

Analytical solutions for the temperature field in a 2D incompressible inviscid flow through a channel with walls of solid fuel

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Abstract: A gas (oxidizer) flows between two parallel walls of solid fuel. A combustion is initiated: the solid fuel is vaporized and a diffusive flame occurs. The hot combustion products are submitted both to thermal diffusion and convection. Analytical solutions can be obtained both for the velocity and temperature distributions by considering an equivalent mean temperature where the density and the thermal conductivity are evaluated. The main effects of heat transfer are due to heat convection at the flame. Because the detailed mechanism of the diffusion flame is not introduced the reference chemical reaction is the combustion of premixed fuel with oxidizer in excess. In exchange the analytical solution is used to define an **ideal quasi-uniform combustion** that could be realized by an adequate control. The given analytical closed solutions prove themselves flexible enough to adjust the main data of some existing experiments and to suggest new approaches to the problem.

Key Words: diffusion flame, heat flux, quasi-stationary approximation, regression speed, injection velocity.

1. INTRODUCTION

The reacting gas flow through a channel of solid fuel is of large interest for many applications, in particular for hybrid rocket engines [1-7]. Because the temperature is very high in the flame region the speed of sound is large leading to small Mach numbers ($M < 0.1$). Therefore the flow can be considered incompressible, more properly expressed, density variations around a mean value has slight effects and several analytical solutions could be obtained, both for 2D and axi-symmetrical inviscid or laminar flow [1;2;8;9;10]. A time variation of the velocity and temperature is taken into account.

In fact, as the experiments prove [3], after an explosive increasing of pressure, temperature and density at ignition, these quantities remain almost constant in time until the end of combustion.

We shall give space and time variable analytical expressions for velocity, pressure and temperature fields in terms of what we call *quasi-stationary approximation*. This is based on the small value of the wall and flame regression speed not only in comparison with the general flow field, but likewise as compared to the injected gas speed.

In addition the *quasi-stationary approximation* will be used to define a *quasi-uniform combustion regime*; indeed this quasi-uniform regime can be installed after a short distance from entrance.

As regards the density and thermal conductivity one will take values at a mean temperature.

2. THE FLOW CONFIGURATION

As one can see from Fig. 2.1, the 2D channel has the length L and the initial width $2a_0$. The wall of grain solid fuel has the initial thickness $(\bar{y}_{sf} - a_0)$. The grain pyrolysis is produced due to the high temperature; the gaseous fuel diffuses to the axis and meets the oxidizer that enters parallel to $O\bar{z}$, at a narrow region that forms the diffusion flame located at a distance a from the axis. The reaction products (gases) are injected towards channel axis at the speed \bar{u}_{inj} much larger than the regression speed \bar{u}_{rg} of the solid wall and flame ($\bar{u}_{inj} \gg \bar{u}_{rg}$). The distance solid fuel - flame is assumed constant in time, being a characteristic of the chemical reaction. The reaction mechanism is complicated, but an overall reaction is of the form:



where the indices “gf”, “ox” and “bp” stand for “gas fuel”, “oxidizer” and “burned products” respectively, and ν denotes stoichiometric coefficients. The corresponding mass fractions N are represented qualitatively in Fig. 2.2.

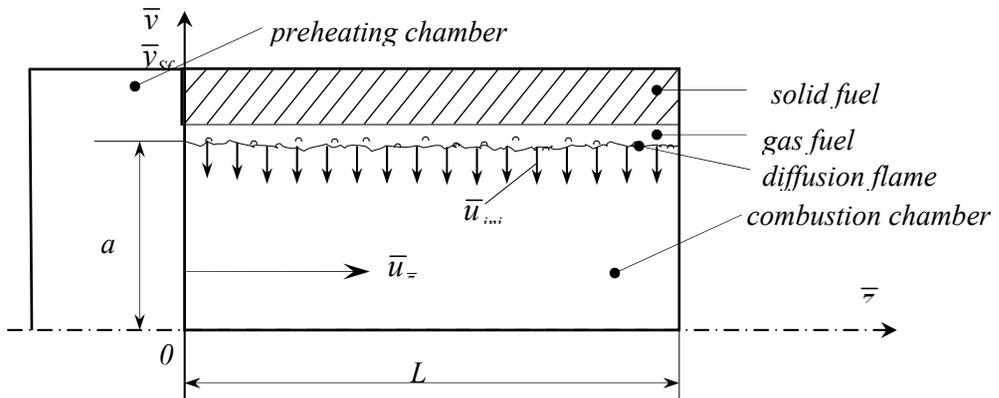


Fig. 1 - Flow configuration

At flame the temperature and the mass fraction N_{bp} of the burned product are at maximum. The heat diffuses to the solid being consumed by pyrolysis, and transported to the axis mainly by convection with the injection velocity at $\bar{y}=a$, \bar{u}_{inj} .

