# The flow of an incompressible electroconductive fluid past a thin airfoil. The parabolic profile

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***Abstract**: We study the two-dimensional steady flow of an ideal incompressible perfectly conducting fluid past an insulating thin parabolic airfoil. We consider the linearized Euler and Maxwell equations and Ohm's law. We use the integral representations for the velocity, magnetic induction and pressure and the boundary conditions to obtain an integral equation for the jump of the pressure across the airfoil. We give some graphic representations for the lift coefficient, velocity and magnetic induction.*

**Key Words:** *linearized system, integral representations, parabolic airfoil.*

## 1. INTRODUCTION

The motion of a wing in an electoconductive fluid was investigated in the second half of the past century, when the researchers were interested in studying the aircraft flow in special meteorogical conditions or at high altitude in an ionized athmosphere. In the papers dedicated to this subject, the lift, drag and moment coefficients were calculated. Recent technological advances claim also for the study of the velocity and electromagnetic fields. We mention two examples: the plasma actuators for aircraft flow control (see [3]) and concealing aircrafts from radar using the interaction between the ionized gas and the electromagnetic radiation. In the present paper we study the steady two-dimensional flow of an ideal perfectly conducting incompressible fluid around a thin insulating parabolic airfoil. We consider the linearized partial differential equations of magnetohydrodynamics consisting of Euler's and Maxwell's equations and Ohm's law.

In [2] we calculated the corresponding fundamental matrix and obtained integral representations of the velocity, the magnetic induction and the pressure fields for arbitrary thin airfoils. We notice that every integral representation has an elliptic as well as a hyperbolic part, the last one being determined by the presence of simple waves bounded by straight characteristics (weak shocks). From the integral representation of the velocity and the boundary conditions (linearized slipping condition and the continuity of the magnetic induction) we rediscover the singular integral equation for the jump of the pressure across the airfoil (see [2],[4], [6], [7]).

We consider the particular case of the parabolic profile for which the solution of the integral equation is analitically calculated. Then we calculate the lift coefficient and perform some numerical integrations to calculate the velocity and the magnetic induction in the points of a two-dimensional grid. We provide graphic representations for the velocity and magnetic induction fields and for the lift coefficient.

## 2. FUNDAMENTAL MATRIX OF THE LINEARIZED SYSTEM FOR THE TWO-DIMENSIONAL INCOMPRESSIBLE FLOW OF PERFECTLY CONDUCTING FLUIDS

The results presented in Sections 2-4 were obtained in [2]. We assume that the plane-parallel motion occurs in the *Oxy* plane and we denote by **i** and **j** the versors of the *Ox* and *Oy* axes. Let **v**,**b** and *p* designate the nondimensional perturbations of the velocity, magnetic induction and pressure, respectively, determined by the presence of a thin insulating airfoil whose equation is

$$y = h_{\pm}(x),\ x \in [0,1],\ |h_{\pm}(x)| << 1,\ |h'_{\pm}(x)| << 1. \tag{1}$$

At infinity upstream, we assume that the unperturbed motion is uniform and parallel to the *Ox* - axis and that there exists a homogeneous magnetic field whose nondimensional expression is

$$\mathbf{B}_0 = (\alpha_x, \alpha_y),\ \alpha_x = \cos\alpha, \alpha_y = \sin\alpha, 0 < \alpha < \pi. \tag{2}$$

