Aeroservoelastic flutter and frequency response interactions on the CL-704 aircraft

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Abstract: *Two methods for the aerodynamic forces conversions from frequency into Laplace domain were conceived and validated: Least Squares LS and Minimum State MS, on the Bombardier CL-604 aircraft. A new feature was added in these two methods consisting in the writing of the error calculated by LS and MS classical methods under an analytical form similar to the LS and MS form of approximated aerodynamic forces; this error was once again minimized. Then, new methods using this new feature were called: Corrected Least Squares CLS and Corrected Minimum State CMS methods (as error was once again corrected). All these four methods were programmed in Matlab to approximate the unsteady aerodynamic forces from frequency domain to Laplace domain.*

Key Words: *aerodynamic forces, Laplace domain, classical methods, flutter algorithm, frequency domain, Matlab codes, aeroservoelasticity studies*

1. INTRODUCTION

Aerodynamic forces in frequency domain were firstly calculated in Nastran by use of the Doublet Lattice Method DLM at one Mach number M = 0.88 on a CL-604 Bombardier Aerospace aircraft. These forces were secondly converted with the above four (two classical and two new corrected) mentioned methods of aerodynamic forces conversions from frequency into Laplace domain by use of Matlab codes developed at ETS and delivered to Bombardier Aerospace team.

First set of flutter frequencies and speeds were calculated by integration of aerodynamic forces initially calculated in Nastran in frequency domain into the flutter algorithm. Second set of flutter speeds and frequencies were calculated by integration of aerodynamic forces calculated by the four different methods of aerodynamic forces conversions from frequency into Laplace domain into the flutter algorithm.

Matlab codes were developed at ÉTS to calculate and compare the second set of flutter speeds and frequencies in the Laplace domain with the first set of initially obtained flutter speeds and frequencies in the frequency domain. The first set of flutter results expressed in terms of flutter speeds and frequencies was compared with the second set of flutter results obtained with aerodynamic forces obtained by four methods, and following this comparison, and in agreement with Bombardier Aerospace team, the LS method with a number of four lag terms was chosen as the best method for aerodynamic forces conversion from frequency into Laplace domain (from execution time and precision point of view) for the CL-604 Bombardier Aerospace aircraft. In addition, a new routine for eigenvalues and eigenvectors studies (necessary for flutter frequencies and speeds calculations) developed at ÉTS gave a very good and a much better flutter analysis results visualization than the previous program used at Bombardier, which is a fact extremely important in flutter analysis of future aircraft at Bombardier Aerospace. Following the comparison between the results obtained with these two methods, the best method was chosen from these two methods in common agreement with Bombardier Aerospace team. The aircraft frequency response was computed in both open loop and closed loop cases for aeroservoelasticity studies. The Matlab codes for these cases were developed at ÉTS separately of Bombardier Aerospace and a comparison between obtained results in both cases was done. Same types of results were obtained with both Matlab codes developed separately by both teams. Theory and results obtained during this period of time are here presented.

2. BIBLIOGRAPHICAL RESEARCH

The aeroservoelastic codes (with respect to Nastran code which is only an aeroelastic code) implemented the unsteady aerodynamic forces conversion from frequency $Q(k,M)$ to Laplace domain $Q(s)$. All the aeroservoelastic codes used mainly the following two methods for the unsteady aerodynamic forces conversion which are:

1. Least Squares LS method is used in the aeroservoelastic computer programs: ADAM [1], ISAC at Nasa Langley Research Center [2, 3] and STARS at NASA Dryden Flight Research Center (Dr Gupta [4, 5] and Mr Marty Brenner [6, 7]).
2. Minimum State MS method is used in the aeroservoelastic computer programs: ASTROS and ZAERO (Dr Moti Karpel [8, 9, 10 , 11, 12, 13] is the main author for the MS method and works in collaboration with NASA Langley Research Center [11] and Zona Technology [14,15,16]). MS method is used also in France at Airbus and Onera (Zimmermann [17] et Porion [18]),

and in addition:

3. A 3rd approximation method is used at Boeing Company in St-Louis (Mr Dale Pitt [19] is the main author for this method) which is still not very popular with respect to the LS and MS methods [20, 21, 22].

These methods were computed in the aeroservoelastic codes as well as in Matlab at NASA DFRC on the F-18 SRA at ETS, LARCASE [23]. Following a comparison between results obtained with the LS, CLS, MS and CMS methods that are described in our previous works [24]- [35], the LS method is used due mainly to its time of execution.

3. INTEGRATION OF THE AERODYNAMIC APPROXIMATION INTO AEROELASTIC EQUATIONS OF MOTION (SELECTION OF THE BEST METHOD)

Two approaches for the LS method integration in the flutter equations were studied and they are described in the following section:

3.1 First formulation of the Least Squares *LS* method applied in the flutter equations of motion (by use of a small A matrix)

The aeroelastic aircraft equations of motion under the aerodynamic forces influences are:

$$\mathrm{M}\ddot{\eta} + \mathrm{C}\dot{\eta} + \mathrm{K}\eta - q_{\mathrm{dyn}}\mathrm{Q}(\mathrm{k})\eta = 0 \tag{1}$$

where M is the mass matrix, C is the damping matrix, K is the stiffness matrix, Q is the aerodynamic forces matrix, q_{dyn} is the dynamic pressure and η is the generalized coordinates vector. The matrix Q(k) is computed in NASTRAN by the Doublet Lattice Method DLM method in the subsonic regime. The aerodynamic forces matrix is computed for a range of reduced frequencies k at one Mach number M which depends on the aircraft true airspeed V and is written under the following form:

$$\mathbf{Q}(k) = \mathbf{Q}_R(k) + j\mathbf{Q}_I(k) \tag{2}$$

where Q_R is the real part of Q and Q_I is the imaginary part of Q. Equation (2) is introduced in equation (1), so that the following system of equations is obtained:

$$M\ddot{\eta} + \left[C - \frac{\bar{c}}{2kV} q_{dyn} \mathrm{Q}_I(k)\right]\dot{\eta} + \left[K - q_{dyn}\mathrm{Q}_R(k)\right]\eta = 0 \tag{3}$$

where no excitation force applies. From equation (3), the second derivative of generalized coordinates is obtained:

$$\ddot{\eta} = -M^{-1}\left[C - \frac{\bar{c}}{2kV} q_{dyn} Q_I(k)\right]\dot{\eta} - M^{-1}\left[K - q_{dyn} Q_R(k)\right]\eta = 0 \tag{4}$$

Equation (4) is rearranged under the following matrix form:

$$\begin{bmatrix}\ddot{\eta}\\ \dot{\eta}\end{bmatrix} = \begin{bmatrix} -M^{-1}\left[C - \frac{\bar{c}}{2kV} q_{dyn} Q_I(k)\right] & -M^{-1}\left[K - q_{dyn} Q_R(k)\right]\eta \\ I & 0 \end{bmatrix}\begin{bmatrix}\dot{\eta}\\ \eta\end{bmatrix} \tag{5}$$