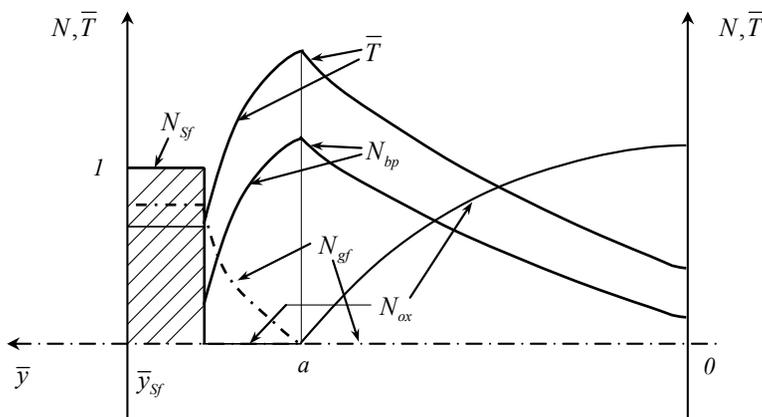


Fig. 2

Qualitative temperature (\bar{T}) and mass fractions (N) distribution

3. THE METHOD OF CALCULATION

A system of partial differential equations for inviscid flow [8], written as below, is to be solved:

$$\frac{\partial \bar{\rho}}{\partial \bar{t}} + \text{div}(\bar{\rho} \mathbf{V}) = 0, (\text{continuity}); \quad (2)$$

$$\frac{\partial \bar{V}}{\partial \bar{t}} + \text{curl} \mathbf{V} \times \mathbf{V} + \text{grad} \left(\frac{V^2}{2} \right) = -\frac{1}{\bar{\rho}} \text{grad} \bar{p}, (\text{momentum}); \quad (3)$$

$$\bar{\rho} \left(\frac{\partial \bar{h}}{\partial \bar{t}} + \bar{u}_z \frac{\partial \bar{h}}{\partial \bar{z}} + \bar{u}_y \frac{\partial \bar{h}}{\partial \bar{y}} \right) = \frac{\partial}{\partial \bar{z}} \left(\bar{\lambda} \frac{\partial \bar{T}}{\partial \bar{z}} \right) + \frac{\partial}{\partial \bar{y}} \left(\bar{\lambda} \frac{\partial \bar{T}}{\partial \bar{y}} \right), (\text{energy}). \quad (4)$$

where $\bar{z}, \bar{y}, \bar{t}$ are space and time coordinates, $\mathbf{V}(\bar{z}, \bar{y}, \bar{t})$ the velocity vector, $\bar{\rho}, \bar{p}, \bar{T}, \bar{h}$, density, pressure, temperature and enthalpy, respectively and $\bar{\lambda}$ [11], the thermal conductivity of the hot gas mixture. Bars stand for dimensional quantities. The motion is considered potential. One also has:

$$\mathbf{V} = \mathbf{k} \bar{u}_z + \mathbf{j} \bar{u}_y; \quad \text{curl} \mathbf{V} = 0, \quad (5)$$

$$\bar{h} = \bar{h}_{rf} + \int_{\bar{T}_{rf}}^{\bar{T}} \bar{c}_p d\bar{T} = \bar{h}_{rf} + \bar{c}_{pav} (\bar{T} - \bar{T}_{rf}), \quad (6)$$

\bar{T}_{rf} is a reference temperature and \bar{c}_{pav} an average specific heat at constant pressure of the hot gas mixture. We consider \bar{c}_{pav} constant at a convenient equivalent temperature \bar{T}_{ech} such that:

$$\frac{\partial \bar{h}}{\partial \bar{t}} \approx \bar{c}_{pav} \frac{\partial \bar{T}}{\partial \bar{t}}; \quad \frac{\partial \bar{h}}{\partial \bar{z}} \approx \bar{c}_{pav} \frac{\partial \bar{T}}{\partial \bar{z}}; \quad \frac{\partial \bar{h}}{\partial \bar{y}} \approx \bar{c}_{pav} \frac{\partial \bar{T}}{\partial \bar{y}}. \quad (7)$$

