\varepsilon > 0$ on $\Omega_+ = \left\{ (x,y) \in R \,\middle|\, x + y > \sqrt{\max\left\{ \ln\frac{1}{\varepsilon}, \ln\frac{c_0\rho_0}{\varepsilon} \right\}} \right\}$ for: $U_0 = 80\ m/s$; $\rho_0 = 1.20\ kg/m^3$; $c_0 = 345\ m/s$; $\varepsilon = 0.1$; $x + y = d$ in the range $[-260,\ldots,50]\,m$ and t in the range $[0,\ldots,0.5]\,s$.

Definition 2.5. The null solution of (4) is ***locally convectively unstable*** on $\Omega \subset R^2$ with respect to the instantaneous perturbation $I = (F, G, P)$ if for the prior given tolerance $\varepsilon > 0$ at any point $(x, y) \notin \Omega$ at every moment of time $t > 0$ the magnitude of the solution of (4), (5) is less than ε and there exists a sequence of points $(x_n, y_n) \in \Omega$, which tends to $+\infty$ (i.e. $x_n^{\,2} + y_n^{\,2} \to +\infty$) and a sequence of moments of times t_n, which tends also to $+\infty$ such that the magnitude of the solution of (4), (5) at the points (x_n, y_n) and moments of times t_n is greater than $\varepsilon > 0$.



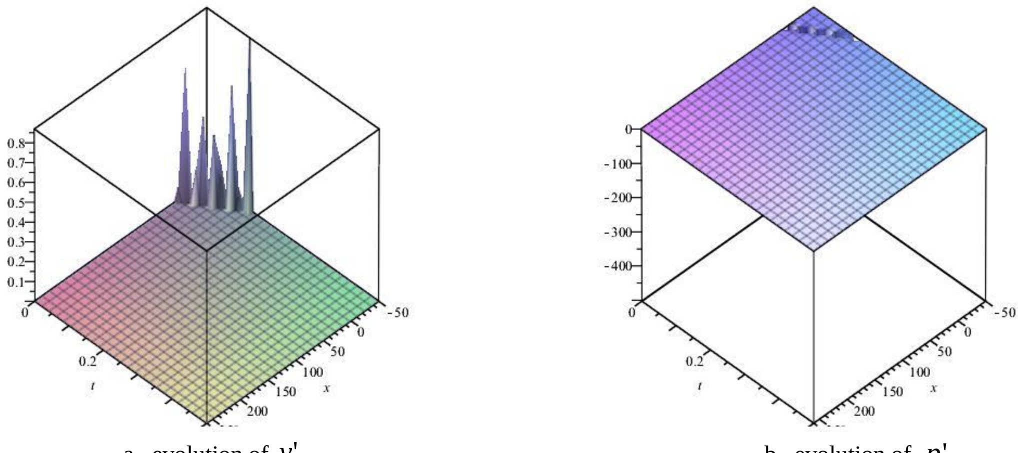
a. evolution of v'_x b. evolution of p'

Figure 3: Local stability with respect to the instantaneous perturbation at an arbitrary point of the line $x + y = d$ in the range $[-260,\ldots,50]\, m$ and t in the range $[0,\ldots,0.5]\, s$

In [7] several statements concerning local convective instability of the null solution with respect to instantaneous perturbations were given.

Numerical illustration of a local convective instability is given by the example:

Example 2.5 is the illustration of the local convective instability of the null solution of equation (4) with respect to the instantaneous perturbation $I = (F, G, P)$ from a phase space $I = (F, G, P) \in Z$ $(k_1 = 1,\ l_1 = 1,\ f_1(\xi) = e^{-\xi^2},\ f_2(\xi) = f_3(\xi) = 0)$ of amplitude $F(x, y) = G(x, y) = e^{-(x+y)^2}$; $P(x, y) = -c_0 \cdot \rho_0 \sqrt{2} \cdot e^{-(x+y)^2}$, and tolerance $\varepsilon > 0$ on $\Omega_- = \left\{ (x, y) \in R^3 \,\middle|\, x + y \le \sqrt{\max\left\{ \ln \frac{1}{\varepsilon},\ \ln \frac{c_0 \rho_0}{\varepsilon} \right\}} \right\}$ for: $U_0 = 80\ m/s$; $\rho_0 = 1.20\ kg/m^3$; $c_0 = 345\ m/s$; $\varepsilon = 0.1$; $x + y = d$ in the range $[-260,\ldots,50]\, m$ and t in the range $[0,\ldots,0.5]\, s$.