Matrix equation (5) may be written as follows:

$$\dot{\mathbf{x}} = \mathbf{A}\cdot\mathbf{x} \tag{6}$$

where

$$\mathbf{x} = \begin{bmatrix}\dot{\eta}\\ \eta\end{bmatrix} \text{and } A = \begin{bmatrix} -M^{-1}\left[C + \frac{\bar{c}}{2kV} q_{dyn} Q_I(k)\right] & -M^{-1}\left[K + \frac{\bar{c}}{2kV} q_{dyn} Q_I(k)\right] \\ I & 0 \end{bmatrix}$$

3.2 Second application of the Least Squares *LS* method in the flutter equations of motion (by use of a big A matrix)

The aerodynamic forces matrix expressed by Pade approximations is introduced into eq. (1) and we obtain:

$$\mathrm{M}\ddot{\eta} + \mathrm{C}\dot{\eta} + \mathrm{K}\eta - q_{\mathrm{dyn}}\left(\mathrm{A}_0 + \mathrm{A}_1\bar{\mathrm{s}} + \mathrm{A}_2\bar{\mathrm{s}}^2 + \sum_{i=1}^{4} A_{i+2}\frac{\bar{s}}{\bar{s} + \frac{V}{b}\beta_i}\right)\eta = 0 \tag{7}$$

where $\bar{s} = \mathrm{j}k$ is modified Laplace domain. The state variable X_i is next defined:

$$X_i = \frac{s}{s + \frac{V}{b}\beta_i}\eta \tag{8}$$

Equations (7) can be written under the following form:

$$\left[\mathrm{M} - q_{dyn}\mathrm{A}_2\left(\frac{b}{V}\right)^2\right]\ddot{\eta} + \left[\mathrm{C} - q_{dyn}\mathrm{A}_1\left(\frac{b}{V}\right)\right]\dot{\eta} + \left(\mathrm{K} - q_{\mathrm{dyn}}\mathrm{A}_0\right)\eta - \mathrm{q}_{\mathrm{dyn}}\mathrm{A}_3\mathrm{X}_1 - \mathrm{q}_{\mathrm{dyn}}\mathrm{A}_4\mathrm{X}_2 - \mathrm{q}_{\mathrm{dyn}}\mathrm{A}_5\mathrm{X}_3 - \mathrm{q}_{\mathrm{dyn}}\mathrm{A}_6\mathrm{X}_4 = 0 \tag{9}$$

Equations (9) may be written as follows:

$$\tilde{M}\ddot{\eta}+\tilde{C}\dot{\eta}+\tilde{K}\eta - q_{dyn}A_3X_2 - q_{dyn}A_4X_2 - q_{dyn}A_5X_3 - q_{dyn}A_6X_4 = 0 \tag{10}$$

where $\tilde{M} = M - q_{dyn}A_2\left(\frac{b}{V}\right)^2$, $\tilde{C} = C - q_{dyn}A_1\left(\frac{b}{V}\right)$ and $\tilde{K} = K - q_{dyn}A_0$

Therefore, we obtain the following matrix system of equations:

$$\begin{pmatrix} \eta \\ \dot{\eta} \\ \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{pmatrix} = \begin{pmatrix} 0 & I & 0 & 0 & 0 & 0 \\ -\tilde{M}^{-1}K & -\tilde{M}^{-1}C & q_{dyn}\tilde{M}^{-1}A_3 & q_{dyn}\tilde{M}^{-1}A_4 & q_{dyn}\tilde{M}^{-1}A_5 & q_{dyn}\tilde{M}^{-1}A_6 \\ 0 & I & -\frac{V}{b}\beta_1 I & 0 & 0 & 0 \\ 0 & I & 0 & -\frac{V}{b}\beta_2 I & 0 & 0 \\ 0 & I & 0 & 0 & -\frac{V}{b}\beta_3 I & 0 \\ 0 & I & 0 & 0 & 0 & -\frac{V}{b}\beta_4 I \end{pmatrix} \begin{pmatrix} \eta \\ \dot{\eta} \\ X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix} \tag{11}$$

which may be again written under the following form:

$$\dot{\mathbf{x}} = \mathbf{A}\cdot\mathbf{x} \tag{12}$$

where

$$\mathrm{x} = \begin{bmatrix} \eta \\ \dot{\eta} \\ X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} \text{ and } A = \begin{pmatrix} 0 & I & 0 & 0 & 0 & 0 \\ -\tilde{M}^{-1}K & -\tilde{M}^{-1}C & q_{dyn}\tilde{M}^{-1}A_3 & q_{dyn}\tilde{M}^{-1}A_4 & q_{dyn}\tilde{M}^{-1}A_5 & q_{dyn}\tilde{M}^{-1}A_6 \\ 0 & I & -\frac{V}{b}\beta_1 I & 0 & 0 & 0 \\ 0 & I & 0 & -\frac{V}{b}\beta_2 I & 0 & 0 \\ 0 & I & 0 & 0 & -\frac{V}{b}\beta_3 I & 0 \\ 0 & I & 0 & 0 & 0 & -\frac{V}{b}\beta_4 I \end{pmatrix} \tag{13}$$

This formulation has been chosen to be the best one to approximate aerodynamic forces in the Laplace domain. In the next section, a comparison of results obtained at ETS versus results obtained at Bombardier Aerospace is realized for the numerical flutter results (damping and frequencies versus flutter speeds).

4. COMPARISON OF THE SET OF NUMERICAL FLUTTER RESULTS

4.1 Comparison of the first set of numerical flutter results (*damping* and *frequencies* versus *flutter speeds*)

At ETS and Bombardier Aerospace, separate codes were developed, and compared. Results were obtained following the Least Squares *LS* application (by use of a big matrix *A*) using the same input data. The comparison of the first set of results gave almost the same flutter results at ETS and at Bombardier Aerospace. Following same input data are used in the computer codes (at ÉTS and Bombardier Aerospace) and were provided to ETS by Bombardier Aerospace for validation purposes. These input data are the following:

- Mass, damping and stiffness matrices M, C, K for 110 modes

  Among these 110 modes, there are 6 rigid modes, 50 elastic anti-symmetric modes, 44 elastic symmetric modes and 10 control modes.
- Reduced frequencies *k*

  k = [0.001 0.1 0.3 0.5 0.7 0.9 1.1 1.4]
- Equivalent airspeed EAS is:

  EAS = [0.01,0.1,0.4,1,2,3.5,5.5,8,11,15,21,29,40:10:1000] knots
- Mach number *Mach* = 0.88;
- *Modes* = [2,4,6,8,9,10,12,13,16,17,20,21,22,24,27,28,31,33,35,39];

Figures 1 and 2 show the frequencies and damping versus Equivalent Air Speed *EAS*. Figure 1 shows the flutter results obtained at Bombardier Aerospace and Figure 2 shows the flutter results obtained at ETS by use of the same input data on the same aircraft as the ones used at Bombardier Aerospace.