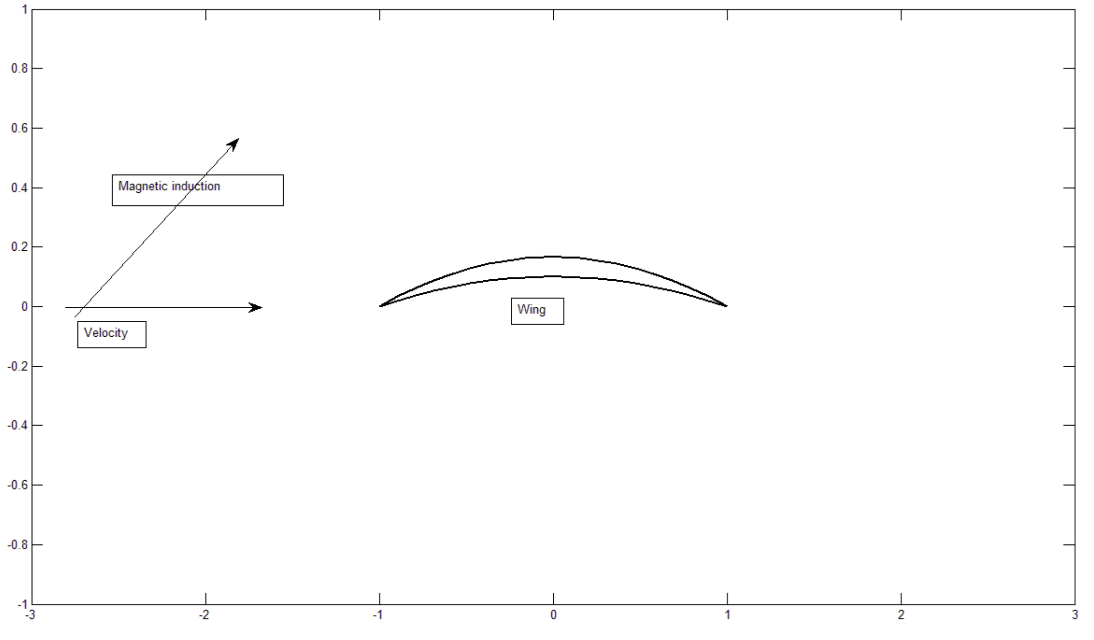


Figure 1: Airfoil and velocity and magnetic induction at infinity

As it is shown in [4], section 5.2, the nondimensional intensity of the electric field is

$$\mathbf{E} = (\alpha_x \mathbf{i} + \alpha_y \mathbf{j}) \times \mathbf{i}.$$

$\mathbf{v} = (v_x, v_y), \mathbf{b} = (b_x, b_y)$ and $p$ satisfy the following system of linear partial differential equations obtained by means of the small perturbations technique:

$$
\begin{aligned}
&\frac{\partial v_x}{\partial x}+\frac{\partial p}{\partial x}+\frac{\alpha_y}{A}\left(\frac{\partial b_y}{\partial x}-\frac{\partial b_x}{\partial y}\right)=0,\\
&\frac{\partial v_y}{\partial x}+\frac{\partial p}{\partial y}-\frac{\alpha_x}{A}\left(\frac{\partial b_y}{\partial x}-\frac{\partial b_x}{\partial y}\right)=0,\\
&\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}=0,\\
&\frac{\partial b_x}{\partial x}+\frac{\partial b_y}{\partial y}=0,\\
&b_y+\alpha_y v_x-\alpha_x v_y=0.
\end{aligned}
\tag{3}
$$

with $A=1/\sqrt{Rh}$ - Alfvén's number ( $Rh$ is the magnetic pressure number).

The first two equations are Euler's equations, the third is the equation of continuity, the fourth equation is Gauss' law for magnetism and the last one is Ohm's law. We introduce like in [1] and [2] the fundamental matrix of the linear system (3):

$$
\begin{pmatrix}
v_x^{(1)} & v_y^{(1)} & b_x^{(1)} & b_y^{(1)} & p^{(1)}\\
v_x^{(2)} & v_y^{(2)} & b_x^{(2)} & b_y^{(2)} & p^{(2)}\\
v_x^{(3)} & v_y^{(3)} & b_x^{(3)} & b_y^{(3)} & p^{(3)}\\
v_x^{(4)} & v_y^{(4)} & b_x^{(4)} & b_y^{(4)} & p^{(4)}
\end{pmatrix},
\tag{4}
$$