a. evolution of v'_x b. evolution of p'

Figure 4: Local convective instability with respect to the instantaneous perturbation at an arbitrary point of the line $x + y = d$ in the range $[-50,\ldots,260]\, m$ and t in the range $[0,\ldots,0.5]\, s$

3. NUMERICAL ILLUSTRATION OF SOME GLOBAL AND LOCAL STABILITIES; ABSOLUTE AND CONVECTIVE INSTABILITIES WITH RESPECT TO SOURCE PRODUCED PERMANENT TIME HARMONIC PERTURBATION IN 2D

Definition 3.1. The null solution of (4) is ***globally stable*** with respect to the source produced, permanent time harmonic perturbation of amplitude $A=(F,G,P)$ and angular frequency ω_f for a given tolerance $\varepsilon>0$ if the magnitude of the solution of (6) (equal to zero for $t<0$) at any point $(x,y)\in R^2$ and moment of time $t>0$ is less than the prior given tolerance $\varepsilon>0$.

In [7] it was shown the following general result: If in a phase space Z the following conditions hold:

i) the supports of the functions F,G,P, defining the amplitude $A=(F,G,P)$ of the time harmonic perturbation are compact

ii) $k_1U_0-c_0\sqrt{k_1^{\,2}+l_1^{\,2}}\neq 0$, $k_2U_0+c_0\sqrt{k_2^{\,2}+l_2^{\,2}}\neq 0$

iii) $\delta=\max\limits_{i=\overline{1,3}}\left\{\sup\limits_{\xi\in[a,b]}\left|f_i(\xi)\right|\right\}<\dfrac{\varepsilon}{M'}$

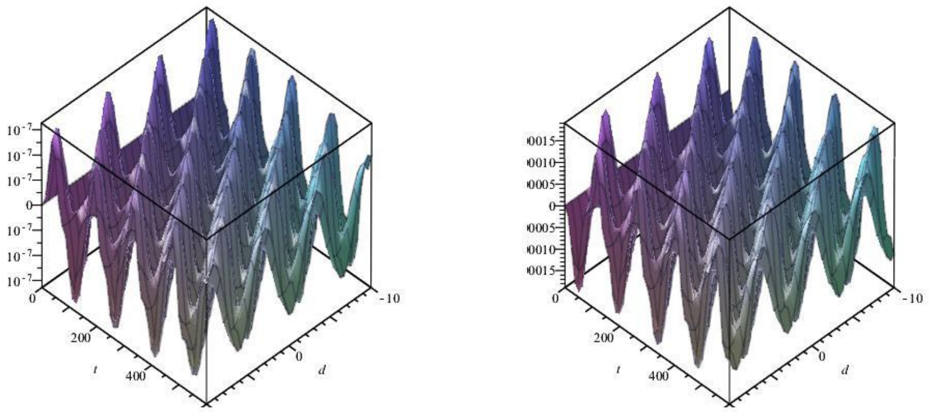
where: *the bounded interval* $[a,b]$ *contains the supports of the functions* f_i, $i=\overline{1,3}$ and