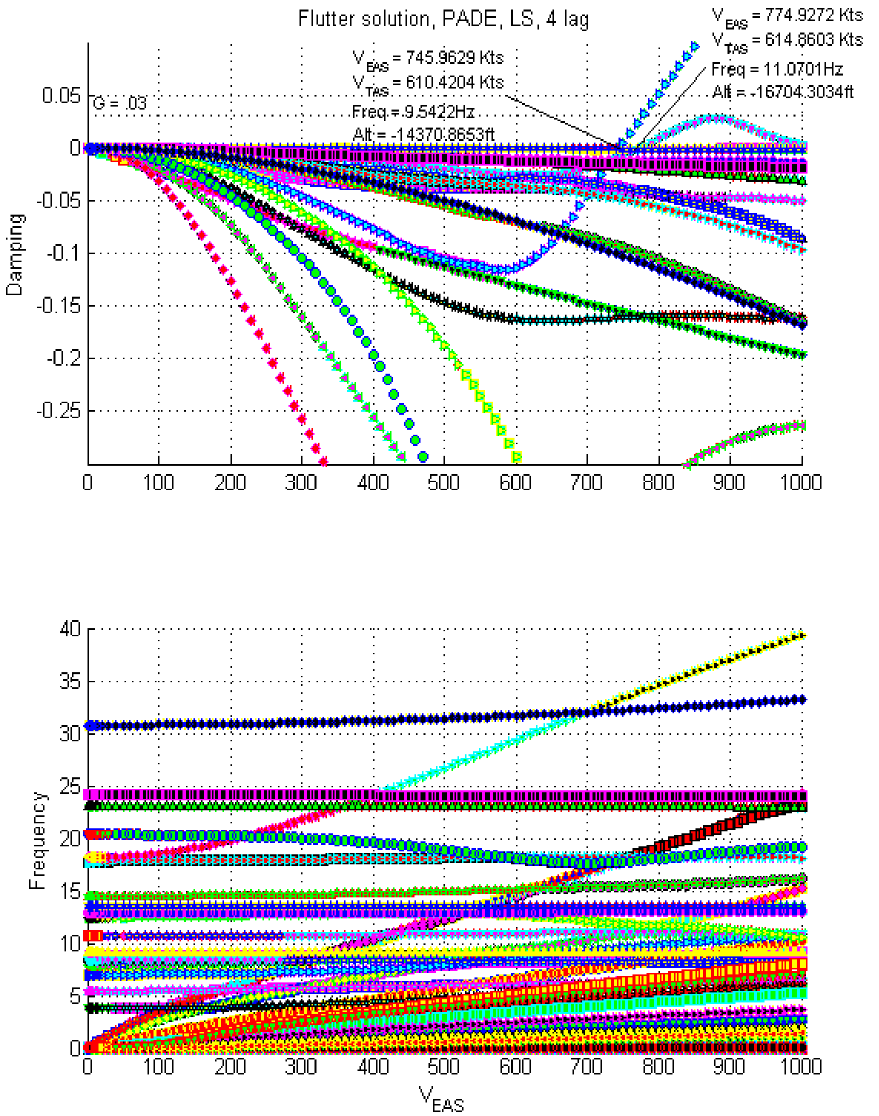

Figure 1. Frequencies and damping versus Equivalent Airspeed (Bombardier Aerospace computer programs)

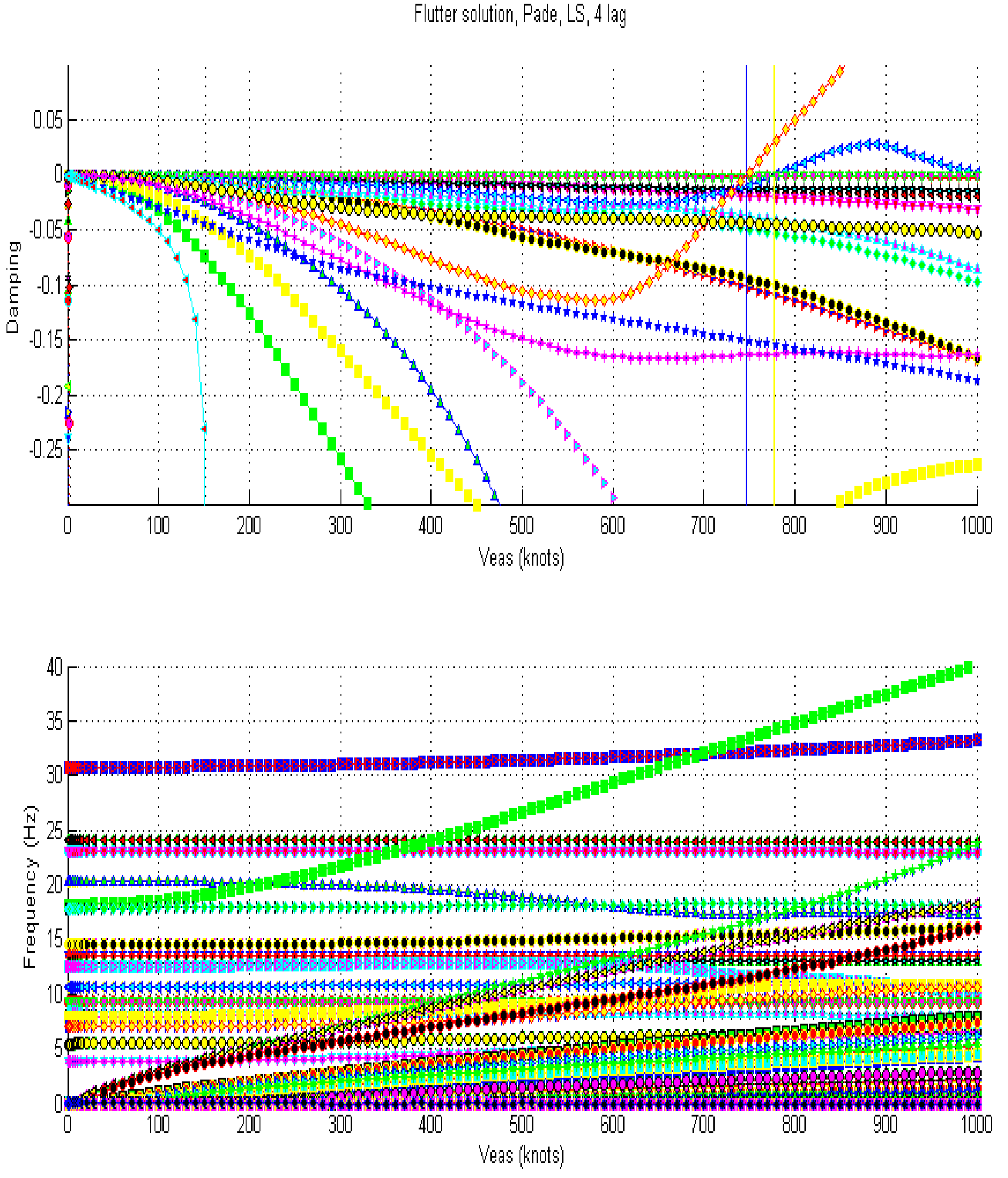

Figure 2. Frequencies and damping versus Equivalent Airspeed (ETS computer programs)

Flutter results shown in Figures 1 and 2 are summarized in Table 1.

Table 1. Comparison of first set of flutter results obtained at Bombardier Aerospace versus flutter results obtained at ETS

	Bombardier Aerospace results		**ETS results**	
	First flutter F1	Second flutter F2	First flutter F1	Second flutter F2
EAS (knots)	745.9629	774.9272	746.6881	777.03
TAS (knots)	610.4204	614.8603	610.5354	615.1799

Frequency (Hz)	9.5422	11.0701	9.5387	11.072
Altitude (ft)	-14370.8653	-16704.3034	-14430.0352	-16871.7193

The minor differences are explained by the optimization codes used for the aerodynamic forces by the *LS* method at ETS and at Bombardier.