whose components are the fundamental solutions of the systems

$$
\begin{aligned}
&\frac{\partial v_x^{(j)}}{\partial x}+\frac{\partial p^{(j)}}{\partial x}+\frac{\alpha_y}{A}\left(\frac{\partial b_y^{(j)}}{\partial x}-\frac{\partial b_x^{(j)}}{\partial y}\right)=\delta_{j1}\delta(x,y),\\
&\frac{\partial v_y^{(j)}}{\partial x}+\frac{\partial p^{(j)}}{\partial y}-\frac{\alpha_x}{A}\left(\frac{\partial b_y^{(j)}}{\partial x}-\frac{\partial b_x^{(j)}}{\partial y}\right)=\delta_{j2}\delta(x,y),\\
&\frac{\partial v_x^{(j)}}{\partial x}+\frac{\partial v_y^{(j)}}{\partial y}=\delta_{j3}\delta(x,y),\\
&\frac{\partial b_x^{(j)}}{\partial x}+\frac{\partial b_y^{(j)}}{\partial y}=\delta_{j4}\delta(x,y),\\
&b_y^{(j)}+\alpha_y v_x^{(j)}-\alpha_x v_y^{(j)}=0,
\end{aligned}
\tag{5}
$$

where $\delta(x,y)$ is Dirac's distribution and

$$
\delta_{ji}=\begin{cases}1, & j=i\\ 0, & j\neq i\end{cases}.
$$

In [2] the following components of the fundamental matrix are calculated

$$
\begin{aligned}
v_x^{(2)}=&\frac{a_{21}}{2\pi}\frac{x}{x^2+y^2}+\frac{b_{21}}{2\pi}\frac{y}{x^2+y^2}-\\
&-\frac{Ac_{21}}{2\alpha_y}\frac{\partial}{\partial x}H\left(x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right)-\frac{Ad_{21}}{2\alpha_y}\frac{\partial}{\partial y}H\left(x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right),
\end{aligned}
\tag{6}
$$

$$v_y^{(2)}=-\frac{a_{30}}{2\pi}\frac{x}{x^2+y^2}-\frac{b_{30}}{2\pi}\frac{y}{x^2+y^2}+$$
$$+\frac{Ac_{30}}{2\alpha_y}\frac{\partial}{\partial x}H\left(x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right)+\frac{Ad_{30}}{2\alpha_y}\frac{\partial}{\partial y}H\left(x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right), \tag{7}$$

$$b_x^{(2)}=\alpha_x v_x^{(2)}+\alpha_y v_y^{(2)}-\frac{A}{2}\frac{\partial}{\partial x}H\left(x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right), \tag{8}$$

$$p_x^{(2)}=-v_x^{(2)}-\frac{\alpha_x}{2A}\frac{\partial}{\partial x}H\left(x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right)-\frac{\alpha_y}{2A}\frac{\partial}{\partial y}H\left(x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right), \tag{9}$$

where

$$H(x)=\begin{cases}1, & x\geq 0\\ 0, & x<0\end{cases}$$

is Heaviside's function and

$$a_{30}=-\frac{A^2\left(A^2+1-2\alpha_x^2\right)}{1+2A^2\left(1-2\alpha_x^2\right)+A^4},\ b_{30}=-\frac{2A^2\alpha_x\alpha_y}{1+2A^2\left(1-2\alpha_x^2\right)+A^4},$$
$$c_{30}=\frac{\alpha_y^2\left(A^2+1+2\alpha_x^2\right)}{1+2A^2\left(1-2\alpha_x^2\right)+A^4},\ d_{30}=\frac{2\alpha_x\alpha_y^3}{1+2A^2\left(1-2\alpha_x^2\right)+A^4}, \tag{10}$$

$$a_{21}=\frac{2A^2\alpha_x\alpha_y}{1+2A^2\left(1-2\alpha_x^2\right)+A^4},\ b_{21}=-\frac{A^2\left(A^2+1-2\alpha_x^2\right)}{1+2A^2\left(1-2\alpha_x^2\right)+A^4},$$
$$c_{21}=\frac{2\alpha_x\alpha_y\left(A^2-\alpha_x^2\right)}{1+2A^2\left(1-2\alpha_x^2\right)+A^4},\ d_{21}=\frac{\alpha_y^2\left(A^2+1-2\alpha_x^2\right)}{1+2A^2\left(1-2\alpha_x^2\right)+A^4}. \tag{11}$$

## 3. INTEGRAL REPRESENTATIONS

In thin airfoil theory the linearized boundary conditions are usually imposed on the segment [-1,1] and the functions we are looking for are defined on $\mathrm{R}^2\setminus[-1,1]$.

