Possible Simple Structures of the Universe to Include General Relativity Effects

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***Abstract**: The general relativity describes the universe properties, the gravity playing a fundamental role. One uses a metric tensor in a Riemann space, $g_{\mu\nu}$, which should be in agreement with a mass (or energy) tensor in order to satisfy the Einstein equation of the general relativity [1]. This equation contains the Ricci curvature as well. In general, applications are done considering that a chosen metric is valid without region limits. In fact, the density of the energy whose distribution is however unknown is variable in universe; therefore, the metrics need to be adapted to different regions. For this reason one suggests to start with a simple, average mass-energy distribution that could represent in a first step the actual universe. This suggestion is in agreement with the symmetrical distribution of equal spheres existing in a model of the early universe given by one of the authors. Two kinds of distribution are given. The possibility of black holes formation is studied and a criterion is given.*

***Key Words:** metrics, photon, refraction index, black hole*

1. INTRODUCTION

In the followings the universe is considered a sphere in expansion from a center C_U , the location of the initial singularity called BIG FLASH, the speed of the frontier being c_{vac}, speed of light in vacuum. One allows a linear velocity distribution from center to the frontier, this being established as equilibrium between the initial momentum at explosion (BIG FLASH) and the gravity forces. There are some reasons to preserve on average a spherical symmetry in the mass-energy distribution in universe. These reasons are related to the spherical symmetry in the early universe formation according to the model proposed by one of the authors [2]. The system of coordinates with center C_U is considered at rest. Other models of universe are given in [3].

2. PROPOSED MODELS

2.1 A model of similar regions.

Let E_U be the admitted total energy of the universe and M_U the corresponding mass. Then:

$$E_U = M_U c_{vac}^2 \tag{1}$$

c_{vac} being the speed of light, finite and constant. Let N_P be a number of parts in which the universe is divided considering equal masses M_P and energies E_P :

$$E_P = E_U / N_P;\ M_P = M_U / N_P \tag{2}$$

If R_U is the radius of universe considered in expansion at the speed c_{vac} at frontier, and ρ_U the average energy density at a time t_U from the BIG FLSH (BIG BANG) one has:

$$R_U = c_{vac} t_U;\ \rho_U = 3E_U / 4\pi R_U^3 \tag{3}$$

The volume of the universe, V_{OLU} , and the volume of a part, V_{OLP} , are:

$$V_{OLU} = \frac{4\pi}{3} R_U^3;\ \ V_{OLP} = \frac{V_{OLU}}{N_P};\ R_U = t_U c_{vac} \tag{4}$$

However the volume V_{OLP} gives only an estimation of the partial volumes because the spheres do not fill the entire space, a void fraction existing. The problem of the spheres distribution will be studied in the following.

One considers a central sphere in the origin C_U and a number of spherical zones having the width $2R_P$ around it, R_P being the radius of a sphere. In the arrangement on the zones we propose for spheres, the centers, C_k, (Fig. 1) are located on a median sphere of radius R_{zk} of zone k given by:

$$R_{zk} = 2kR_P,\ k = 1; k_{\max} \tag{5}$$

If N_{zk} is the numbers of centers on a circle of radius R_{zk} in a plane passing by the Oz axis (Fig. 1), one has with an accuracy which increases with *k:*

$$N_{zk} = \text{int}(2\pi R_{zk} / 2R_P) = \text{int}(2\pi k) \tag{6}$$

where int(…) is the function "(integer part of)".

Now, one takes a circle of radius R_{zk} in a plane normal to the Oz axis, divided in N_{zk} equal parts; a number of $\text{int}(N_{zk}/2)$ circles of radius R_{zk} will pass through a pair of diameter ends. On the other hand, the two points on the Oz axis are common to all of the $\text{int}(N_{zk}/2)$ circles. Then the total number of sphere centers, (therefore the total numbers of spheres) is:

$$N_{ztot} = \text{int}(N_{zk}^2 / 2 - 2(N_{ztot}/2 - 1)) = \text{int}(2\pi^2 k^2 - 2\pi k + 2) \tag{7}$$

The void fraction, ε_{VK} , is:

$$\varepsilon_{VK} = 1 - \frac{volume\,of\ N_{ztot} spheres}{volume\,of\ zone\,k} = 1 - \frac{\text{int}(2\pi^2 k^2 - 2\pi k + 2)}{24k^2 + 1} \tag{8}$$

For k=1; 2, 10; 100 one obtains: $\varepsilon_{V1} = 0.600$; $\varepsilon_{V2} = 0.299$; $\varepsilon_{V10} = 0.203$; $\varepsilon_{V100} = 0.1801$. The minimum value is $\varepsilon_{\min} = 0.1775$.