4.2 Comparison of second set of results (*damping* and *frequencies* versus *flutter speeds*)

Following the corrections of the results presented in the previous section, much closest results are obtained between Bombardier and ETS (see Table 1).

Table 2 Comparison of second set of flutter results obtained at Bombardier Aerospace versus flutter results obtained at ETS

	Bombardier Aerospace results		**ETS results**	
	First flutter F1	Second flutter F2	First flutter F1	Second flutter F2
EAS (knots)	745.9629	774.9272	745.9622	774.926
TAS (knots)	610.4204	614.8603	610.4223	614.8623
Frequency (Hz)	9.5422	11.0701	9.5422	11.0701
Altitude (ft)	-14370.8653	-16704.3034	-14370.8078	-16704.2062

Firstly, differences in these results in the previous section were mainly dues to the ***format of the initial lag terms*** β_0 provided by the Bombardier team to ETS (only 4 decimals were considered):

$$\boldsymbol{\beta_0} = \mathbf{0.1000, 0.4633, 0.8266, 1.1900.}$$

We should have considered β_0 under the long format, as the Bombardier team wrote a computer code to calculate β_0's and obtained from his code the following values:

$$\boldsymbol{\beta_0} = \mathbf{0.10000000000000, 0.46333333333333, 0.82666666666667, 1.19000000000000.}$$

With these corrections, the same values of optimal lag terms β are obtained at ETS as the ones obtained by the Bombardier team, which are expressed in the long format as follows:

$$\boldsymbol{\beta} = \mathbf{0.02028342615263, 0.71738957626484, 0.82035613775127, 0.84113466008217}.$$

Secondly, differences were in the way of calculating the optimal lag terms β's, as Bombardier team has chosen to obtain them in increasing order while ETS has not considered any rule to obtain them.

The same values are obtained for the following coefficients A_0, A_1, A_2, A_3, A_4, A_5 and A_6, and for the *C* matrix. The values of C's are compared from the command in both codes (Bombardier and ETS):

C = LSaeroelastic(beta0,Qr(:,:,:,i),Qi(:,:,:,i),freq,flag)

and the differences between C calculated at ETS and C calculated at Bombardier are found equal to zero, which showed the fact that the A coefficients of C matrix are the same in both cases.

Figure 3 shows the flutter results (frequencies and damping) versus the equivalent airspeed for the second set of comparison) obtained at ETS that are the same as the ones obtained at Bombardier.

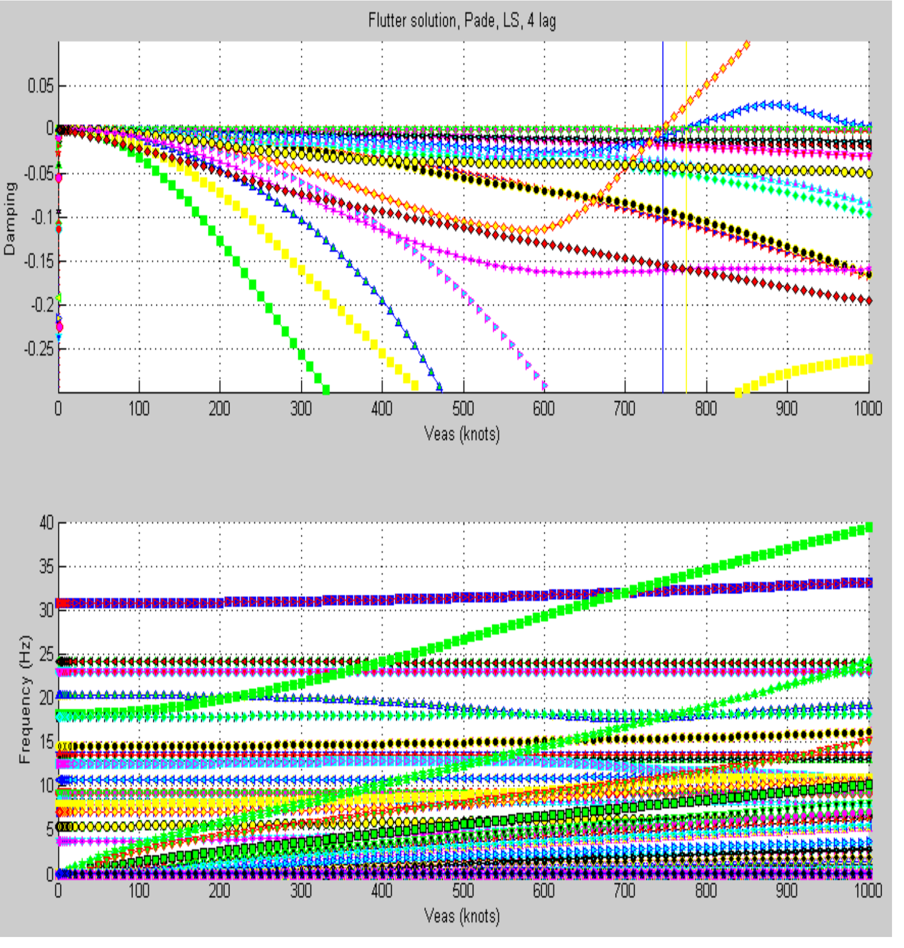

Figure 3. Frequencies and damping versus Equivalent Airspeed – second set of flutter results comparison (ETS computer programs)

The here described formulation is recommended to apply for an aeroservoelastic closed loop system and therefore is recommended in the future work.

5. INTEGRATION OF AERODYNAMIC APPROXIMATIONS AND CONTROLS IN AEROSERVOELASTIC EQUATIONS

In this section, the frequency response theory used by Bombardier Aerospace team and by ETS team are described.

5.1 Frequency response theory used by Bombardier Aerospace team

The aeroelastic equation of motion of the aircraft under the aerodynamic unsteady forces influences is expressed with the following equation:

$$\mathrm{M}\ddot{\eta} + \mathrm{C}\dot{\eta} + \mathrm{K}\eta - q_{\mathrm{dyn}}\mathrm{Q}(k)\eta = 0 \tag{14}$$

where **M** is the mass matrix, **C** is the damping matrix, **K** is the stiffness matrix, **Q** is the aerodynamic forces matrix, q_{dyn} is the dynamic pressure and η is the generalized coordinates vector. The unsteady aerodynamic forces **Q**(k) matrix is computed in NASTRAN by the