Since $v_x, v_y, b_x, b_y$ and $p$ are integrable functions, they may be regarded as regular distributions.

Taking into account the boundary conditions ([4], Chapter 5)

$$[\mathbf{b}](x)=0,\ v_y(x,\pm 0)=h_{\pm}'(x),\ x\in[-1,1], \tag{12}$$

we obtain the following linearized system for the distributions $v_x, v_y, b_x, b_y$ and $p$ :

$$\begin{aligned}
&\tfrac{\partial v_x}{\partial x}+\tfrac{\partial p}{\partial x}+\tfrac{\alpha_y}{A}\left(\tfrac{\partial b_y}{\partial x}-\tfrac{\partial b_x}{\partial y}\right)=0=f_1,\\
&\tfrac{\partial v_y}{\partial x}+\tfrac{\partial p}{\partial y}-\tfrac{\alpha_x}{A}\left(\tfrac{\partial b_y}{\partial x}-\tfrac{\partial b_x}{\partial y}\right)=[p]\delta_{[-1,1]}=f_2,\\
&\tfrac{\partial v_x}{\partial x}+\tfrac{\partial v_y}{\partial y}=[h']\delta_{[-1,1]}=f_3,\\
&\tfrac{\partial b_x}{\partial x}+\tfrac{\partial b_y}{\partial y}=0=f_4,\\
&b_y+\alpha_y v_x-\alpha_x v_y=0,
\end{aligned}\tag{13}$$

where $[\mathbf{b}](x)$ and $[p](x)$ represent the jumps of $\mathbf{b}$ and $p$ over the segment [-1,1] and $[p]\delta_{[-1,1]},[h']\delta_{[-1,1]}$ are simple layer distributions. We may easily verify that

$$\begin{aligned}
&v_x=\sum_{j=1}^{4}v_x^{(j)}\otimes f_j,\ v_y=\sum_{j=1}^{4}v_y^{(j)}\otimes f_j,\\
&b_x=\sum_{j=1}^{4}b_x^{(j)}\otimes f_j,\ p=\sum_{j=1}^{4}p^{(j)}\otimes f_j,
\end{aligned}\tag{14}$$

where $\otimes$ stands for the convolution product. We shall consider, for the sake of simplicity, the case of zero thickness wing, i.e. $h_+(x)=h_-(x)$. Hence $f_1=f_3=f_4=0$ and

$$\begin{aligned}
&v_x=v_x^{(2)}\otimes[p]\delta_{[-1,1]},\ v_y=v_y^{(2)}\otimes[p]\delta_{[-1,1]},\\
&b_x=b_x^{(2)}\otimes[p]\delta_{[-1,1]},\ p=p^{(2)}\otimes[p]\delta_{[-1,1]}.
\end{aligned}$$

In [2] the convolutions were calculated and the following integral representations were obtained:

$$\begin{aligned}
v_x(x,y)=&\frac{a_{21}}{2\pi}\int_{-1}^{1}[p](\xi)\frac{x-\xi}{(x-\xi)^2+y^2}d\xi+\frac{b_{21}}{2\pi}\int_{-1}^{1}[p](\xi)\frac{y}{(x-\xi)^2+y^2}d\xi-\\
&-\frac{Ac_{21}}{2\alpha_y}\frac{\partial}{\partial x}\int_{-1}^{1}[p](\xi)H\left(x-\xi-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right)d\xi-\\
&-\frac{Ad_{21}}{2\alpha_y}\frac{\partial}{\partial y}\int_{-1}^{1}[p(\xi)]H\left(x-\xi-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right)d\xi=\\
=&\frac{a_{21}}{2\pi}\int_{-1}^{1}[p](\xi)\frac{x-\xi}{(x-\xi)^2+y^2}d\xi+\frac{b_{21}}{2\pi}\int_{-1}^{1}[p](\xi)\frac{y}{(x-\xi)^2+y^2}d\xi-\\
&-\frac{A}{2\alpha_y^2}\left(c_{21}\alpha_y-d_{21}\alpha_x-d_{21}A\frac{y}{|y|}\right)\times\\
&\times\begin{cases}[p](x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|)\ \text{for}\ \left|x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right|<1,\\
0\ \text{for}\ \left|x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right|>1,\end{cases}
\end{aligned}\tag{15}$$

$$v_y(x,y)=$$

$$-\frac{a_{30}}{2\pi}\int_{-1}^{1}[p](\xi)\frac{x-\xi}{(x-\xi)^2+y^2}d\xi-\frac{b_{30}}{2\pi}\int_{-1}^{1}[p](\xi)\frac{y}{(x-\xi)^2+y^2}d\xi+$$

$$+\frac{A}{2\alpha_y^2}\left(c_{30}\alpha_y-d_{30}\alpha_x-d_{30}A\frac{y}{|y|}\right)\times \tag{16}$$

$$\times\begin{cases}[p](x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|) \text{ for } \left|x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right|<1,\\ 0 \text{ for } \left|x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right|>1,\end{cases}$$

$$b_x(x,y)=\alpha_x v_x(x,y)+\alpha_y v_y(x,y)-$$

$$-\frac{A}{2}\times\begin{cases}[p](x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|) \text{ for } \left|x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right|<1,\\ 0 \text{ for } \left|x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right|>1,\end{cases} \tag{17}$$

$$p(x,y)=-v_x(x,y)+$$

$$+\frac{y}{2|y|}\times\begin{cases}[p](x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|) \text{ for } \left|x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right|<1,\\ 0 \text{ for } \left|x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right|>1.\end{cases} \tag{18}$$

## 4. THE INTEGRAL EQUATION FOR THE JUMP OF THE PRESSURE

Using the Plemelj formulas and the linearized boundary conditions (12), we get from (16) the integral equation for $-1< x < 1$:

$$h'(x)=v_y(x,\pm 0)=-\frac{a_{30}}{2\pi}\,p.v.\int_{-1}^{1}\frac{[p(\xi)]}{x-\xi}d\xi+\frac{A\left(c_{30}\alpha_y-d_{30}\alpha_x\right)}{2\alpha_y^2}[p(x)], \tag{19}$$

which is equivalent to

$$-\beta[p](x)+\frac{k}{\pi}\,p.v.\int_{-1}^{1}\frac{[p](\xi)}{\xi-x}d\xi=-h'(x), \tag{20}$$

where

$$k=-\frac{a_{30}}{2},\ \beta=\frac{A\left(c_{30}\alpha_y-d_{30}\alpha_x\right)}{2\alpha_y^2}.$$

$$k=A^2+1-2\alpha_x^2,\quad \beta=\alpha_y\left(1+A^2\right)A^{-1}\ \text{ and }\ \chi=2\left(4\alpha_x^2+A^{-2}-A^2\right).$$

As it is shown in [1], [2], [6] and [4], chapter 5, the solution of equation (20) is

$$[p](x) = \frac{-\beta\chi}{\beta^2+k^2}h'(x) - \frac{k\chi}{\pi(\beta^2+k^2)}\left(\frac{1-x}{1+x}\right)^{\theta} p.v.\int_{-1}^{1}\left(\frac{1+\xi}{1-\xi}\right)^{\theta}\frac{h'(\xi)}{\xi-x}d\xi + $$
$$+\frac{2C\sin\theta\pi}{(1-x)^{1-\theta}(1+x)^{\theta}},\ \tan\theta\pi = \frac{k}{\beta},\ 0\le\theta<1. \tag{21}$$

We may take $C=0$ if we impose Kutta-Joukovsky's condition. Other choices of the constant $C$ were considered by Stewartson [7].

