Numerical Methods in Fluid Dynamics (Metode Numerice in Dinamica Fluidelor)

Sterian DANAILA, Corneliu BERBENTE

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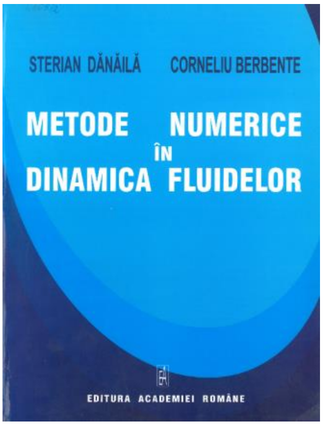

This book is addressed to all professionals engaged in research & design, education, ranging from PhD students to graduates as well as students from Engineering Sciences (Aerospace, Mechanics, Power, Applied Sciences, Automotives, Medical Applications and Biotechnical Systems), Mathematics and Physics. Its large amount of information, thoroughly explained within a 715 A4 size pages volume, is structured in 2 parts: FUNDAMENTALS (Chapters 1÷6) and APPLICATIONS (Chapters 7÷12). The topics are the mathematical modeling and numerical simulation of complex and intricate problems of fluid dynamics. Fully details are given for the hereby presented mathematical models expressed by partial derivatives equations or partial derivatives equations systems that allow us to study different cases of fluid flows, such as: the steady and non-steady regimes, incompressible and compressible flows (developed for subsonic, transonic and supersonic velocities), laminar and/or turbulent flows, for inviscid as well as viscous fluids.

A thorough study of the **Partial Differential Equations** PDE' is presented in **Chapter 1**; the approach consists in presenting real phenomena with their associate mathematical model expressed by PDE's. A classification based on physical properties (§1.2) followed by the study of first order partial differential equations (§1.3), first order linear differential systems (§1.4) and first order non-linear differential equations (§1.6), first order non-linear differential systems (§1.7), as well as the explanation of the properties of non-linear hyperbolic systems (§1.8), the detailing of the hyperbolic, parabolic and elliptic equations within §1.9, integral representations of solutions (§1.10) for the Laplace equation and Prandtl-Glauert equation, introduces the reader to these topics and enables a better understanding due to the very good organization of the material, according to criteria and a sharp logic & argumentation. Following this approach, the authors express more clearly the appropriate modeling of fluid flows and body-flow interactions, in case of intricate geometry for both solid bodies and boundaries. The non-linearity and non-steady feature of the flow considered by the authors, allow an improved mathematical model of the real study cases.

Chapter 2 is dedicated to the **Finite Differences Methods** FDM'; the basics: explicit & implicit finite differences and coordinates are presented in §2.1, which allows the discretization of the partial differential equations into finite difference equations (§2.2); fully details are provided for the study of the consistency (§2.4), stability (§2.5) and convergence (§2.9), with significant examples that have been chosen, such as the convective transport equation and equation of diffusion. There also have been expressed out details about: boundary and initial conditions, matrix representation of finite differences scheme, explicit versus implicit schemes, multidimensional problems, when associated to the Von Neumann method (§2.6), matrix method (§2.7) and method of modified equation (§2.8). There are presented comparatively many schemes and possible means of their improvement, for the linear convection equation (§2.10 – the explicit and implicit Euler schemes, upwind, Lax, Leapfrog, Lax-Wendroff, Mac Cormack, Beam-Warming and third order schemes) and the equation of diffusion (§2.11 – Crank-Nicolson, Richtmyer & Morton, DuFort-Frankel schemes); for the multidimensional problems (§2.11) the numerical solution can be obtained by using either the alternating-directions implicit ADI method, the alternating-directions explicit ADE method or the Keller's box method.

Chapter 3 deals with the **Finite Elements Methods** FEM'; basics of the weighted residuals method (§3.1 – collocation, method of moments, Galerkin), are presented by comparison, with complete details and significant examples provided by the authors; a prodigious description regarding the application of the algorithm (§3.4) for one-dimensional (§3.6), two-dimensional (§3.7) and three-dimensional (§3.8) finite elements is also given; the convergence is studied in §3.9, while in §3.10 several propagation issues are presented, such as the diffusion equation and the wave equation.

Within **Chapter 4** the **Boundary Elements Methods** BEM' are presented; theirs application requires the presentation of the integral equation weighted by the fundamental solutions of the partial derivatives equation (i.e. the Laplace equation) to be solved, followed by the discretization of the domain's boundaries; the interpolating functions allow us to obtaining the numerical solution of the equations system.