Doublet Lattice Method DLM method for one Mach number and a range of reduced frequencies k and is a complex aerodynamic matrix – so that is written as follows:

$$\mathbf{Q}(k)=\mathbf{Q}_R(k)+\mathrm{j}\mathbf{Q}_I(k) \tag{15}$$

where $\mathbf{Q}_R$ is the real part of **Q** and $\mathbf{Q}_I$ is the imaginary part of **Q.** Aerodynamic forces **Q**(k) are approximated with the LS method with 4 lag terms β_1, β_2, β_3 and β_4 by use of Padé polynomials as follows :

$$\mathrm{Q}(k) = \mathrm{A}_0 + \mathrm{j}k\mathrm{A}_1 + (\mathrm{j}k)^2 A_2 + \frac{\mathrm{j}k}{\mathrm{j}k+\beta_1}A_3 + \frac{\mathrm{j}k}{\mathrm{j}k+\beta_2}A_4 + \frac{\mathrm{j}k}{\mathrm{j}k+\beta_3}A_5 + \frac{\mathrm{j}k}{\mathrm{j}k+\beta_4}A_6 \tag{16}$$

where A_0, A_1, A_2, A_3, A_4, A_5 and A_6 are Padé polynomials coefficients of dimensions equal to the **Q**(k) matrix dimensions and are calculated from the Least Squares *LS* algorithm, and where β_1, β_2, β_3 and β_4 are the four lag terms calculated by the *fmincon* Matlab function optimization algorithm. Unsteady forces matrix given by equation (16) is replaced in equation (14) and following equation is obtained:

$$\mathrm{M}\ddot{\eta} + \mathrm{C}\dot{\eta} + \mathrm{K}\eta - q_{\mathrm{dyn}}\left(\mathrm{A}_0 + \mathrm{A}_1\bar{s} + \mathrm{A}_2\bar{s}^2 + \sum_{i=1}^{4} A_{i+2}\frac{\bar{s}}{\bar{s}+\frac{V}{b}\beta_i}\right)\eta = 0 \tag{17}$$

where $\mathbf{j}k$ is replaced by

$$\bar{s} = \mathrm{j}k = \mathbf{j}\,\omega b / V \tag{18}$$

which is called the modified Laplace domain variable and where i = 1,2,3,4.

Generalized coordinates η are written under the oscillatory form as $\eta = e^{\mathbf{j}\omega t}$ then the first and second derivatives of generalized coordinates are written as:

$$\dot{\eta} = \mathrm{j}\omega^{ej\omega t} = \mathrm{j}\omega\eta \text{ and } \ddot{\eta} = -(\omega^2)^{ej\omega t} = -\omega^2\eta . \tag{19}$$

We next calculate, from equations (18) and (19):

$$\bar{s}\eta = \frac{\mathrm{j}\omega b}{V}\eta = \frac{b}{V}\mathrm{j}\omega\eta = \frac{b}{V}\dot{\eta} \tag{20}$$

and

$$\bar{s}^2\eta^2 = \left(\frac{\mathrm{j}\omega b}{V}\right)^2\eta^2 = -\omega^2\frac{b}{V}\eta^2 \tag{21}$$

The state variable X_i is next defined :

$$\mathrm{X}_i = \frac{\bar{s}}{\bar{s}+\frac{V}{b}\beta_i}\eta \text{ where } i = 1,2,3,4 \tag{22}$$

From where, by inverse Laplace transform, we obtain following equation:

$$\dot{X}_i + \frac{V}{b}\beta_i\mathrm{X}_i = \dot{\eta}_i \tag{23}$$

Equations (20), (21) and (23) where i = 1,2,3,4 for the four lag terms are replaced in equations (17) which are further written under the following concise form:

$$\tilde{\mathrm{M}}\ddot{\eta} + \tilde{\mathrm{C}}\dot{\eta} + \tilde{\mathrm{K}}\eta - q_{dyn}\mathrm{A}_3\mathrm{X}_1 - q_{dyn}\mathrm{A}_4\mathrm{X}_2 - q_{dyn}\mathrm{A}_5\mathrm{X}_3 - q_{dyn}\mathrm{A}_6\mathrm{X}_4 = 0 \tag{24}$$

where

$$\tilde{\mathrm{M}} = \mathrm{M} - q_{dyn}\mathrm{A}_2\left(\frac{b}{V}\right)^2,\ \tilde{\mathrm{C}} = \mathrm{C} - q_{dyn}\mathrm{A}_1\left(\frac{b}{V}\right) \text{ and } \tilde{\mathrm{K}} = \mathrm{K} - q_{dyn}\mathrm{A}_0 \tag{25}$$

Equations (23) and (24) are used to obtain the following matrix system of equations (after pre-multiplying both sides of equation (24) by $\tilde{\mathrm{M}}^{-1}$:

$$\begin{bmatrix} \dot{\eta} \\ \ddot{\eta} \\ \dot{\mathrm{X}}_1 \\ \dot{\mathrm{X}}_2 \\ \dot{\mathrm{X}}_3 \\ \dot{\mathrm{X}}_4 \end{bmatrix} = \begin{bmatrix} 0 & \mathrm{I} & 0 & 0 & 0 & 0 \\ -\breve{\mathrm{M}}^{-1}\tilde{\mathrm{K}} & -\breve{\mathrm{M}}^{-1}\tilde{\mathrm{C}} & -q_{dyn}\tilde{\mathrm{M}}^{-1}\mathrm{A}_3 & -q_{dyn}\tilde{\mathrm{M}}^{-1}\mathrm{A}_4 & -q_{dyn}\tilde{\mathrm{M}}^{-1}\mathrm{A}_5 & -q_{dyn}\tilde{\mathrm{M}}^{-1}\mathrm{A}_6 \\ 0 & \mathrm{I} & -\frac{V}{b}\beta_1\mathrm{I} & 0 & 0 & 0 \\ 0 & \mathrm{I} & 0 & -\frac{V}{b}\beta_2\mathrm{I} & 0 & 0 \\ 0 & \mathrm{I} & 0 & 0 & -\frac{V}{b}\beta_3\mathrm{I} & 0 \\ 0 & \mathrm{I} & 0 & 0 & 0 & -\frac{V}{b}\beta_4\mathrm{I} \end{bmatrix} \tag{26}$$

which is written under the following concise form:

$$\dot{\boldsymbol{x}} = \boldsymbol{A} \cdot \boldsymbol{x} \tag{27}$$

where

$$\mathrm{x} = \begin{bmatrix} \eta \\ \dot{\eta} \\ \mathrm{X}_1 \\ \mathrm{X}_2 \\ \mathrm{X}_3 \\ \mathrm{X}_4 \end{bmatrix} \text{ and } \mathrm{A} = \begin{bmatrix} 0 & \mathrm{I} & 0 & 0 & 0 & 0 \\ -\tilde{\mathrm{M}}^{-1}\tilde{\mathrm{K}} & -\tilde{\mathrm{M}}^{-1}\tilde{\mathrm{C}} & -q_{dyn}\tilde{\mathrm{M}}^{-1}\mathrm{A}_3 & -q_{dyn}\tilde{\mathrm{M}}^{-1}\mathrm{A}_4 & -q_{dyn}\tilde{\mathrm{M}}^{-1}\mathrm{A}_5 & -q_{dyn}\tilde{\mathrm{M}}^{-1}\mathrm{A}_6 \\ 0 & \mathrm{I} & -\frac{V}{b}\beta_1\mathrm{I} & 0 & 0 & 0 \\ 0 & \mathrm{I} & 0 & -\frac{V}{b}\beta_2\mathrm{I} & 0 & 0 \\ 0 & \mathrm{I} & 0 & 0 & -\frac{V}{b}\beta_3\mathrm{I} & 0 \\ 0 & \mathrm{I} & 0 & 0 & 0 & -\frac{V}{b}\beta_4\mathrm{I} \end{bmatrix} \tag{28}$$