The velocity distribution. The oxidizer is introduced at given high pressure, temperature and entrance velocity, a preheating chamber being used to start the combustion. At ignition a high pressure and temperature are rapidly installed in the channel. Then a quasi-stationary regime of burning takes place until all fuel is consumed; the duration of that quasi-stationary regime is from the existing experiments of the order of 20-30 sec. [3;4]. This will be the time interval for which we intend to give analytical solutions for velocity, pressure and temperature.

First one introduces the dimensionless coordinates and velocities z, y, t, u_z, u_y , as follows:

$$\bar{z} = Lz; \quad \bar{z} = a_0 y; \quad \bar{t} = \bar{t}_{rf} t; \quad \bar{t}_{rf} = a_0 / \bar{u}_{rf}, \quad (8)$$

\bar{t}_{rf} being a reference time and \bar{u}_{rf} a reference velocity (constant). The solid fuel regression velocity, \bar{u}_{rg} , is small ($\bar{u}_{rg} \in (0.25 - 1) \text{ mm/sec}$) the volume of the gas resulting gas being two orders of magnitude larger, compared to the volume of the solid where from it was produced. One considers the regression velocity constant and adopts for the channel width the following law of variation:

$$a = a_0 (1 + \bar{u}_r \bar{t} / a_0) = a_0 (1 + u_r t); \quad u_r = \bar{u}_r / \bar{u}_{rf}. \quad (9)$$

Now one uses the new variables y_1 and τ defined below:

$$y_1 = y/(1+u_r t), \quad \tau = 1+u_r t, \quad (10)$$

in order to have the fixed frontier $y_1=1$. In the new coordinates, one assumes the time derivatives small, and the time functions of the group τ .

For the velocity components, one looks for expressions of the form:

$$u_z = (\alpha_0(\tau) + \alpha_1(\tau)z)/\tau; \quad u_y = -y_1 u_{inj}, \quad (11)$$

satisfying the boundary conditions:

$$z=0, u_z = u_{z0}(\tau) = \alpha_0(\tau)/\tau; \quad y_1=0, u_y=0; \quad y_1=1, u_y = -u_{inj}. \quad (12)$$

The above velocity distribution satisfies the condition of potential motion. If the density depends on τ only, the continuity equation gives:

$$u_{rg} \frac{\partial(\ln \bar{\rho})}{\partial(\ln \tau)} + \alpha_1(\tau) \frac{a_0}{L} - u_{inj} = 0. \quad (13)$$

Because $\bar{u}_{rg} / \bar{u}_{inj} \ll 1$ one can neglect the term in u_{rg} , to obtain:

$$\alpha_1(\tau) = \frac{L}{a_0} u_{inj} = \text{const.} \quad (14)$$

If the entrance velocity at $t=0$ is taken as reference and the oxidizer rate is constant, one yields:

$$\alpha_0(\tau) = 1 = \text{const.} \quad (\text{constant oxidizer rate}). \quad (15)$$

The pressure distribution. By integrating the momentum equation (3) between entrance and channel exit, for potential flow, along a line $y=\text{const.}$, one obtains:

$$\frac{\bar{p}_{en} - \bar{p}_{ex}}{\bar{\rho}_{av} u_{rf}^2} = \frac{\bar{\rho}(\tau)}{\bar{\rho}_{av}} \left(\frac{u_{zex}^2}{2} - \frac{u_{zen}^2}{2} + \int_{en}^{ex} \frac{\partial u_z}{\partial t} dz \right) \quad (16)$$

As one can see, the pressure difference $(\bar{p}_{en} - \bar{p}_{ex})$ is not large as the difference between the entrance and exit velocities and the time derivative of axial velocity are small.