$$M'=(b-a)\cdot\max\left\{\left|\frac{k_1}{k_1U_0-c_0\sqrt{k_1^{\,2}+l_1^{\,2}}}\right|+\left|\frac{k_2}{k_2U_0+c_0\sqrt{k_2^{\,2}+l_2^{\,2}}}\right|+\left|\frac{l_3}{k_3^{\,2}U_0}\right|;\right.$$
$$\left|\frac{l_1}{k_1U_0-c_0\sqrt{k_1^{\,2}+l_1^{\,2}}}\right|+\left|\frac{l_2}{k_2U_0+c_0\sqrt{k_2^{\,2}+l_2^{\,2}}}\right|+\left|\frac{1}{k_3U_0}\right|;$$
$$\left.\frac{c_0\rho_0\sqrt{k_1^{\,2}+l_1^{\,2}}}{\left|k_1U_0-c_0\sqrt{k_1^{\,2}+l_1^{\,2}}\right|}+\frac{c_0\rho_0\sqrt{k_2^{\,2}+l_2^{\,2}}}{\left|k_2U_0+c_0\sqrt{k_2^{\,2}+l_2^{\,2}}\right|}\right\},$$

then the null solution of (4) is globally stable with respect to the source produced permanent time harmonic perturbation of amplitude $A=(F,G,P)$ and arbitrary angular frequency ω_f for the given tolerance $\varepsilon>0$.

Numerical illustration of a global stability is given by the example:

Example 3.1 is the illustration of the global stability of the null solution of equation (4) with respect to the source produced permanent time harmonic perturbation which amplitude $A=(F,G,P)$ is from a phase space Z ($k_1=1, l_1=1,\ f_1(\xi)=\sin(\xi),\ f_2(\xi)=f_3(\xi)=0$) and is given by $F(x,y)=G(x,y)=K\cdot\sin(x+y)$; $P(x,y)=-K\cdot c_0\cdot\rho_0\sqrt{2}\cdot\sin(x+y)$ of angular frequency $\omega_f>0$ and prior given tolerance $\varepsilon>0$ for: $U_0=80\ m/s$; $\rho_0=1.20\ kg/m^3$; $c_0=345\ m/s$; $\varepsilon=0.1$ and $K=0.0001$, $\omega_f=100$.



a. evolution of v'_x b. evolution of p'

Figure 5: Global stability with respect to a source produced permanent time harmonic perturbation during the first $600s$ at an arbitrary point of the line $x + y = d$ during the first $600s$.

Definition 3.2. The null solution of (4) is ***globally absolutely unstable*** with respect to the permanent source produced time harmonic perturbation of amplitude $A = (F, G, P)$ and angular frequency ω_f if for any point $(x, y) \in R^2$ any real numbers $M > 0,\ N > 0$ (M and N big) there exists a moment of time $t > N$ such that the magnitude of the solution of problem (6) (which is equal to zero for $t < 0$) at (x, y) and at moment of time t is greater than M.

In [7] the following result were proven:

In a phase space Z for a source produced permanent time harmonic perturbation of amplitude $A = (F, G, P) \in Z$ and angular frequency ω_f the following statements hold:

i) if $F(x, y) = k_1 \cdot e^{k_1 x + l_1 y} \cdot \sin(k_1 x + l_1 y)$; $G(x, y) = l_1 \cdot e^{k_1 x + l_1 y} \cdot \sin(k_1 x + l_1 y)$;

$P(x, y) = -c_0\rho_0\sqrt{k_1^{\ 2} + l_1^{\ 2}} \cdot e^{k_1 x + l_1 y} \cdot \sin(k_1 x + l_1 y)$ and $\omega_f = k_1 U_0 - c_0\sqrt{k_1^{\ 2} + l_1^{\ 2}} < 0$, then the null solution of (4) is globally absolutely unstable with respect to the perturbation.