Separation of the rigid η_r and elastic modes η_e (which is written as a common vector η_1) from control modes η_c is applied in equation (24) and following equation is obtained:

$$\begin{aligned} &\tilde{\mathrm{M}}_1\ddot{\eta}_1 + \tilde{\mathrm{C}}_1\dot{\eta} + \tilde{\mathrm{K}}_1\eta_1 - q_{dyn}\mathrm{A}_3\mathrm{X}_1 - q_{dyn}\mathrm{A}_4\mathrm{X}_2 - q_{dyn}\mathrm{A}_5\mathrm{X}_3 - q_{dyn}\mathrm{A}_6\mathrm{X}_4 = \\ &-\tilde{\mathrm{M}}_2\ddot{\eta}_c - \tilde{\mathrm{C}}_2\dot{\eta}_c - \tilde{\mathrm{K}}_2\eta_c \end{aligned} \tag{29}$$

where $\eta_1 = \begin{pmatrix} \eta_r \\ \eta_e \end{pmatrix}$ is the common vector for rigid and elastic modes, and η_c is the control modes vector, $\tilde{\mathrm{M}}_1 = \begin{pmatrix} \tilde{\mathrm{M}}_{rr} & \tilde{\mathrm{M}}_{re} \\ \tilde{\mathrm{M}}_{er} & \tilde{\mathrm{M}}_{ee} \end{pmatrix}$, $\tilde{\mathrm{C}}_1 = \begin{pmatrix} \tilde{\mathrm{C}}_{rr} & \tilde{\mathrm{C}}_{re} \\ \tilde{\mathrm{C}}_{er} & \tilde{\mathrm{C}}_{ee} \end{pmatrix}$ and $\tilde{\mathrm{K}}_1 = \begin{pmatrix} \tilde{\mathrm{K}}_{rr} & \tilde{\mathrm{K}}_{re} \\ \tilde{\mathrm{K}}_{er} & \tilde{\mathrm{K}}_{ee} \end{pmatrix}$ are the structural mass, damping and stiffness matrices for rigid and elastic modes interactions, and $\tilde{\mathrm{M}}_2 = \begin{pmatrix} \tilde{\mathrm{M}}_{rc} \\ \tilde{\mathrm{M}}_{ec} \end{pmatrix}$, $\tilde{\mathrm{C}}_2 = \begin{pmatrix} \tilde{\mathrm{C}}_{rc} \\ \tilde{\mathrm{C}}_{ec} \end{pmatrix}$ and $\tilde{\mathrm{K}}_2 = \begin{pmatrix} \tilde{\mathrm{K}}_{rc} \\ \tilde{\mathrm{K}}_{ec} \end{pmatrix}$ are the structural mass, damping and stiffness matrices for control modes. Matrices $\tilde{\mathrm{M}}_1$, $\tilde{\mathrm{C}}_1$ and $\tilde{\mathrm{K}}_1$ are square matrices with $(r + e)$ lines and $(r + e)$ columns, and matrices $\tilde{\mathrm{M}}_2$, $\tilde{\mathrm{C}}_2$ and $\tilde{\mathrm{K}}_2$ are square matrices with $(r + e)$ lines and c columns.

$$\begin{pmatrix} I & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{M}_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{pmatrix} \begin{pmatrix} \dot{\eta}_1 \\ \ddot{\eta}_1 \\ \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{pmatrix} = \begin{pmatrix} 0 & I & 0 & 0 & 0 & 0 \\ -\tilde{K}_1 & -\tilde{C}_1 & -q_{dyn}A_3 & -q_{dyn}A_4 & -q_{dyn}A_5 & -q_{dyn}A_6 \\ 0 & \begin{bmatrix} I_{r+e,r+e} \\ 0_{c,r+e} \end{bmatrix} & -\frac{V}{b}\beta_1 I & 0 & 0 & 0 \\ 0 & \begin{bmatrix} I_{r+e,r+e} \\ 0_{c,r+e} \end{bmatrix} & 0 & -\frac{V}{b}\beta_1 I & 0 & 0 \\ 0 & \begin{bmatrix} I_{r+e,r+e} \\ 0_{c,r+e} \end{bmatrix} & 0 & 0 & -\frac{V}{b}\beta_1 I & 0 \\ 0 & \begin{bmatrix} I_{r+e,r+e} \\ 0_{c,r+e} \end{bmatrix} & 0 & 0 & 0 & -\frac{V}{b}\beta_1 I \end{pmatrix} \begin{pmatrix} \eta_1 \\ \dot{\eta}_1 \\ X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix} + \tag{30}$$

$$+ \begin{pmatrix} 0 & 0 & 0 \\ -\tilde{K}_2 & -\tilde{C}_2 & -\tilde{M}_2 \\ 0 & \begin{bmatrix} 0_{r+e,c} \\ 0_{c,c} \end{bmatrix} & 0 \\ 0 & \begin{bmatrix} 0_{r+e,c} \\ I_{c,c} \end{bmatrix} & 0 \end{pmatrix} \begin{pmatrix} \eta_c \\ \dot{\eta}_c \\ \ddot{\eta}_c \end{pmatrix}$$

might also be expressed under the following form :

$$\begin{pmatrix} \dot{\eta}_1 \\ \ddot{\eta}_1 \\ \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{pmatrix} = \begin{pmatrix} 0 & I & 0 & 0 & 0 & 0 \\ -\tilde{M}_1^{-1}\tilde{K}_1 & -\tilde{M}_1^{-1}\tilde{C}_1 & -\tilde{M}_1^{-1}q_{dyn}A_3 & -\tilde{M}_1^{-1}q_{dyn}A_4 & -\tilde{M}_1^{-1}q_{dyn}A_5 & -\tilde{M}_1^{-1}q_{dyn}A_6 \\ 0 & \begin{bmatrix} I_{r+e,r+e} \\ 0_{c,r+e} \end{bmatrix} & -\frac{V}{b}\beta_1 I & 0 & 0 & 0 \\ 0 & \begin{bmatrix} I_{r+e,r+e} \\ 0_{c,r+e} \end{bmatrix} & 0 & -\frac{V}{b}\beta_1 I & 0 & 0 \\ 0 & \begin{bmatrix} I_{r+e,r+e} \\ 0_{c,r+e} \end{bmatrix} & 0 & 0 & -\frac{V}{b}\beta_1 I & 0 \\ 0 & \begin{bmatrix} I_{r+e,r+e} \\ 0_{c,r+e} \end{bmatrix} & 0 & 0 & 0 & -\frac{V}{b}\beta_1 I \end{pmatrix} \in \begin{pmatrix} \eta_1 \\ \dot{\eta}_1 \\ X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix} + \tag{31}$$

$$+ \begin{pmatrix} 0 & 0 & 0 \\ -\tilde{M}_1^{-1}\tilde{K}_2 & -\tilde{M}_1^{-1}\tilde{C}_2 & -\tilde{M}_1^{-1}\tilde{M}_2 \\ 0 & \begin{bmatrix} 0_{r+e,c} \\ 0_{c,c} \end{bmatrix} & 0 \\ 0 & \begin{bmatrix} 0_{r+e,c} \\ I_{c,c} \end{bmatrix} & 0 \end{pmatrix} \begin{pmatrix} \eta_c \\ \dot{\eta}_c \\ \ddot{\eta}_c \end{pmatrix}$$

This equation is $\dot{X} = AX + BU$, with the solution in frequency domain:

$$\left(j\omega I - A\right)X = B\,u \rightarrow X = \left(j\,\omega I - A\right)^{-1} B\,u \tag{32}$$

A solution of equation (32) for each frequency in the interval (0.1…20) Hz is calculated with a **step of 0.01 Hz**.

