The rotational velocities evaluation for the engine mounts gyroscopic loads

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Abstract: The default values for the maximum pitch and yaw speeds from CS 23.371, seem to be too conservative that would result in overstressing of the structure. A simplified dynamic simulation is proposed to evaluate more confident velocities for a specific aircraft. The yawing condition is related to the “sudden rudder deflection” and a maneuver with “lateral gust”. The pitching conditions are a result of a “sudden elevator deflection”. The model takes into account the nonlinear effects of the aerodynamic coefficients and controls efficiencies.

Key Words: aircraft rotational speeds, pitching moment, gyroscopic loads, aircraft flight response, differential equation

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>wing reference span,</td>
</tr>
<tr>
<td>S</td>
<td>wing reference area,</td>
</tr>
<tr>
<td>C_L</td>
<td>lift coefficient,</td>
</tr>
<tr>
<td>C_m</td>
<td>pitching moment coefficient,</td>
</tr>
<tr>
<td>C_mq</td>
<td>pitch damping derivatives,</td>
</tr>
<tr>
<td>C_n</td>
<td>yawing moment coefficient,</td>
</tr>
<tr>
<td>C_nr</td>
<td>yaw damping derivatives,</td>
</tr>
<tr>
<td>I_x, I_y, I_z, I_prop</td>
<td>moment of inertia in roll, pitch and yaw or propeller,</td>
</tr>
<tr>
<td>M_g</td>
<td>gyroscopic moment,</td>
</tr>
<tr>
<td>L, D, T, G</td>
<td>lift, drag, thrust, weight,</td>
</tr>
<tr>
<td>M_a</td>
<td>aerodynamic pitching moment</td>
</tr>
<tr>
<td>M_gust</td>
<td>yawing moment due to the gust load on vertical tail,</td>
</tr>
<tr>
<td>p</td>
<td>propeller rotation speed,</td>
</tr>
<tr>
<td>V</td>
<td>airplane speed,</td>
</tr>
<tr>
<td>V_A</td>
<td>design maneuver speed,</td>
</tr>
<tr>
<td>V_B</td>
<td>design speed for maximum gust velocity,</td>
</tr>
<tr>
<td>q</td>
<td>pitch speed,</td>
</tr>
<tr>
<td>α, θ, γ</td>
<td>incidence, pitch angle, slope angle</td>
</tr>
<tr>
<td>r</td>
<td>yaw speed,</td>
</tr>
<tr>
<td>β</td>
<td>airplane sideslip angle,</td>
</tr>
<tr>
<td>δ_e, δ_f</td>
<td>control surface deflection, elevator or rudder</td>
</tr>
<tr>
<td>t, k</td>
<td>time, delay time factor.</td>
</tr>
</tbody>
</table>
1. GYROSCOPIC ACTION DUE TO THE AIRCRAFT ROTATIONAL SPEEDS

The rotating components on the aircraft: propeller, engine turbines or compressors are similar to a gyroscope and thus have similar properties.

All practical applications of the gyroscope are based upon two fundamental properties of gyroscopic action: rigidity in space and precession.

Precession is the resultant action, or deflection, of a spinning rotor when a deflecting force or moment is applied to its rim.

As it can be seen in Figure 1, when a force (or moment) is applied, the displacement is normal to the force direction.

Gyroscope moments:
\[ \tilde{M}_g, q \] is the yawing moment due to the pitching maneuver
\[ \tilde{M}_g, r \] is the pitching moment due to the yawing maneuver

Figure 1. Gyroscopic moments on the engine mount due to rotational speeds

It can be said that as a result of gyroscopic action on the rotating propeller (\( \tilde{p} \)): any yawing (\( \tilde{r} \)) around the vertical axis results in a pitching moment (\( \tilde{M}_g, r \)) and any pitching (\( \tilde{q} \)) around the lateral axis results in a yawing moment (\( \tilde{M}_g, q \)).

The detailed analysis of the gyroscopic moments from [2] and the requirements from [1], CS 23, AMC 23.371 give the equation 1 for the maximum gyroscopic pitching couple and the equation 2 for the yawing couple.

Pitching couple: \[ \tilde{M}_g, r = I_{prop} * \tilde{p} \times \tilde{r} \] (1)
The rotational velocities evaluation for the engine mounts gyroscopic loads

Yawing couple: \( \ddot{M}g, q = I_{prop} \dot{p} \times \dot{q} \) \hspace{1cm} (2)

where \( I_{prop} \) is the polar moment of inertia of the rotating items (propeller, engine rotors).

The relations 1 and 2 show that the gyroscopic moment (loads) are proportional to the values of the aircraft maneuver rotational speeds \( \dot{r} \) or \( \dot{q} \).

2. GENERAL REQUIREMENTS FOR THE GYROSCOPIC LOADS

The following paragraph is a summary from § CS 23 related to the critical flight conditions that are important in evaluating the gyroscopic loads on the engine mount.