The temperature distribution. In the new dimensionless coordinates z, y_1, t , the energy equation (4) becomes:

$$\delta_0 \left(\frac{\partial T}{\partial t} - \frac{u_r y_1}{\tau} \frac{\partial T}{\partial y_1} + \frac{u_z a_0}{L} \frac{\partial T}{\partial z} + \frac{u_y}{\tau} \frac{\partial T}{\partial y_1} \right) = \frac{a_0^2}{L^2} \frac{\partial^2 T}{\partial z^2} + \frac{1}{\tau^2} \frac{\partial^2 T}{\partial y_1^2}; \quad \delta_0 = \frac{\bar{\rho}_{av}}{\lambda_{av}} c_{p,av} u_{rf} a_0. \quad (17)$$

For the thermal conductivity the following approximations will be used:

$$\bar{\lambda}(T) \approx \bar{\lambda}(\bar{T}_{av}) = \bar{\lambda}_{300} = \bar{\lambda}_{300} \left(\frac{\bar{T}_{av}}{300} \right)^{n1}; \quad n1 \approx 1, \quad \bar{T}_{av} \approx \bar{T}_{ech}, \quad (18)$$

\bar{T}_{av} being an average temperature and $\bar{\lambda}_{300}$ the thermal conductivity at 300 K. The density is also calculated for the burned gases at \bar{T}_{av} and at a given pressure of these burned gases.

One looks for an analytical solution of the form:

$$T(z, y_1, t) = \frac{1}{\tau} (\alpha_0(\tau) + \alpha_1 z) C_1(\tau) g(\xi_1) + C_0(\tau); \alpha_0(\tau) = \alpha_0 = 1, \quad (19)$$

$\theta(y_1, t)$ being the unknown temperature function and $C_0(\tau), C_1(\tau)$ time functions to be determined from boundary and initial conditions. Introducing (16) in (14) leads to:

$$\delta_0 \left(-\frac{\alpha_0 u_r}{\tau^2} \theta(y_1, t) + u_r \frac{dC_0}{d\tau} + \frac{(\alpha_0 + \alpha_1 z) u_r y_1}{\tau^2} \frac{\partial \theta(y_1, t)}{\partial y_1} + \frac{(\alpha_0 + \alpha_1 z)}{\tau^2} u_{inj} - \frac{(\alpha_0 + \alpha_1 z) u_{inj} y_1}{\tau^2} \frac{\partial \theta(y_1, t)}{\partial y_1} \right) = \frac{(\alpha_0 + \alpha_1 z)}{\tau^3} \frac{\partial^2 \theta(y_1, t)}{\partial y_1^2}. \quad (20)$$

By neglecting the terms in u_{rg} in (20) in comparison with the ones in u_{inj} , and estimating a small absolute value for the derivative $\frac{dC_0}{d\tau}$ (a fact verified later), one obtains the basic differential equation of our quasi-stationary approximation as follows:

$$\frac{\partial^2 \theta(y_1, \tau)}{\partial y_1^2} + y_1 \frac{\partial \theta(y_1, \tau)}{\partial y_1} \delta u_{inj} - \theta(y_1, \tau) \delta u_{inj} = 0, \quad \delta = \delta_0 \tau. \quad (21)$$

Further one uses a new independent variable, ξ , to yield a simpler differential equation as bellow:

$$\frac{\partial^2 \theta(\xi, \tau)}{\partial \xi^2} + \xi_1 \frac{\partial \theta(\xi, \tau)}{\partial \xi} - \theta(\xi, \tau) \delta u_{inj} = 0; \quad \xi = y_1 \omega; \quad \omega = \sqrt{\delta u_{inj}}. \quad (22)$$

The general solution of eq. (19) is:

$$\theta(\xi, \tau) = C_1(\tau) g(\xi) + C_2(\tau) \xi,$$

where $C_1(\tau), C_2(\tau)$ are functions on time only (eventually pure constants); $g(\xi)$ and its derivative have the expressions:

$$g(\xi) = e^{-\xi^2/2} + \xi \int_0^\xi e^{-\xi^2/2} d\xi; \quad \frac{dg}{d\xi} \equiv gp(\xi) = \int_0^\xi e^{-\xi^2/2} d\xi. \quad (23)$$

The boundary conditions for $\theta(\xi, \tau)$ are discussed further. One condition is **the null conduction flux at the channel axis**. This leads to:

$$y_1 = 0; \quad \xi = 0, \quad \frac{\partial T}{\partial y_1} = 0; \quad \frac{\partial \theta}{\partial \xi} = 0; \quad C_2(\tau) = 0. \quad (24)$$