5.2 Frequency response theory used by ETS team

Eq. (14) can be further written as follows:

$$\begin{pmatrix} M_{rr} & M_{re} & M_{rc} \\ M_{er} & M_{ee} & M_{ec} \\ M_{cr} & M_{ce} & M_{cc} \end{pmatrix}\begin{pmatrix} \ddot{\eta}_r \\ \ddot{\eta}_e \\ \ddot{\eta}_c \end{pmatrix} + \begin{pmatrix} C_{rr} & C_{re} & C_{rc} \\ C_{er} & C_{ee} & C_{ec} \\ C_{cr} & C_{ce} & C_{cc} \end{pmatrix}\begin{pmatrix} \dot{\eta}_r \\ \dot{\eta}_e \\ \dot{\eta}_c \end{pmatrix}$$
$$+\begin{pmatrix} K_{rr} & K_{re} & K_{rc} \\ K_{er} & K_{ee} & K_{ec} \\ K_{cr} & K_{ce} & K_{cc} \end{pmatrix}\begin{pmatrix} \eta_r \\ \eta_e \\ \eta_c \end{pmatrix} - q_{dyn}\begin{pmatrix} Q_{rr} & Q_{re} & Q_{rc} \\ Q_{er} & Q_{ee} & Q_{ec} \\ Q_{cr} & Q_{ce} & Q_{cc} \end{pmatrix}\begin{pmatrix} \eta_r \\ \eta_e \\ \eta_c \end{pmatrix} = 0 \tag{33}$$

Separating the rigid and elastic modes from control modes, we obtain:

$$\begin{pmatrix} M_{rr} & M_{re} \\ M_{er} & M_{ee} \end{pmatrix}\begin{pmatrix} \ddot{\eta}_r \\ \ddot{\eta}_e \end{pmatrix} + \begin{pmatrix} C_{rr} & C_{re} \\ C_{er} & C_{ee} \end{pmatrix}\begin{pmatrix} \dot{\eta}_r \\ \dot{\eta}_e \end{pmatrix} + \begin{pmatrix} K_{rr} & K_{re} \\ K_{er} & K_{ee} \end{pmatrix}\begin{pmatrix} \eta_r \\ \eta_e \end{pmatrix} - q_{dyn}\begin{pmatrix} Q_{rr} & Q_{re} \\ Q_{er} & Q_{ee} \end{pmatrix}\begin{pmatrix} \eta_r \\ \eta_e \end{pmatrix} =$$
$$= -\begin{pmatrix} M_{rc} \\ M_{ec} \end{pmatrix}\ddot{\eta}_c - \begin{pmatrix} C_{rc} \\ C_{ec} \end{pmatrix}\dot{\eta}_c - \begin{pmatrix} K_{rc} \\ K_{ec} \end{pmatrix}\eta_c + q_{dyn}\begin{pmatrix} Q_{rc} \\ Q_{ec} \end{pmatrix}\eta_c \tag{34}$$

Calling left-hand matrices with the subscript 1 and right-hand matrices with subscript 2, we obtain:

$$M_1\ddot{\eta}_1 + C_1\dot{\eta}_1 + K_1\eta_1 - q_{dyn}Q_1(k)\eta_1 = -M_2\ddot{\eta}_2 - C_2\dot{\eta}_2 - K_2\eta_2 + q_{dyn}\,Q_2(k)\eta_2 \tag{35}$$

For a specific frequency, the generalized coordinates η are written as function of ω:

$$\dot{\eta} = j\omega\eta$$
$$\ddot{\eta} = j\omega\dot{\eta} = -\omega^2\eta \tag{36}$$

We replace $\omega = \frac{kV}{b}$ and eq. (35) in eq. (36) and we obtain:

$$\left(-\left(\frac{V}{b}\right)^2 k^2 M_1 + j\frac{V}{b}kC_1 + K_1 - q_{dyn}Q_1(k)\right)\eta_1 = \left(\left(\frac{V}{b}\right)^2 k^2 M_2 - j\frac{V}{b}kC_2 - K_2 + q_{dyn}Q_2(k)\right)\eta_2 \tag{37}$$

or:

$$A\,\eta_1 = B\,\eta_2 \tag{38}$$

The unknowns in eq. (38) is η_1. The solution is:

$$\eta_1 = A^{-1}\;B\,\eta_2 \tag{39}$$

The input η_2 is given by the control surfaces positions applied through a control system with transfer function H_c provided by Bombardier Aerospace. Eq. (39) becomes:

$$\eta_1 = A^{-1}\,B\,H_c\;\eta_c \tag{40}$$

where η_c are positions of aircraft's control surfaces.

6. VALIDATION OF EQUATIONS (FOR BOTH OPEN AND CLOSED LOOP)

Results are presented under the form of Bode plots, more specifically under the form of magnitude (dB) and phase (deg) versus frequency (Hz) for frequency response analysis on a CL-604 aircraft at one flight condition: Mach number = 0.88 and Altitude = 35,000 ft.

Aerodynamic forces were approximated with the Padé approximations by use of the Least Squares LS method.

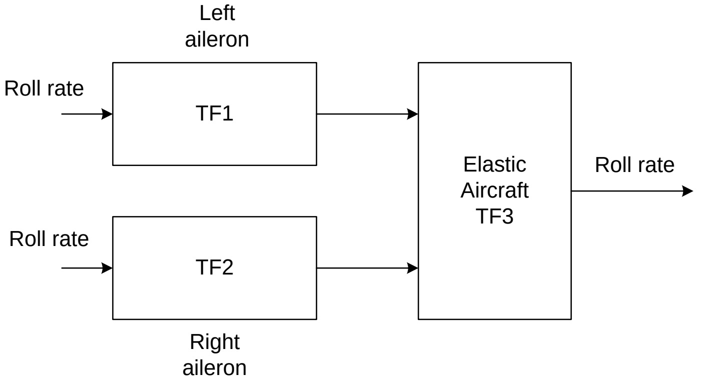

Figure 4. Block diagram of the aircraft system with left and right ailerons control

Frequency responses are calculated for the aircraft by use of five different transfer functions corresponding to control surfaces outputs versus inputs which are represented with different types of lines. These transfer functions are the following (please refer to Figure 4 for TF1, TF2 and TF3 definitions):

- Right aileron/ (Right aileron & Left aileron) Systems response for right aileron; flexible time control law response: the Bode plot of the product TF2*TF3.
- Left aileron/ (Right aileron & Left aileron) Systems response for left aileron; flexible time control law response: the Bode plot of the product TF1*TF3.
- Right aileron/ Roll rate Right aileron control law response to a roll rate input: the Bode plot of TF2.
- Left aileron/ Roll rate Left aileron control law response to a roll rate input: the Bode plot of TF1.
- Roll rate/ (Right aileron & Left aileron) Flexible A/C response; roll rate caused by ailerons excitation: the Bode plot of TF3.