“§23.371 Gyroscopic and aerodynamic loads

(a) Each engine mount and its supporting structure must be designed for the gyroscopic, inertial, and aerodynamic loads that result, with the engine(s) and propeller(s), if applicable, at maximum continuous r.p.m., under either:
(1) The conditions prescribed in §23.351 and §23.423; or
(2) All possible combinations of the following—
   (i) A yaw velocity of 2.5 radians per second;
   (ii) A pitch velocity of 1.0 radian per second;
   (iii) A normal load factor of 2.5; and
   (iv) Maximum continuous thrust….”

The conditions prescribed in paragraph §23.351 are related to the “Yawing conditions” and are detailed in:
 - §23.441 Maneuvering loads for Vertical Tail due to the “RUDDER MANEUVER”;
 - §23.443 Vertical Tail loads due to LATERAL GUST.

It can be seen that the maximum chosen yaw velocity used for the gyroscopic loads evaluation would be either the recommended value of 2.5 rad/s or a more realistic value according to the specific aircraft flight response from §23.441 and §23.443.

The conditions prescribed in paragraph §23.423 are related to the “Pitching conditions” due to “SUDDEN ELEVATOR MANEUVER”.

The maximum pitch velocity would be either the recommended value of 1 rad/s or a more realistic value according to the specific aircraft flight response from §23.423.

3. YAWING CONDITIONS

All the input data used in this analysis are an example consistent with other specific data for a real CS 23 Commuter Aircraft. These data are related to:
 - Aerodynamic data based on numerical and experimental analysis.
 - Mass and Inertia definition similar to those of a real aircraft.
 - Design Speeds according to the Flight Envelopes.

A dynamic simulator was made to estimate the evolution of the sideslip \( \beta(t) \) and the yawing speed, \( r(t) \), assuming two scenarios: sudden rudder deflection and lateral gust.

Due to the character of the motion, “sudden” rudder deflection or “sudden gust”, the analysis took into account only one differential equation in yaw, [4] and [5].

The formal equation is presented in 3.
\[ \dot{\beta} = A \dot{\beta} + B \beta + C \delta_r(t) + E \]  
(3)

The coefficients from (3) are shown in the relations 4, 5, 6 and 7:

\[ A = \frac{\rho V b^2 C_{nr}(C_L) }{4 I_{ZZ}} \]  
(4)

\[ B = -\frac{\rho V^2 S b Cn_{\beta}(\beta)}{2 I_{ZZ}} \]  
(5)

\[ C = -\frac{\rho V^2 S b Cn_{\delta_r}(\delta_r)}{2 I_{ZZ}} \]  
(6)

for rudder deflection or \( C = 0 \) for lateral gust

\[ E = 0 \text{ for rudder deflection or } E = -\frac{M_{\text{gust}}}{I_{ZZ}} \text{ for lateral gust} \]  
(7)

The real simulation assumes two cases for the inertia properties \( I_{zz} \): case E with low inertia and case A with high inertia.

A summary of the results for SUDDEN RUDDER DEFLECTION are given in Table 1.

<table>
<thead>
<tr>
<th>CASES: Flight Conditions</th>
<th>Inertia Izz (kg m(^2))</th>
<th>Lateral Gust Load (N)</th>
<th>Pilot Control ( \delta_r ) (deg)</th>
<th>Sideslip ( \beta ) (deg)</th>
<th>Yawing Speed (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: Sudden rudder</td>
<td>148734</td>
<td>-</td>
<td>28</td>
<td>23.7</td>
<td>-0.521</td>
</tr>
<tr>
<td>E: Sudden rudder</td>
<td>100092</td>
<td>-</td>
<td>28</td>
<td>23.14</td>
<td>-0.598</td>
</tr>
</tbody>
</table>

Figure 2 presents the variation of the yawing speed in SUDDEN RUDDER DEFLECTION.

**Case E: CS 23/441 (a), (1) + (2) SUDDEN RUDDER DEFLECTION**

**YAWING VELOCITY at SUDDEN RUDDER DEFLECTION**

![Figure 2. Yawing velocity for sudden rudder deflection](image-url)
Lateral Gust loads on the Vertical Tail are evaluated according to the procedure from CS 23.443. The results for LATERAL GUST conditions are given in Table 2.

<table>
<thead>
<tr>
<th>CASES: Flight Conditions</th>
<th>Inertia Izz (kg m²)</th>
<th>Lateral Gust Load (N)</th>
<th>Pilot Control δr (deg)</th>
<th>Sideslip β (deg)</th>
<th>Yawing Speed (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: Lateral gust</td>
<td>148734</td>
<td>17380</td>
<td>OFF</td>
<td>20.6</td>
<td>-0.437</td>
</tr>
<tr>
<td>E: Lateral gust</td>
<td>100092</td>
<td>16232</td>
<td>OFF</td>
<td>21.7</td>
<td>-0.518</td>
</tr>
</tbody>
</table>

Figure 3 presents the variation of the yawing speed in LATERAL GUST conditions.