For a given number N_P the following condition giving $k_{\max}$ must be satisfied:

$$N_P = 1 + \sum_{1}^{k_{\max}} \mathrm{int}(2\pi^2 k^2 - 2\pi k + 2) \tag{9}$$

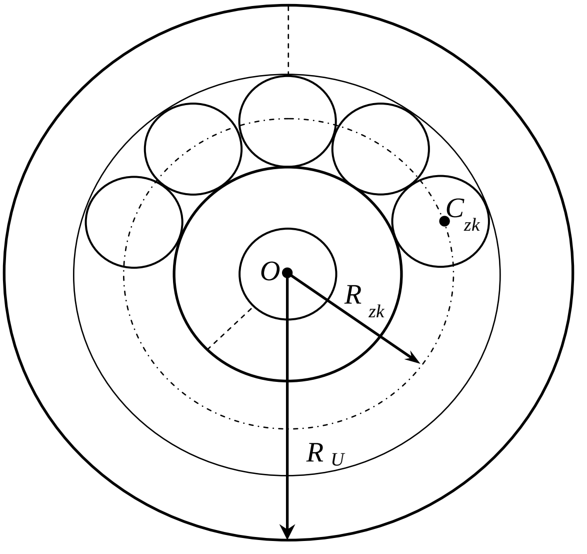

Fig. 1 The sphere distribution in universe

To satisfy the condition (9) the value of $k_{\max}$ is arranged, to obtain N_P and the radius R_P, the maximum radius being given. For example, for the universe of radius R_U a safety length equal to $2R_P$ is left from the frontier where the velocity is c_{vac}. Then one has:

$$R_U - 2R_p = R_p\left(1 + 2k_{\max}\right);\; R_p = R_U / \left(3 + 2k_{\max}\right) \tag{10}$$

In this scheme applied to a Sun-like model one has $N_P = N_{Psun} = 1.9284E23$; in case of a Galaxy model one obtains $N_P = N_{PG} = 1.9284E11$. The results are given in Table 1 for the age of universe equal to 9.15 billion of years.

In the case of the Galaxy model an arrangement of spheres (Suns) can also be done, the radius being obtained by arranging 10^{12} Suns in zones as above .One obtains $k_{\max}$ = *5338* and R_p =1.314*E18* meter. For the actual universe characteristics, one admits: E_U =3.473E70 J; M_U =3.857E53 kg; R_U =1.83 *E10* light years [2]; M_{Sun} =2*E30* kg;

Other values for R_U are: 1.35 *E10* light years and *20.E10* light years, the last figure corresponding to a Hubble constant of 50 km/sec/Mpc. Because one expects transformations in time of the universe structure, one has selected as moment of our discussion the value R_U =0.915 *E10* light years (half of the actual value in [2]).

Table 1 Some region characteristics for R_U =0.915 E10 l.ys.

M_P, kg	N_P	R_P, l.ys./meter	$k_{\max}$	r_f, m	Vol_f / Vol_U	M_P / r_f
2E30	1.928E23	1.484 E2/ 1.404E18	3083.E4	6.955 E8	4.266E-24	2.876 E21
2E42	1.928E11	1.483 E6/ 1.403E22	3083.	1.314 E22	4.258E-12	1.425 E20
7.894E36	3.947E6	7.318E-9/ 0.6923E8	84	1.170E10	-	6.747 E26

Relativistic correction of mass. Until now, the system of references was the fixed system with center in the point where the BIG FLASH took place. Therefore a body at a radius say R_{zk} (see eq. (5)) is moving with the speed:

$$v_{zk} = c_{vac} R_{zk} / R_U \tag{11}$$

proportional to the speed at frontier. This velocity is constant in time. Therefore if the mass is M_P in the fixed system of reference, the proper mass, M_{PROPK} , for zone *k* is according to the special theory of relativity [9]:

$$M_{PROPK} = M_P \sqrt{1 - \left(\frac{R_{zk}}{R_U}\right)^2} \tag{12}$$

This fact can strongly diminish the mass near frontier: up to a factor 0.000312 for Sun-like model and 0.0312 for Galaxy model.