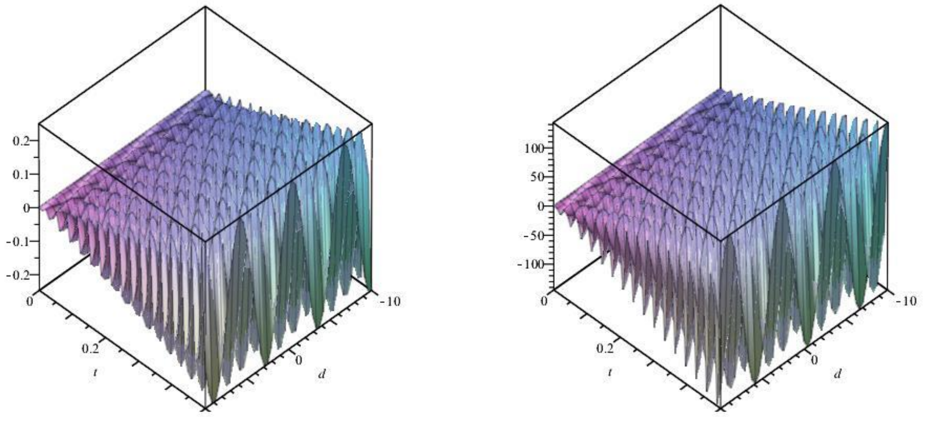
ii) if $F(x, y) = k_2 \cdot e^{k_2 x + l_2 y} \cdot \sin(k_2 x + l_2 y)$; $G(x, y) = l_2 \cdot e^{k_2 x + l_2 y} \cdot \sin(k_2 x + l_2 y)$;

$P(x, y) = c_0\rho_0\sqrt{k_2^{\ 2} + l_2^{\ 2}} \cdot e^{k_2 x + l_2 y} \cdot \sin(k_2 x + l_2 y)$ and $\omega_f = k_2 U_0 + c_0\sqrt{k_2^{\ 2} + l_2^{\ 2}} < 0$, then the null solution of (4) is globally absolutely unstable with respect to the perturbation.

iii) if $F(x, y) = \frac{l_3}{k_3} \cdot e^{k_3 x + l_3 y} \cdot \sin(k_3 x + l_3 y)$; $G(x, y) = -e^{k_3 x + l_3 y} \cdot \sin(k_3 x + l_3 y)$; $P(x, y) = 0$; $\omega_f = k_1 U_0 < 0$ and $l_3 \neq 0$, then the null solution of (4) is globally absolutely unstable with respect to the perturbation.

Numerical illustration of a global absolute instability is given by the example:

Example 3.2 is the illustration of the global absolute instability of the null solution of equation (4) with respect to the source produced permanent time harmonic perturbation which amplitude $A = (F, G, P)$ is from a phase space Z ($k_1 = 1,\ l_1 = 1,\ f_1(\xi) = \sin(\xi),\ f_2(\xi) = f_3(\xi) = 0$) and is given by $F(x, y) = G(x, y) = K \cdot \sin(x + y)$; $P(x, y) = -K \cdot c_0 \cdot \rho_0\sqrt{2} \cdot \sin(x + y)$; of angular frequency ω_f for: $U_0 = 80\ m/s$; $\rho_0 = 1.20\ kg/m^3$; $c_0 = 345\ m/s$; $K = 1$, $\omega_f = U_0 - c_0\sqrt{2}$.



a. evolution of v'_x b. evolution of p'

Figure 6: Global absolute instability with respect to a source produced permanent time harmonic perturbation at an arbitrary point of the line $x + y = d$ during the first $0.5\,s$.

Definition 3.3. The null solution of (4) is ***locally stable*** on $\Omega \subset R^2$, with respect to the source produced permanent time harmonic perturbation of amplitude $A = (F, G, P)$ and angular frequency ω_f if the magnitude of the solution of the problem (6) (which is equal to zero for $t < 0$) at any point $(x, y) \in \Omega$ and moment of time $t > 0$ is less than a prior given value $\varepsilon > 0$.