Then the temperature field expression is reduced to:

$$\theta(y_1, t) = \frac{C_1(\tau)}{\tau} (\alpha_0 + \alpha_1 z) g(\xi) + C_0(\tau); \quad \xi = y_1 \omega \quad (25)$$

The conductive heat fluxes q_y, q_z in the directions y and z are (in Watt/m):

$$\frac{1}{\bar{T}_{rf}} (q_y) = -\frac{\bar{\lambda}_{av} \omega C_1(\tau)}{a_0 \tau^2} (\alpha_0 + \alpha_1 z) gp(\xi); \quad \frac{1}{\bar{T}_{rf}} (q_z) = -\frac{\bar{\lambda}_{av} \alpha_1 C_1(\tau)}{L \tau} g(\xi) \quad (26)$$

It remains to determine the functions of time, $C_0(\tau), C_1(\tau)$.

Boundary conditions for temperature. One boundary condition: *the null conduction flux at the channel axis* was already implemented. A second boundary condition is **the flame temperature** \bar{T}_f . It is to be expected that \bar{T}_f , as a reaction characteristic is constant in time

for a quasi- uniform combustion. To this aim the heat produced by combustion must be evacuated (mainly by convection), from the flame (the heat conduction is not expected to play an important role in this region). On the other hand the oxidizer mass rate is required to ensure combustion rate by diffusion, an extra quantity being necessary, however avoiding the flame cooling or even its blowing. Important hydrodynamic and thermal parameters are the regression and injection velocities, u_{rg}, u_{inj} . These are related such that the heat is not explosively accumulated.

Let the solid fuel density be ρ_{sf} , the combustion duration be t_D [sec] and $(\bar{y}_{sf} - a_0)$ the solid fuel thickness. Then for a *quasi-uniform combustion regime* is natural to impose a ***quasi-constant in time temperature and flux of heat released at flame***.

One can write the mass balance for a *minimum injection velocity* when only the gas resulted from pyrolysis is evacuated:

$$\begin{aligned} \bar{\rho}_{sf} L \bar{u}_{rg} &= \bar{\rho}_{bp} L (\bar{u}_{inj})_{\min} ; (\bar{u}_{inj})_{\min} = \bar{u}_{rg} \bar{\rho}_{sf} / \bar{\rho}_{bp} = \\ &= \bar{u}_{rg} \bar{\rho}_{sf} R_{bp} \bar{T}_f / \bar{p}_f, z = 1/2. \end{aligned} \quad (27)$$

In (27) $\bar{\rho}_{bp}$ represents the density of burned products. Thus one has obtained a connection between the injection velocity $(\bar{u}_{inj})_{\min}$ and the wall regression \bar{u}_{rg} .

According to expression (25), the flame temperature, \bar{T}_f , will vary linearly with z ; then one considers the value at $z = 0.5$ and one will combine the relation (27) with ***a value for the average exit temperature, $\bar{T}_{av,ex}$*** .

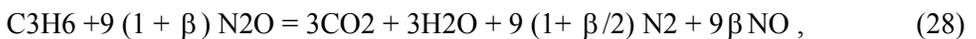
A fourth boundary condition is the constant entrance temperature of oxidizer, $\bar{T}_{ox,en}$, related to the temperature necessary for pyrolysis ($\bar{T}_{ox,en}$ in the interval 700-900 K [4], by using the preheating chamber). This condition cannot be satisfied by our analytical solution even at $z = 0$, but after a short distance that will not be large if this solution is appropriate, the mixture of hot gases and oxidizer will approach the temperature distribution (25). Anyhow a distance for the mixing of oxidizer with the hot combustion gases is necessary.

4. RESULTS AND COMPARISONS

As application we consider the combustion of polypropylene (as solid fuel) with N₂O used like oxidizer. One has several data from [3] ($\bar{\rho}_{sf} = 900$ kg/m.c ; $t_D = 30$ sec.; $\bar{u}_{rg} = 0.001$ m/sec.; $\bar{p}_f = 26$ bar; $L = 0.30$ m; $a_0 = 0.01$ m; $\bar{y}_{sf} = 0.04$ m). The combustion of HTPB (polybutadiene, C₄H₆) in gaseous oxygen considered in [4] will be also used for comparison, as the reaction heat for C₃H₆ and C₄H₆ are very closed.