Chapter 5 is focused on the **Finite Volumes Methods**; basic notions of the conservative discretization (§5.2) such as the numerical sources, numerical flux and convergence are introduced; the integral approximations (§5.3) represented by the Helmholtz equation, finite volume schemes (§5.4) and the diffusion equation (§5.5) are studied; a thorough analysis of the stationary convection-diffusion equation (§5.6) is given, with a detailed presentation of numerical schemes: centered space, upwind with artificial viscosity, exponential, hybrid, power law, Patankar, QUICK and multidimensional.

The **integration of conservation laws** is studied in **Chapter 6**; the Burgers equation (§6.1), as well as the complete Burgers equation (§6.2) are considered as representative examples and analyzed.

Chapter 7 prepares the ground for the applications in the second part, being dedicated to the **conservation laws**; by the aid of the transfer theorem (§7.2), there are presented the mass conservation (§7.3), the momentum conservation (§7.4), the energy conservation (§7.5) and their appropriate logical deduction.

Large classes of **mathematical models** used in **fluid dynamics** are detailed within **Chapter 8**, such as: the Navier-Stokes model (§8.1), Reynolds Averaged Navier-Stokes RANS (§8.2), Euler model (§8.3), Boundary Layer model (§8.4), the potential model (§8.6) and the perturbation potential model (§8.7). The selection (of one or another) of these models to describe a real flow issue is done according to the desired numerical accuracy, i.e. with the appropriate consideration of work hypothesis; the less the number of simplifying assumptions, the more accurate numerical results, but this feature comes in pair with the increased effort during computations. The turbulence models described in §8.5 are necessary to complete the description of the flow issue considered as study case. The following chapters (9÷12) present with fully details the way of integration of different flow models; the work is reader-oriented, the topics are gradually introduced i.e. from the rather simpler (in relative terms) flow model to the most complicate flow model. Therefore, the **integration of the perturbation model** is explained in **Chapter 9**; for such purpose, one can select either the panel method (§9.3) or the Murman-Cole method (§9.4); these methods can be proven significantly accurate for rather complex applications, like the two-dimensional incompressible steady flows, three-dimensional supersonic steady flows and the three-dimensional compressible unsteady flows.

The **integration of the potential model** is presented in **Chapter 10**; the model discretization is focused in §10.2 on the subsonic regime and in §10.3 on the transonic flow. The authors provide detailed comparisons for the numerical formulations (e.g. the finite difference formulations versus the finite volume formulations, both used for subsonic flows); in case of transonic flows, there have been introduced new concepts: artificial flux, artificial compressibility and artificial viscosity; The authors present efficient techniques purposed for solving the algebraic system of equation (§10.5), like: the successive line over-relaxation methods and alternating directions methods.

A thorough attention for the details is given in **Chapter 11**, dedicated to the **integration of Euler model**. The mathematical properties of the Euler model are exposed in a large extent in §11.2, for both integral and conservative differential formulations; the most frequently used schemes are presented as follows: centered spaces schemes (§11.3), upwind schemes with vector flux splitting (§11.4), Godunov type upwind schemes (§11.5), second order upwind schemes (§11.6 – TVD, MUSCL), TVD schemes for Euler system (§11.7 – for one-and two-dimensional cases), ENO and WENO schemes (§11.8); the last but not least are the boundary conditions, whose implementation has been explained in §11.9, for multidimensional cases. The Euler model contains non-linear terms of the convective transport of the fluid's energy and includes the effects of the compressibility; for these reasons, the Euler model can be used with convenient accuracy for describing real issues, as the shock waves flows. The limitations of the Euler model are due to the omission of the viscous terms (that are responsible for generating the dissipation of energy); at wall vicinity and for restricted but significant areas with discontinuities, it is important to consider the viscous terms, for which the Euler model is not appropriate.

The **integration of the Navier-Stokes model**, the most complex of all models used in Fluid Dynamics for the numerical solving of intricate problems, is analyzed in **Chapter 12**. The discretization in compressible regime is fully detailed in §12.3, focusing the numerical explicit schemes (Mac Cormack, vector flux splitting, Runge-Kutta) and implicit schemes (one time step, first and second order convective flux splitting) and the LU factorization. The formulation of Navier-Stokes equations with pseudo-compressibility and the pressure field correction based methods introduce in a more direct way the discretization in incompressible regime (§12.4). A large **References list** with more than 300 entries provides thus a thorough documentation and knowledge.

This book represents a major accomplishment, proven by its structure and organized information; both the theoretical frame and applications as worked examples presented in full details, introduce the reader and prospective user to complex issues (such as aircraft and inner flow aerodynamics) of significant interest from the Fluid Dynamics.

PhD. Eng. Irina Carmen ANDREI,
INCAS – National Institute for Aerospace Research "Elie Carafoli"
Flow Physics Department/ Numerical Simulation Unit
B-dul Iuliu Maniu 220, Bucharest 061126, Romania
andrei.irina@incas.ro