## 5. THE PARABOLIC AIRFOIL

In this case, $h(x)=\varepsilon(1-x^2)$, $h'(x)=-2\varepsilon x$ and

$$[p](x) = \frac{2\varepsilon\beta\chi x}{\beta^2+k^2} + \frac{2\varepsilon k\chi x}{\pi(\beta^2+k^2)}\left(\frac{1-x}{1+x}\right)^{\theta} p.v.\int_{-1}^{1}\left(\frac{1+\xi}{1-\xi}\right)^{\theta}\frac{d\xi}{\xi-x}$$
$$+\frac{2\varepsilon k\chi}{\pi(\beta^2+k^2)}\left(\frac{1-x}{1+x}\right)^{\theta}\int_{-1}^{1}\left(\frac{1+\xi}{1-\xi}\right)^{\theta}d\xi. \tag{22}$$

Taking into account that

$$p.v.\int_{-1}^{1}\left(\frac{1+\xi}{1-\xi}\right)^{\theta}\frac{1}{\xi-x}d\xi = \frac{\pi}{\sin\theta\pi}\left\{1-\left(\frac{1+x}{1-x}\right)^{\theta}\cos\theta\pi\right\}, \tag{23}$$

and

$$\int_{-1}^{1}\left(\frac{1+\xi}{1-\xi}\right)^{\theta}d\xi = \frac{2\pi\theta}{\sin\pi\theta} \tag{24}$$

we get:

$$[p](x) = \frac{2\varepsilon\chi}{\sqrt{\beta^2+k^2}}(x+2\theta)\left(\frac{1-x}{1+x}\right)^{\theta}. \tag{25}$$

The lift is ([4], 5.2.6)

$$L = -\int_{-1}^{1}[p](x)dx.$$

From (24) and from the relation

$$\int_{-1}^{1}\left(\frac{1-\xi}{1+\xi}\right)^{\theta}\xi d\xi = -\frac{2\pi\theta^2}{\sin\pi\theta}, \tag{26}$$