Our idea is to compare the actual process with the combustion of the same but premixed gases, at an oxidizer excess that leads to the same flame temperature.

One considers the combustion of propylene (gas) in N₂O, and O₂ according to the reactions:



β being an excess coefficient of oxidizer. In (25) one accepts the NO as pollutant, consuming some heat by dissociation. One idea for a good combustion is to avoid temperatures above 2800 K, when the dissociation becomes important. In fact, the reaction mechanism is more complicated than given by reaction (28) and (29), but it gives some supplementary information. The problem for the reactions (28) and (29) is solved by using the data from [12] and the method of calculation from [13]. *Taking the reactions (28) and (29) with premixed fuel and oxidizer as an ideal case, one can evaluate the efficiency of existing case of a diffusive flame, where the dependence on diffusion processes will lead to a smaller heat quantity released by combustion and to a smaller flame temperature.*

The mass ratio between oxidizer (m_{ox}) and gas fuel ($m_{C_3H_6}$) is compared to these premixed gases reactions. In this way a *minimum oxidizer/fuel mass ratio*, $\min r_{ox}$, as below:

$$\min r_{N_2O} \equiv \min \left(\frac{m_{N_2O}}{m_{C_3H_6}} \right) = \frac{66(1+\beta)}{7}; \quad \min r_{O_2} \equiv \min \left(\frac{m_{O_2}}{m_{C_3H_6}} \right) = \frac{48(1+\beta)}{7} \quad (30)$$

The real ratio, r_{ox} , is certainly larger, by a factor K_{extra} larger than unity:

$$r_{ox} = K_{extra} \min r_{ox}; \quad K_{extra} \geq 1 \quad \bar{u}_{rf} = m_{C_3H_6} r_{ox} / (t_D \rho_{oxin} a_0) = \bar{u}_{rg} \rho_{Sf} L r_{ox} / (\rho_{oxin} a_0) \quad (31)$$

One can now define a maximum injection speed: to this aim one takes into account that besides the fuel (gaseous C3H6), the burning products also contain a percentage, let it be K_{ox1} or K_{ox2} , from oxidizer. The values of m_{bp} are (see also (30)):

$$m_{bpN_2O} = m_{C_3H_6} (1 + K_{ox1} r_{N_2O}); \quad m_{bpO_2} = m_{C_3H_6} (1 + K_{ox2} K_{O_2}); \quad 0 \leq K_{ox1,2} \leq 1. \quad (32)$$

Then the injection velocity is:

$$\bar{u}_{inj} = (1 + K_{ox} \min r_{ox}) (\bar{u}_{inj})_{\min}; \quad (\bar{u}_{inj})_{\min} = \bar{u}_{rg} \bar{\rho}_{Sf} R_{bp} \bar{T}_f / \bar{p}_f, \quad \text{at } z = 1/2. \quad (33)$$

The calculations for the reactions (28) and (29) are given in Table 1; q [J/kg] is the heat released per kg of gas mixture, leading to a caloric capacity equal to 45.769 J/kgC3H6. In [3] about 0.1857 of this q [J/kg] is reported. It seems that ***the diffusive combustion of polypropylene is much less efficient in comparison with the combustion of premixed C3H6/N2 or O2.*** The explanation has several reasons: 1) the diffusion covers only partly the possible burning quantity of the gas fuel; 2) a quantity of the released heat is consumed for pyrolysis. Will use the thermodynamic functions for burning products, R_{bp} , given in Table 1, and Table 2, calculated by using data from [5].

Table 1 The combustion of premixed C3H6 in N2O and O2; $\bar{u}_{rg} = 0.001$ m/sec. $\bar{T}_{ech} = 2425$ K

β	$\bar{T}_f 0.5$	$q \cdot 10^{-6}$	$\min m_{ox} / m_{fuel}$	R_{bp}	$\min \bar{u}_{rf} 0$	$\min \bar{u}_{inj}$	Oxidizer
1.30	2726.4	2.672	21.686	284.12	29.788	0.2682	N2O
3.79	2727.2	2.629	32.846	261.96	62.037	0.2473	O2