In [7] the following result were proven concerning local stability with respect to the source produced permanent time harmonic perturbation. In a phase space Z for a source produced permanent time harmonic perturbation of amplitude $A = (F, G, P) \in Z$ and an arbitrary angular frequency ω_f the following statements hold:

i) if $F(x, y) = k_1 \cdot e^{k_1 x + l_1 y} \cdot \sin(k_1 x + l_1 y)$; $G(x, y) = l_1 \cdot e^{k_1 x + l_1 y} \cdot \sin(k_1 x + l_1 y)$;
$P(x, y) = -c_0\rho_0\sqrt{k_1^2 + l_1^2} \cdot e^{k_1 x + l_1 y} \cdot \sin(k_1 x + l_1 y)$; $k_1 U_0 - c_0\sqrt{k_1^2 + l_1^2} > 0$; and $k_1 \cdot l_1 \neq 0$,
then the null solution of (4) is locally stable on

$$\Omega = \left\{ (x, y) \in R^2 \,\middle|\, k_1 x + l_1 y < \min\left\{ \ln\frac{\varepsilon(k_1 U_0 - c_0\sqrt{k_1^2 + l_1^2})}{|k_1|}, \ln\frac{\varepsilon(k_1 U_0 - c_0\sqrt{k_1^2 + l_1^2})}{|l_1|}, \ln\frac{\varepsilon(k_1 U_0 - c_0\sqrt{k_1^2 + l_1^2})}{c_0\rho_0\sqrt{k_1^2 + l_1^2}} \right\} \right\}$$

for the given tolerance $\varepsilon > 0$.

ii) if $F(x, y) = k_2 \cdot e^{k_2 x + l_2 y} \cdot \sin(k_2 x + l_2 y)$; $G(x, y) = l_2 \cdot e^{k_2 x + l_2 y} \cdot \sin(k_2 x + l_2 y)$;
$P(x, y) = c_0\rho_0\sqrt{k_2^2 + l_2^2} \cdot e^{k_2 x + l_2 y} \cdot \sin(k_2 x + l_2 y)$; $k_2 U_0 + c_0\sqrt{k_2^2 + l_2^2} > 0$; and $k_2 \cdot l_2 \neq 0$,
then the null solution of (4) is locally stable on

$$\Omega = \left\{ (x, y) \in R^2 \,\middle|\, k_2 x + l_2 y < \min\left\{ \ln\frac{\varepsilon(k_2 U_0 + c_0\sqrt{k_2^2 + l_2^2})}{|k_2|}, \ln\frac{\varepsilon(k_2 U_0 + c_0\sqrt{k_2^2 + l_2^2})}{|l_2|}, \ln\frac{\varepsilon(k_2 U_0 + c_0\sqrt{k_2^2 + l_2^2})}{c_0\rho_0\sqrt{k_2^2 + l_2^2}} \right\} \right\}$$

for the given tolerance $\varepsilon > 0$.

iii) if $F(x,y)=\frac{l_3}{k_3}\cdot e^{k_3x+l_3y}\cdot\sin(k_3x+l_3y)$; $G(x,y)=-e^{k_3x+l_3y}\cdot\sin(k_3x+l_3y)$; $P(x,y)=0$; $k_3U_0>0$; and $l_3\neq 0$, then the null solution of (4) is locally stable on

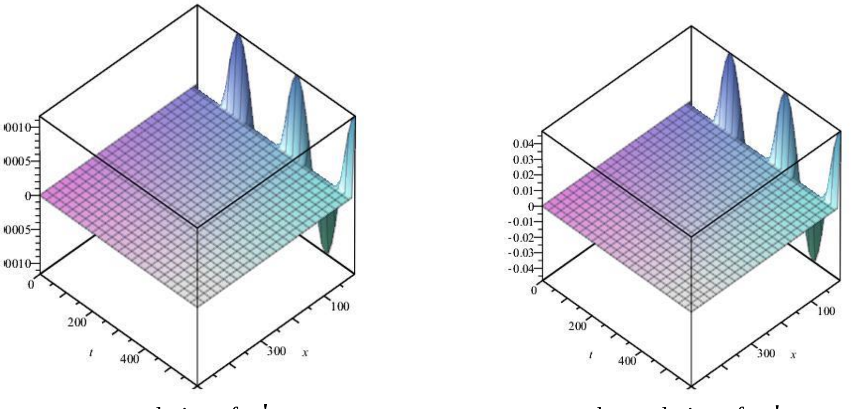
$$\Omega=\left\{(x,y)\in R^2\left|k_3x+l_3y<\min\left\{\ln\frac{\varepsilon\cdot k_3U_0\cdot k_3}{|l_3|},\ln\varepsilon\cdot k_3U_0\right\}\right.\right\}$$ for the given tolerance $\varepsilon>0$.