Results (expressed in terms of frequency response) obtained by Bombardier Aerospace team are represented in Figure 5, while the same type of results obtained by the ETS team are represented in Figure 6. TF1 and TF2 give similar results (as represent the control laws acting on the aircraft) obtained by both teams, and for this reason, the only differences between the two sets of results represented in Figures 5 and 6 obtained by the ETS and Bombardier teams were found to arise from the TF3 calculations. The frequency response results represented under the form of Bode plots with TF3 obtained by the Bombardier Aerospace team and by the ETS team are shown in Figures 7 and 8. Magnitudes are represented in Figure 7, while phases are represented in Figure 8. These results were found to be very close (see Figure 7.a and Figure 8.a), and for this reason, the differences between them was further calculated. In Figures 7.b and 7.c, the differences in magnitudes were represented in dB and in absolute values, respectively. In Figure 8.b, the differences in phases were represented. The minor differences between results obtained by both teams arise from the LS approximations for the aerodynamic unsteady forces, more precisely, from the lag terms optimizations routines. Very small differences in lags optimizations (less than 10^{-5}) lead in the small differences seen in these figures.

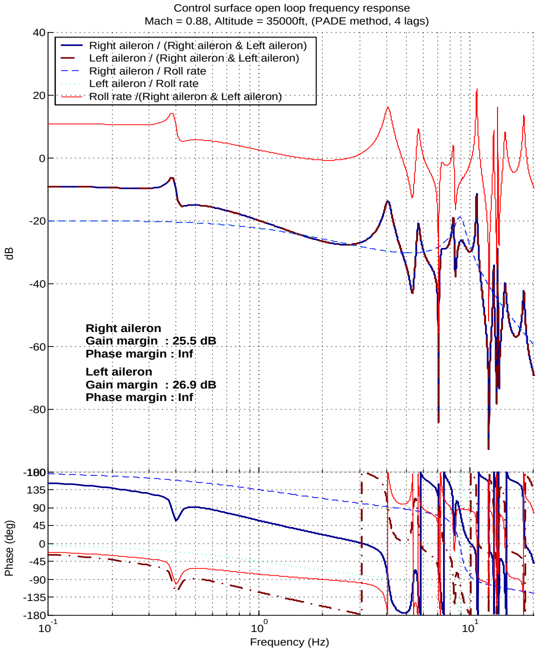

Figure 5. Magnitude and phase plots for aircraft frequency response. Results by Bombardier Aerospace Team

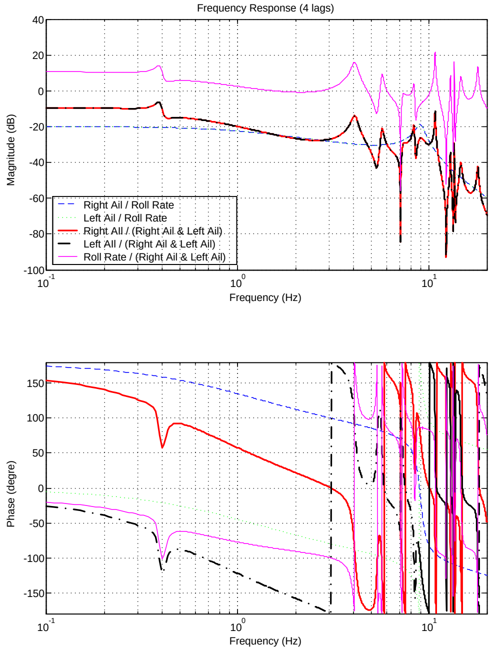

Figure 6. Magnitude and phase plots for aircraft frequency response. Results by ETS Team

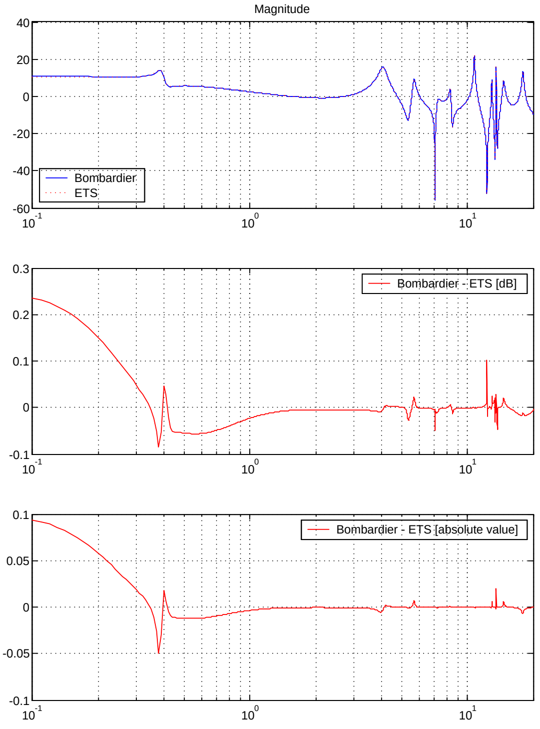

Figure 7. Magnitude comparison (a), differences in magnitudes: dB (b) and absolute value (c)

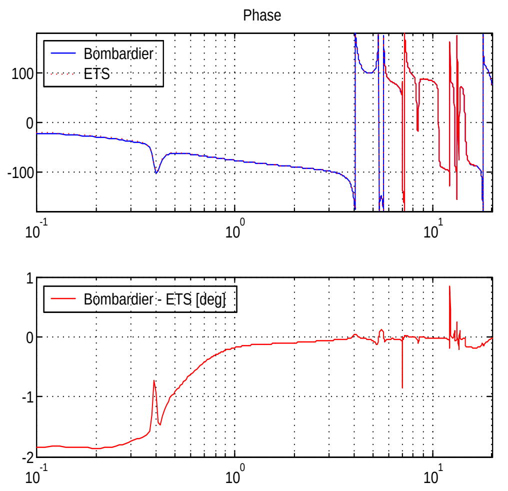

Figure 8. Phase comparisons (a) and differences b)

Although the two algorithms are different, the obtained results are found to be very close.

7. DISCUSSION ON THE ADVANTAGES AND LIMITATIONS OF THIS WORK

For the CL-604 aircraft here studied, we have chosen among four methods LS, MS, CLS and CMS as the best method – the method Least Squares with 4 lag terms for the aerodynamic forces conversions from frequency into Laplace domain.

This choice was mainly based mainly on the comparison of flutter analysis results obtained by use of this method with respect to the flutter analysis results initially obtained with the *pk* flutter standard method. This method has been chosen as is faster and easier to integrate with the flutter equations of motion for aeroservoelastic interactions studies. Two approaches were studied and one of these approaches (use of *A* big matrix) was selected as being much better than the other (use of *A* small matrix).

In future works, the LS method should be chosen for aeroservoelasticity studies, but we might need a different number of lag terms (than 4) depending on the aircraft type. Each time the finite elements aircraft model will be different, number of optimal lags might be different. On the other hand, our computer programs are written in such a way that the number of optimal lags will take very little time to choose for each aircraft time.

The frequency response was analyzed in this work only by use of ailerons inputs. As future work, we will need to calculate and analyze the frequency response by use also of the other control surfaces inputs such as elevators and rudder. Control laws were simple in the work here performed. In future work, we will need to analyze full control laws and non-linearities – so that computer programs will become heavier.

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