*Case E: CS 23/443 LATERAL GUST at VB*

**YAWING VELOCITY in LATERAL GUST SIMULATION**

For the lower inertia value the maximum yawing speed is 0.598 (rad/s) in “SUDDEN RUUDER DEFLECTION”. It seems that this value is about 24% from the default maximum yaw speed of 2.5 rad/s from CS 23.371 (2).

The analysis made in this paper is proper for a specific aerodynamics and inertia data related to a commuter aircraft with a weight of 19000 lbs. In this case, the default values for the maximum yaw speed seem to be too conservative that would result in overstressing of the structure.

4. SUDDEN ELEVATOR DEFLECTION

The evaluation of the maximum pitching speed, \( q \), due to a sudden elevator deflection takes into account the classical equations of the flight mechanics written in ground axis, [5]. These are presented in relation 8.
\[
\begin{align*}
\{ 
    m \cdot \ddot{x}_G &= F_x & \text{eq. of motion along } x \text{ axis}, \\
    m \cdot \ddot{z}_G &= F_z & \text{eq. of motion along } z \text{ axis}, \\
    J_{yy} \cdot \ddot{\theta} &= M_y & \text{rotation equation around } CG,
\}
\]

(8)

The additional relations 9 will complete the system:
\[
\begin{align*}
    V &= \sqrt{\dot{x}_G^2 + \dot{z}_G^2} & \text{eq. of motion along } x \text{ axis}, \\
    \gamma &= \arctg \frac{\dot{z}_G}{\dot{x}_G} & \text{eq. of motion along } z \text{ axis}, \\
    \theta &= \gamma + \alpha & \text{rotation equation around } CG,
\end{align*}
\]

(9)

The right side of the system (8) has the well know definition for the forces as it is shown in 10.
\[
\begin{align*}
    F_x &= T \cos \theta - L \sin \gamma - D \cos \gamma \\
    F_z &= T \sin \phi + L \cos \lambda - G \\
    M_y &= M_a - T^* z_t
\end{align*}
\]

(10)

According to CS 23.423 and /3/ related to the recommended conditions for “sudden elevator deflection”, \( \delta e(t) \) is treated as input formal function given in (11).
\[
\delta e(t) = \delta_0 (1 - e^{-kt})
\]

(11)

The typical input “sudden elevator command” that takes into account a realistic delay time constant, \( k \), is presented in figure 4.

![Figure 4. Sudden elevator control command](image)

Due to that “sudden elevator deflection” will imply large pitch angles and, as a consequence, large incidence angles, the classical linear definition for the aerodynamic coefficients (\( C_L, C_D \)) will give non-realistic results for the trajectory and load factor.

The nonlinear construction was based on Wind Tunnel Results, CFD evaluation and some experimental data presented in “Aerofoil Sections” by F. W. Riegels. The polar for NACA 0012 for 360 deg incidence range, reference [7], is presented in figure 5.
So, it is proposed that the lift coefficient, $C_L$, and the drag coefficient $C_D$ to have fully nonlinear forms at large incidence angles that exceed the linear zone (more 15 deg), figure 6.

Figure 5. NACA 0012 polar for 360 deg incidence range

Figure 6. Lift and Drag, qualitative nonlinear representation
The results for “sudden elevator deflection” are presented in figure 7.
There are shown general data related to airplane kinematics and dynamics. These results are summarised as follows:

- speed, $V$
- pitch rotational speed, $q$
- load factor, $N_z$
- pitch angle, $\theta$
- incidence, $\alpha$
- flight slope, $\gamma$
- elevator deflection, $\delta_e$
- flight trajectory

The maximum pitch velocity is 0.67 rad/s that is less than the default value from CS 23.371 (a), (2), (ii) of 1 rad/s.
These results are proper to the all aerodynamic and inertial properties for a specific given aircraft. More accurate input data will lead to more realistic output results.

5. CONCLUSIONS
The real dynamic response of the aircraft to the rudder and elevator sudden maneuvers was simulated to obtain more specific results.
A special attention was given to the level of accuracy for the input data: more accurate input data will lead to more realistic output results.
The dynamic simulation takes into account the nonlinear effects of the aerodynamic coefficients and controls efficiencies.

The sensitivity of the inertial properties $I_{zz}$ on the yawing rotational speed was presented.

The rotational speeds, specific to this Commuter Aircraft definition, gives lower values than the default values from regulation.

The yawing speed is 24% and the pitching speed is 68% from the recommended values from CS 23.371.

It seems that the maximum default rotational speeds proposed in CS 23.371 (yaw of 2.5 rad/s and pitch of 1 rad/s, and [6]) would cover all “unexpected” flight conditions but will lead to a higher level for the gyroscopic loads on the engine mount and, as a consequence, a penalty on the airplane weight.

REFERENCES