3. GENERAL RELATIVITY EFFECTS

To introduce the general relativity effects one considers punctual masses and the metric tensor $g_{\mu\nu}$ of Rastall [4], the universe element of length having the form:

$$dS^2 = g_{\mu\nu} dx^\mu dx^\nu = \frac{1}{n}\left(c_{vac} dt\right)^2 - n\left(d\vec{r}\right)^2 ;\ \mu, \nu = \left(0;3\right) \tag{13}$$

where *t* is the time, $\vec{r}$ is the position vector ***in the proper system of each considered part; c_{vac} being the speed of light in vacuum:***

$$x^0 = t,\ x^1 = x,\ x^2 = y, x^3 = z \tag{13-a}$$

n is the refraction index given by the relation:

$$\ln\left(n\right) = \frac{2 f_N M_0}{c_{vac}^2\, r} \tag{14}$$

In the above relation (14) M_0 is the proper mass creating (mainly) the gravitation and f_N =6.67E-11 N $m^2 / \left(kg\right)^2$. At large distances from M_0 the refraction index is very close to unity but the general relativity effects are not negligible. As one can see from Table 1, the radii R_P are ***large enough to consider every mass separately.***

The equation (13) also satisfies the equation of Einstein [1;7]:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi f_N}{c_{vac}^2} T_{\mu\nu} \tag{15}$$

where $R_{\mu\nu}, R, T_{\mu\nu}$ are: the tensor of curvature of Ricci [7], the static curvature and the tensor of mass, respectively. First one uses the Lagrange function L given by [5]:

$$L = -m_0 c_{vac} \frac{dS}{dt} \tag{16}$$

to obtain the total energy, E_{tot}, of a particle of proper mass m_0, moving around M_0:

$$E_{tot} = \vec{v}\frac{\partial L}{\partial \vec{v}} - L;\ \vec{v} = \frac{d\vec{r}}{dt} \tag{17}$$

From (16) and (17) one obtains:

$$E_{tot} = \frac{m_0 c_{vac}^2}{\sqrt{n\left(1-\left(n\,v/c_{vac}\right)^2\right)}};\ \frac{dE_{tot}}{dt} = 0. \tag{18}$$

the total energy being constant along the particle trajectory.

As one can infer from (18), in case the particle is a photon, one has m_0 = 0; therefore to obtain a total energy different from zero, the speed of photon, v_{ph}, in a gravitational field with the refraction index *n*, one should have the expression:

$$v_{ph} = \frac{c_{vac}}{n} \tag{19}$$

As a consequence, the speed of photon is variable with the distance from the mass M_0. In the following one considers two masses: 1) one mass equal to our Sun mass (M_{Psun} = 2*E30* kg, in the fixed system of reference); 2) a second mass of a galaxy mass order (M_{PG} = 2.*E42* kg, in the fixed system of reference). One important parameter is the ratio M_P / r_f, r_f being the radius at frontier. The radii r_f are given in Table1.

If one takes the corresponding values for Sun-like and Galaxy-like cases in formula (14), one obtains refraction indexes very close to unity (with an error smaller then 10^{-5}).

3.1 An analogue of the second law of Mechanics.

In order to obtain an analogue of the second law of Mechanics one minimizes the integral of the Lagrange function (16):

$$L = -m_0 c_{vac} \frac{dS}{dt};\ \delta \int L\,dt = 0. \tag{20}$$

By using the Euler-Lagrange relations for minimum one obtains the differential equation:

$$\frac{d}{dt}\left(n^2\, m\vec{v}\right) + (1+n^2\beta^2)\frac{f_N M_0 m}{r^2}\frac{\vec{r}}{r} = 0;\ \beta^2 = \frac{v^2}{c_{vac}^2};\ m = \frac{E_{tot}}{c_{vac}^2} \tag{21}$$

m being the mass of particle in the system of reference with the center in M_0.

One can see the similarity of equation (21) with the Newton equation of gravitational interaction between masses M_0 and *m*. The differences are related to the modification of momentum with a factor n^2 and the amplification of the Newtonian force with the relativistic factor $(1+n^2\beta^2)$ which takes the value two for the photon.