Numerical illustration of a local stability is given by the example:

Example 3.3 is the illustration of the local stability of the null solution of equation (4) on the set

$$\Omega_+=\left\{(x,y)\in R^2\left|x+y>\max\left\{\ln\frac{1}{\varepsilon\left(c_0\sqrt{2}-U_0\right)},\ \ln\frac{c_0\rho_0\sqrt{2}}{\varepsilon\left(c_0\sqrt{2}-U_0\right)}\right\}\right.\right\}$$

with respect to the source produced permanent time harmonic perturbation which amplitude is $A=(F,G,P)$ from a phase space Z ($k_1=1,\ l_1=1,\ f_1(\xi)=e^{-\xi},\ f_2(\xi)=f_3(\xi)=0$) and is given by $F(x,y)=G(x,y)=K\cdot e^{-(x+y)}$; $P(x,y)=-K\cdot c_0\cdot\rho_0\sqrt{2}\cdot e^{-(x+y)}$ of angular frequency ω_f and prior given tolerance $\varepsilon>0$ for: $U_0=80\ m/s$; $\rho_0=1.20\ kg/m^3$; $c_0=345\ m/s$; $\varepsilon=0.1$ and $\omega_f=100$.



a. evolution of v'_x b. evolution of p'

Figure7: Local stability on Ω_+ of a permanent source produced time harmonic perturbation at an arbitrary point of the line $x+y=d$, d in the range $[2.6099,...,500]\ m$, during the first $600\ s$.

Definition 3.4. The null solution of (4) is ***locally convectively unstable*** on $\Omega\subset R^2$ with respect to the source produced permanent time harmonic perturbation of amplitude $A=(F,G,P)$ and angular frequency ω_f if for the prior given tolerance $\varepsilon>0$ at any point $(x,y)\notin\Omega$ at every moment of time $t>0$ the magnitude of the solution of (6) is less than ε and there exists a sequence of points $(x_n,y_n)\in\Omega$, which tends to $+\infty$ (i.e. $x_n^2+y_n^2\to+\infty$) and a sequence of moments of times t_n, which tends also to $+\infty$ such that the magnitude of the solution of (6) at the points (x_n,y_n) and moments of times t_n is greater than $\varepsilon>0$. In [7] the following result were proven concerning local stability with respect to the source produced permanent time harmonic perturbation. In a phase space Z for a source produced permanent time harmonic perturbation of amplitude $A=(F,G,P)\in Z$ and an arbitrary angular frequency ω_f the following statements hold:

i) if $F(x,y)=k_1\cdot e^{-(k_1x+l_1y)}$; $G(x,y)=l_1\cdot e^{-(k_1x+l_1y)}$; $P(x,y)=-c_0\rho_0\sqrt{k_1^{\,2}+l_1^{\,2}}\cdot e^{-(k_1x+l_1y)}$; $k_1U_0-c_0\sqrt{k_1^{\,2}+l_1^{\,2}}<0$; $k_1>0$ and $l_1\neq 0$ then the null solution of (4) is locally convectively unstable on

$$\Omega=\left\{(x,y)\in R^2\left|k_1x+l_1y<\max\left\{\ln\frac{k_1}{\varepsilon(c_0\sqrt{k_1^{\,2}+l_1^{\,2}}-k_1U_0)},\ln\frac{|l_1|}{\varepsilon(c_0\sqrt{k_1^{\,2}+l_1^{\,2}}-k_1U_0)},\ln\frac{c_0\rho_0\sqrt{k_1^{\,2}+l_1^{\,2}}}{\varepsilon(c_0\sqrt{k_1^{\,2}+l_1^{\,2}}-k_1U_0)}\right\}\right.\right\}$$

for the given tolerance $\varepsilon>0$.