In order to complete the boundary conditions, the flame temperature at $z = 0.5$, $\bar{T}_f 0.5$, is chosen according to the maximum temperature obtained in experiments [4]. To these mean temperatures around 1750 K for N2O and 1685 for O2 are reached at channel exit. This leads to $\bar{T}_f \approx 2726$ K, that (see Table1) corresponds to a dilution $\beta = 1.3$ in case of C3H6, and

$\beta = 3.79$ in case of O₂. According to (30) the injection velocity at pressure $\bar{p}_f \approx 26$ bar is, under these circumstances, $\min \bar{u}_{inj} = 0.2682$ m/sec. for combustion in N₂O and $\min \bar{u}_{inj} = 0.2473$ m/sec. for combustion in O₂. Therefore the quasi-stationary assumption is valid ($\bar{u}_r / \bar{u}_{inj} < 4.04(E-3)$). The time functions $C_0(\tau)$ are the same for N₂O and O₂ and $C_1(\tau)$ are very close (see Table2).

The minimum entrance velocities are large as compared to the ones used in Ref.[4] for O₂ because the regression velocities are different: here $\bar{u}_{rg} = 1(E-3)$ m/sec. as reported in Ref.[3]; in Ref.[4] $\bar{u}_{rg} = 2.5(E-4)$ m/sec.(four times smaller). The obtained values are given in Table 2. The injection velocities xxx for the two oxidizers, N₂O and O₂, are pretty close. As the oxidizer flux is increased, the flame temperature is diminished, as well as the exit average velocity $\bar{T}_{av,ex}$.

Table 2 The combustion of premixed polypropylene in N₂O and O₂ (diffusion flame)

$$\bar{u}_{rg} = 10^{-3}, \text{ m/sec, } \bar{p}_{av} = 26 \text{ bar, } C_1(\tau) = b_1 \tau^{\epsilon_1}, C_0(\tau) = 0.467 \tau^{0.15}$$

$\min r_{ox}$	$\min \bar{u}_{inj}$	$\max \bar{u}_{inj}$ m/sec	\bar{c}_{pech} J/kg.K	$\bar{T}_{av,ex}$	$b_1; \epsilon_1$	Oxidizer
21.686	0.2682	6.0844	1389.52	1750	0.267; 0.475	N ₂ O
32.846	0.2473	8.3701	1377.91	1685	0.288; 0.475	O ₂

In Fig. 4.1 the transverse temperature function $g(\xi)$ is represented, pointing out an almost linear variation in the dimensionless variable ξ defined in relation (22). In Fig. 4.2 the transverse temperature time - variation at channel exit is exhibited (after 0; 10; 20 and 30 sec.), showing an almost constant in time temperature. In order to make the curves distinct some multiplication factors were used. In Fig. 4.3 the average conduction y-heat flux (Watt/m) points out a decreasing of this flux in time; however the main part of the total heat flux is due to convection. The curves in Figs. 4.1, 4.2 and 4.3 are given for the case of N₂O in Table 2.

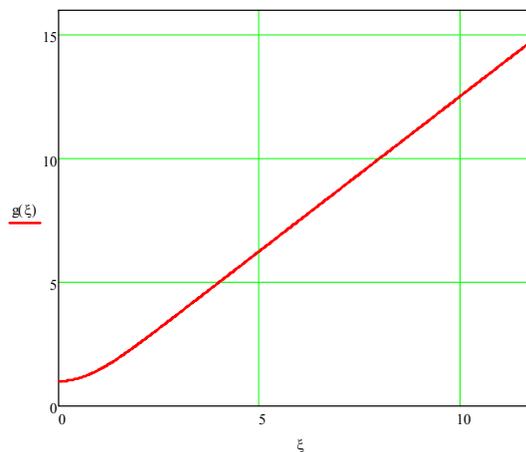


Fig. 4.1 - The transverse temperature function (N₂O)