Taking into account that the total mass *m* is constant, together with the total energy, one obtains the simplified equation:

$$\frac{d}{dt}\left(n^2\,\vec{v}\right)+(1+n^2\beta^2)\frac{f_N M_0}{r^2}\frac{\vec{r}}{r}=0 \tag{21-a}$$

Because on the radius intervals $\left(r_f\,;R_P\right)$ one has with a very good approximation $n=1$ the momentum has the classical expression; the speed of photon is c_{vac} and the amplification of the Newton force is $(1+\beta^2)$ equal to 2 for photon, but very close to 1 for bodies with small velocities. From Table 1 one can see that the part of the universe volume where $n>1$ is comparatively very small (see the volume ratios $Vol_f\,/Vol_U$).

For *n=1*, the general relativity element of universe of Rastall (13) is identical to Minkowski invariant of the special theory of relativity:

$$dS^2=\left(c_{vac}dt\right)^2-\left(d\vec{r}\,\right)^2 \tag{22}$$

valid almost in the entire volume of the universe both for the Sun-like and Galaxy-like models (see the volume ratios in Table 1). However the general relativity effects last because of the time derivatives of the refraction index:

$$\frac{1}{n}\frac{dn}{dt}=-2\frac{f_N M_0}{c_{vac}^2 r^2}\frac{dr}{dt} \tag{23}$$

Thus for *n=1*, equation (21) becomes:

$$\frac{d\vec{v}}{dt}+\frac{f_N M_0}{r^2}((1+\beta^2)\frac{\vec{r}}{r}-4\beta^2\vec{\tau}\,sign(\frac{dr}{dt}))=0, \tag{24}$$

$\vec{\tau}$ being the velocity unit vector.

As one can see from (24) there are two components of the acceleration $\frac{d\vec{v}}{dt}$: one after the position vector and another after the velocity direction. For $\beta^2 << 1$, equation (24) is reduced to the Newton law of gravity. For the photon one has:

$$\vec{v}=\vec{v}_{ph}=\frac{c_{vac}}{n}\vec{\tau};n^2\beta^2=1 \tag{25}$$

and for *n=1* equation (21) becomes

$$c_{vac}\frac{d\vec{\tau}}{dt}+\frac{2f_N M_0}{r^2}(\frac{\vec{r}}{r}-\vec{\tau}\,sign\left(\frac{dr}{dt}\right))=0 \tag{26}$$

If the photon is moving after a straight line passing through the center of M_0 equation (26) is clearly satisfied ($\vec{\tau}$ =const.)

Application to a photon passing by the body of mass M_0 (Fig.2). By integrating equation (21) with respect to time, one obtains:

$$\left(n\vec{\tau}\right)_{\infty}-\left(n\vec{\tau}\right)_{-\infty}=-2\frac{f_N M_0}{c_{vac}}\int_{-\infty}^{\infty}\frac{\vec{r}}{r^3}dt \tag{27}$$

One considers the trajectory pretty close to a straight line to approximate ***for integration only*** and the position vector and *n* as follows:

$$\vec{r}=r_0\,\vec{i}+\frac{c_{vac}}{n}\,\vec{j}\,;n\approx 1 \tag{28}$$

Then because $n_{\infty}=n_{-\infty}=1$, one obtains finally the :deviation angle of the photon δ :

$$\delta=\vec{\tau}_{\infty}-\vec{\tau}_{-\infty}=4\frac{f_N M_0}{c_{vac}^2 r_0} \tag{29}$$

The formula (29) is well-known; it was used to find the curvature of the light ray passing near the Sun, representing one of the confirmations of the general theory of relativity in 1919. This value is two times the one given by the Newton formula.

Remark. Because the refraction index *n* depends on the ratio M/r and these values are very close at frontiers for the Sun and Galaxy model (see Table 1), the general relativity effects are similar for the two models.