ii) if $F(x,y)=k_2\cdot e^{-(k_2x+l_2y)}$; $G(x,y)=l_2\cdot e^{-(k_2x+l_2y)}$; $P(x,y)=c_0\rho_0\sqrt{k_2^{\,2}+l_2^{\,2}}\cdot e^{-(k_2x+l_2y)}$; $k_2U_0+c_0\sqrt{k_2^{\,2}+l_2^{\,2}}<0$; $l_2>0$ and $k_2\neq 0$, then the null solution of (4) is locally convectively unstable on

$$\Omega=\left\{(x,y)\in R^2\left|k_2x+l_2y<\max\left\{\ln\frac{|k_2|}{\varepsilon\left|c_0\sqrt{k_2^{\,2}+l_2^{\,2}}+k_2U_0\right|},\ln\frac{l_2}{\varepsilon\left|c_0\sqrt{k_2^{\,2}+l_2^{\,2}}+k_2U_0\right|},\ln\frac{c_0\rho_0\sqrt{k_2^{\,2}+l_2^{\,2}}}{\varepsilon\left|c_0\sqrt{k_2^{\,2}+l_2^{\,2}}+k_2U_0\right|}\right\}\right.\right\}$$

for the given tolerance $\varepsilon>0$.

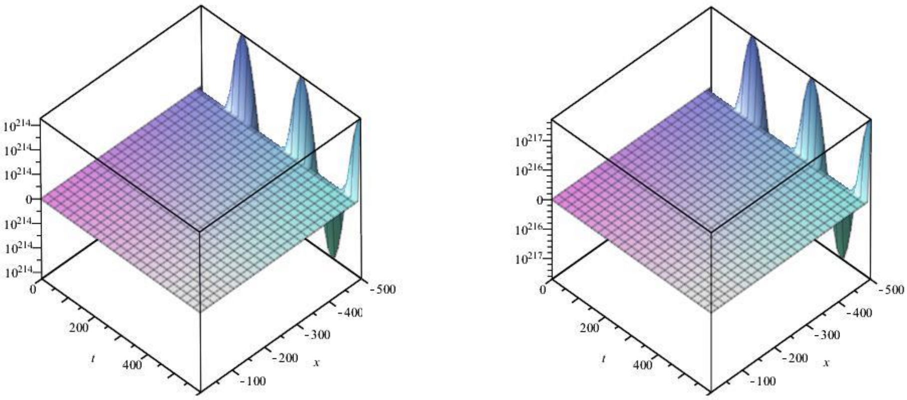
iii) if $F(x,y)=\frac{l_3}{k_3}\cdot e^{-(k_3x+l_3y)}$; $G(x,y)=-e^{-(k_3x+l_3y)}$; $P(x,y)=0$; $k_3U_0<0$; and $l_3>0$, then the null solution of (4) is locally convectively unstable on

$$\Omega=\left\{(x,y)\in R^2\left|k_3x+l_3y<\max\left\{\ln\frac{l_3}{\varepsilon\cdot|k_3U_0|\cdot|k_3|},-\ln\varepsilon\cdot|k_3U_0|\right\}\right.\right\}$$

for the given tolerance $\varepsilon>0$.

Numerical illustration of a local stability is given by the example:

Example 3.4 is the illustration of the local convective instability of the null solution of equation (4). The null solution of (4) is locally convectively unstable with respect to the permanent source produced time harmonic perturbation given in Example 3.3 on Ω_-, given by: $\Omega_-=\{(x,y,z)\in R^2\,|x+y<-3\}$.



a. evolution of v'_x b. evolution of p'

Figure 8: Local convective instability with respect to the permanent source produced time harmonic perturbation at an arbitrary point of the line $x+y=d$, d in the range $[-500,\ldots,-3]$ m during the first 600 s.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this paper.

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