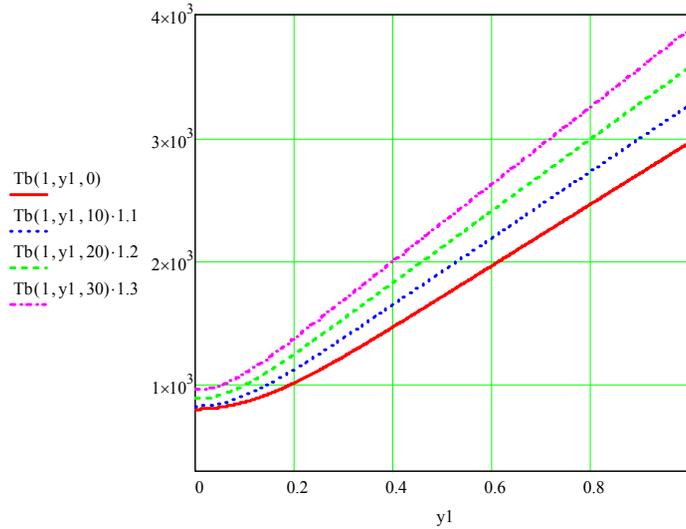


Fig. 4.2 - The transverse temperature time - variation at exit (N2O)

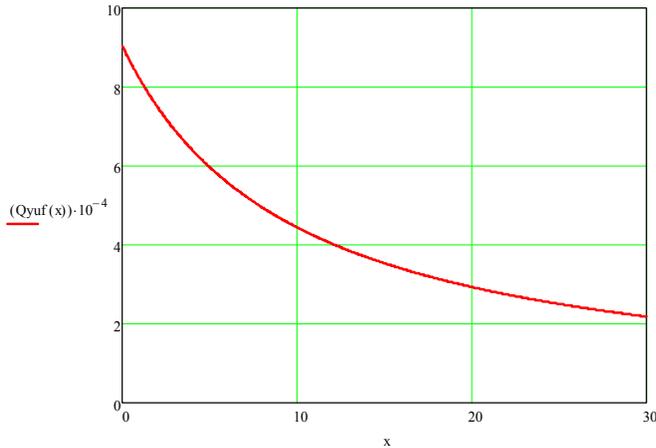
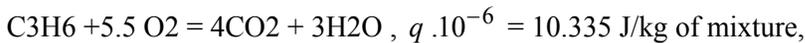
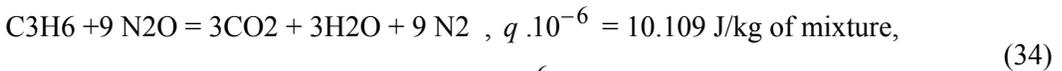


Fig. 4.3 - The average y- conduction heat flux (x- time, sec; (N2O))

Remark. The heats q released by polypropylene (C3H6; [3]) and HTPB (C4H6; [4]) in the stoichiometric reactions of premixed fuel and gaseous O₂, namely the reactions:



are very close (see(34)). This make possible an easier comparison with data from Ref.[4]. The geometry is however different: in the present case there is an axis of symmetry.

5. CONCLUSIONS

The analytical solutions given for the velocity and temperature fields prove themselves to adjust very well the available experimental results. Moreover by using the obtained analytical expressions for the temperature distribution one can formulate several general conditions for a reasonably efficient combustion in a channel with solid fuel walls. The contributions of this paper could be considered the following: 1) closed simple analytical

expressions for the temperature field are given; 2) using the obtained analytical expressions the definition of a quasi-stationary (quasi-uniform) burning is possible; 3) the connection between the solid wall regression and the injection velocity of the resulting burning products, minimal and maximal values being pointed out; 4) the evaluation of the combustion efficiency by comparison with a selected premixed gas combustion what has evidenced the small efficiency of the combustion in the analyzed system; 5) the emphasizing of a similarity of flows with various oxidizers that can be described with similar time functions $C_0(\tau)$ and $C_1(\tau)$. The slow variation of time function exponents in $C_0(\tau)$ and $C_1(\tau)$ indicates a good representation of the temperature distribution by our analytical solution. As well the proposed solution proves the possibility to maintain an almost constant in time flame temperature. The presence of dimensionless parameters like $\delta_0 = \frac{\bar{\rho}_{av} \bar{c}_{p,av}}{\lambda_{av}} \bar{u}_{rf} a_0$ - a combination between an entrance flux $\bar{u}_{rf} a_0$ and the thermal diffusivity $\bar{\lambda}_{av} / (\bar{\rho}_{av} \bar{c}_{p,av})$ is also interesting. The product $\delta_0 \tau$, ($\tau = 1 + \bar{t} \bar{u}_{rf} / a_0$), is able to include the time behaviour of the temperature distribution.

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