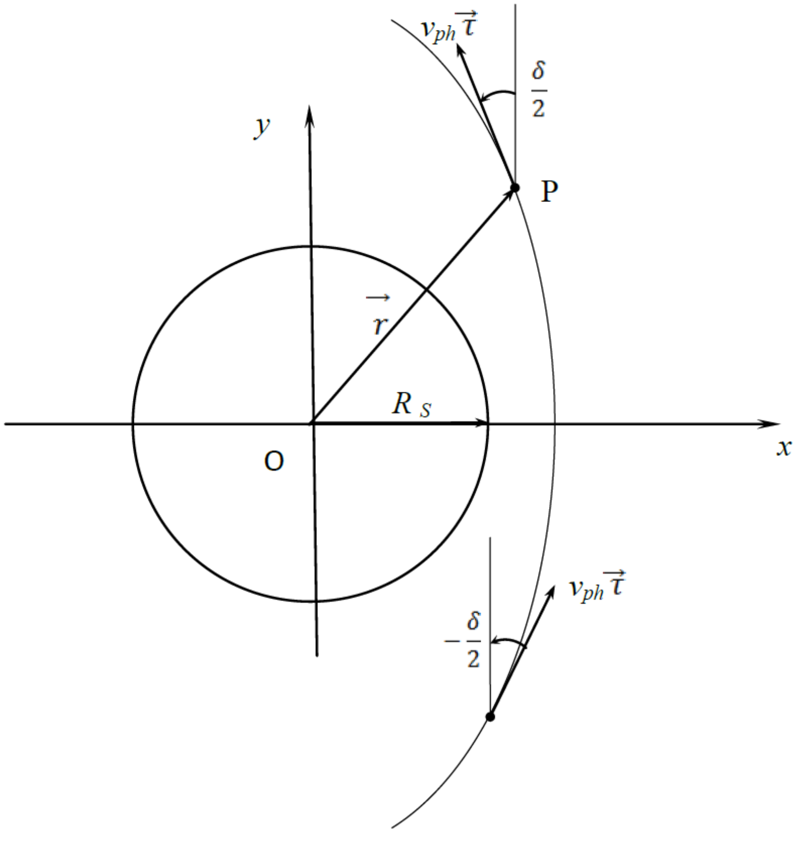

Fig. 2 The passing of a photon near the Sun; R_S - sun radius; δ - the deviation angle; v_{ph} – the speed of photon ; $\vec{\tau}$ - the tangent unit vector.

4. POSSIBLE FORMATION OF BLACK HOLES. DARK MATTER

From Table 1, one can see that the numbers of parts, N_P, are big enough for various situations to appear. One of them could be the ***black holes formation*** as a consequence of the mass concentration. The main condition for the definition of a black hole is the impossibility to emit photons [8]. Then, denoting by E_{ph} the energy of a photon at the black hole frontier of the radius R_{Bh} and by using the expression of the gravity force given by equation (21) for photon and considering the mechanical work of this force from R_{Bh} to infinity, one obtains the condition:

$$E_{ph} \leq 2\frac{f_N M_0}{R_{Bh}}\frac{E_{ph}}{c_{vac}^2};\ \frac{M_0}{R_{Bh}} \geq \frac{c_{vac}^2}{2\, f_N} = 6.747E26 = M_P / r_f \ \text{kg/m} \tag{30}$$

Thus one has obtained ***a critical value for the ratio*** M_0 / R_{Bh} for the black hole formation. According to Newton formula this ratio is half only; therefore the general relativity effects help the black hole formation. The critical radius is the largest radius for a given mass to form a black hole. The refraction index at the black hole frontier is $n_{Bh} = e^1 = 2.718$. Of course black holes of higher densities can be formed at smaller values of the frontier radii, the refraction index increasing exponentially. A suggestion can be made with respect to so called ***dark matter.*** This could be given by black hole formations close to the critical value $6.747E26 = M_P / r_f$; thus the black hole is weak, not able to act visible to surroundings; in addition at such a body surface the speed of a coming photon is sensibly reduced due to the big values of the refraction index. From Table 1 it results that both the Sun-like and the Galaxy models presented, comparing the ratios M_P / r_f, are far from a state of black hole. The same behavior has a Sun- like body within a Galaxy: $M_P / r_f = 1.522E18$ *Kg/ m.*

In the last line of Table 1 the characteristics of a black hole made from 3.947*E6* Sun masses at the limit for a black hole formation are given. Such a mass would correspond to a super-massive black hole supposed to exist in the center of our Galaxy. At limit the density of the weakest black hole is about 8245 times the actual Sun density. The calculation suggests a possible formation of a black hole in more steps: first the Sun-like body is concentrated at about the tenth of its radius; then a big number (3.947*E6*) of the obtained body are clustered to form a weak black hole. Further the cluster is concentrated to form a final strong black hole. Thus it results a black hole similar to the one supposed to exist in the center of our Galaxy (named ***Sagittarius*** *A*, [10]), having the Sun diameter (about 17 times smaller than the weakest black hole). Its density is 3.947*E6* times the actual Sun density!