it follows

$$L = -\frac{4\pi\varepsilon\chi\theta^2}{k}. \tag{27}$$

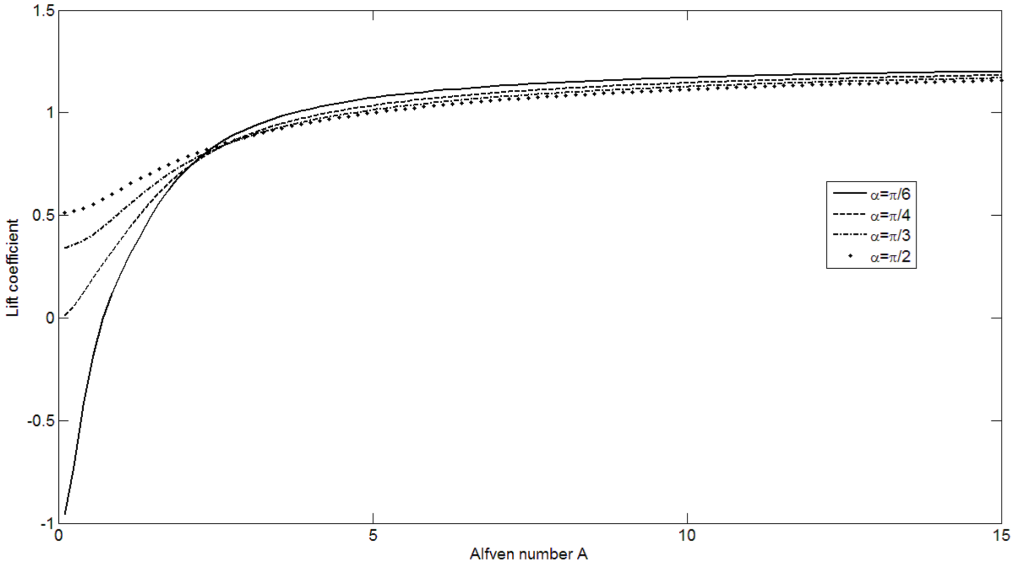


Figure 2: Lift coefficient versus Alfvén's number

In Figure 2 we present the lift coefficient against Alfvén's number $A$ for various values of the angle α made by the directions of the magnetic induction and the velocity at infinity upstream. We notice that for a strong magnetic field ( $A \to 0$ ) the lift coefficient may have negative values.

## 6. THE VELOCITY AND THE MAGNETIC INDUCTION

From (15), (16) and (21) we get

$$v_x(x,y) = \frac{\varepsilon\chi a_{21}}{\pi\sqrt{\beta^2+k^2}}\left[I_5(x,y)+2\theta I_3(x,y)\right] + \frac{\varepsilon\chi b_{21}}{\pi\sqrt{\beta^2+k^2}}\left[I_6(x,y)+2\theta I_4(x,y)\right]$$
$$-\frac{A}{2\alpha_y^2}\left(c_{21}\alpha_y - d_{21}\alpha_x - d_{21}A\frac{y}{|y|}\right)\times \tag{28}$$
$$\times\begin{cases}[p](x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|) \text{ for } \left|x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right|<1,\\ 0 \text{ for } \left|x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right|>1,\end{cases}$$

$$v_y(x,y) = -\frac{\varepsilon\chi a_{30}}{\pi\sqrt{\beta^2+k^2}}\left[I_5(x,y)+2\theta I_3(x,y)\right] - \frac{\varepsilon\chi b_{30}}{\pi\sqrt{\beta^2+k^2}}\left[I_6(x,y)+2\theta I_4(x,y)\right]$$
$$+\frac{A}{2\alpha_y^2}\left(c_{30}\alpha_y - d_{30}\alpha_x - d_{30}A\frac{y}{|y|}\right)\times \tag{29}$$
$$\times\begin{cases}[p](x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|) \text{ for } \left|x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right|<1,\\ 0 \text{ for } \left|x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right|>1,\end{cases}$$