One more remark could be done with respect to the possibility for the universal coefficient of attraction, f_N, not to be constant but depending on the age of the universe. Indeed, as given by our hydro-dynamical model of Newton law of gravity [6], f_N is increasing slowly in time (about two times in a billion of years from now on); this increasing favors the black holes formation in time. Unlike the amplification factor $(1+n^2\beta^2)$ of the general relativity which depends on velocity and of the refraction index, the increasing in time of the universal coefficient of attraction, f_N, proposed by the hydro-dynamical model affects equally all bodies irrespective of their properties and surroundings.

5. CONCLUSIONS

Two simple models of spherical bodies of different masses and radii, one of the Sun size and another one of a Galaxy size were proposed. An arrangement was done by using spherical zones in a fixed system of reference with the origin at the point of the initial singularity (Big Bang or Big Flash) of the universe. The vicinity of any body is big enough to apply the simple formula for the length of universe given by Rastall that satisfies the Einstein equation of general relativity (15). The metric tensor of Rastall form contains a refraction index that accounts for the general relativity effects, having the value 1 at the frontier regions and outside. Thus the formula (13) of Rastall is valid on parts of universe. The relativistic correction of masses [9] is necessary, the universe being in expansion. Thus some particularities of the zones near the frontier of universe can be evidenced.

Despite the big differences in masses there are some important ratios like the ratio between the mass and the body radius which are small enough for both the Sun and the Galaxy type models. One consequence is that the bodies are pretty far from the state of black hole. A scenario for the stepwise formation of black holes is given. A suggestion is done regarding the possibility to assimilate them to black holes of smaller intensity to form a kind of dark matter.

A comparison with the Newton form of the equation of motion permits to establish differences and similarities.

In conclusion, the proposed models are simple and useful. By combinations and redistribution one can obtain information regarding more complicate structures.

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REFERENCES

[1] A. Einstein, Die Grundlage der allgemeinen Relativitätstheorie, *Annalen der Physik*, vol. **35**4, issue 7, pages 769-822, 1916.

[2] C. Berbente, A Physical - Geometrical Model of an Early Universe, *INCAS BULLETIN*, vol. **6**, Issue 4, (online) ISSN 2247–4528, (print) ISSN 2066–8201, ISSN–L 2066–8201, DOI: 10.13111/2066-8201.2014.6.4.1, p. 3-12, Bucharest, Romania, 2014.

[3] Ya. B. Zeldovitch, I. D. Novikov, *1. Relativistic astrophysics, 2. Structure and evolution of the universe,* Univ. of Chicago Press,Chicago, 1983.

[4] P. Rastall, An improved theory of gravitation, *Canad. J. of Physics*, vol. **46**, nr. 19, oct. 1968.

[5] N. Ionescu-Pallas, *General Relativity and Cosmology* (in Romanian), Bucureşti, Ed. Şt. şi Enciclop., 1980.

[6] C. Berbente, A Hydro-Dynamical Model for Gravity, *INCAS BULLETIN*, vol. **8**, Issue 1, (online) ISSN 2247–4528, (print) ISSN 2066–8201, ISSN–L 2066–8201, DOI: 10.13111/2066-8201.2014.8.1.5, p.39- 47, Jan.- March. 2016.

[7] L. Jalbă, O. Stănăşilă, *Vectors, tensors, fields,* (in Romanian), Bucureşti, Fundaţia Floarea Darurilor, 2015.

[8] S. W. Hawking, Particle Creation by Black Hole, *Communications in Mathem. Physics*, **43**, (3), p.199-220, 1975.

[9] M. S. Rogalski, S. B. Palmer, *Advanced University Physics*, Chapman & Hall / CRC, 2006.

[10] * * * http://nasa.gov/.../k4/, June 2014.