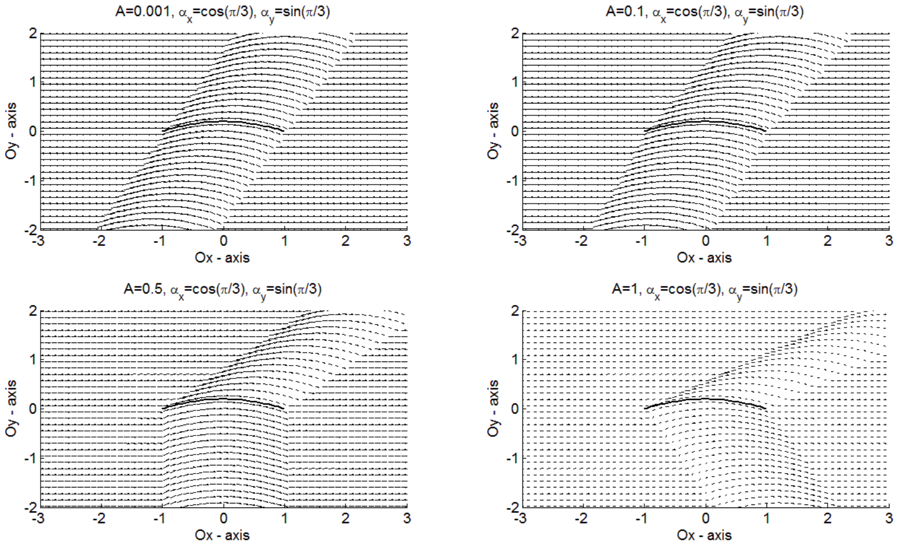


Figure 3: Velocity field and the parabolic profile

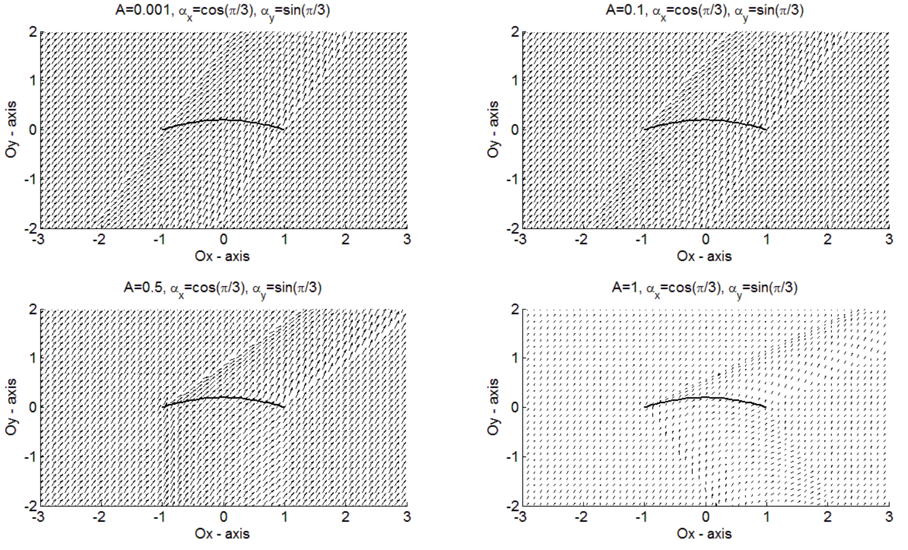


Figure 4: Magnetic induction and the parabolic profile

For calculating the integrals

$$I_3(x,y)=\int_{-1}^{1}\left(\frac{1-\xi}{1+\xi}\right)^{\theta}\frac{x-\xi}{(x-\xi)^2+y^2}d\xi,\ I_4(x,y)=\int_{-1}^{1}\left(\frac{1-\xi}{1+\xi}\right)^{\theta}\frac{y}{(x-\xi)^2+y^2}d\xi,$$

$$I_5(x,y)=\int_{-1}^{1}\left(\frac{1-\xi}{1+\xi}\right)^{\theta}\frac{\xi(x-\xi)}{(x-\xi)^2+y^2}d\xi,\ I_6(x,y)=\int_{-1}^{1}\left(\frac{1-\xi}{1+\xi}\right)^{\theta}\frac{\xi y}{(x-\xi)^2+y^2}d\xi,$$

we use the quadrature formulas

$$\int_{-1}^{1}\left(\frac{1-\xi}{1+\xi}\right)^{\theta} F(x,y,\xi)d\xi=\sum_{k=1}^{N} W_k F(x,y,t_k),$$

where

$$W_k=-\frac{4\Gamma\left(N+\theta+1\right)\Gamma\left(N-\theta+1\right)}{\left[\left(N+1\right)!\right]^2 P_{N-1}^{(\theta+1,-\theta+1)}\left(t_k\right)P_{N+1}^{(\theta,-\theta)}\left(t_k\right)}.$$

$P_{N-1}^{(\theta+1,-\theta+1)}\left(t_k\right), P_{N+1}^{(\theta,-\theta)}\left(t_k\right)$ are Jacobi polynomials and $t_k, k=1,...,N$ are the roots of the Jacobi polynomials $P_N^{(\theta,-\theta)}\left(t_k\right)$. We considered ε = 0.2.

In Figure 3 we present the velocity field for various values of Alfvén's number and the parabolic profile. We observe the waves generated by the magnetic field and notice that the normal component of the velocity on the profile vanishes. In Figure 4 we present the magnetic induction field for various values of Alfvén's number and the parabolic profile. We notice that the condition for continuity of the magnetic induction across the insulating profile is satisfied.

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