A numerical solution for the equation of the lifting surface in subsonic flow including tunnel effects

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DOI: 10.13111/2066-8201.2011.3.3.4

Abstract: *We employ the images method to establish the lifting surface equation including tunnel effects. We obtain a 2d hypersingular integral and we utilize Gauss-type quadrature formulas to discretize it. Numerical calculations are performed for the elliptical and rectangular wings in order to calculate the jump of the pressure.*

Key Words: *lifting surface, images method, hypersingular integral equation, tunnel effects, discretization, aerodynamic coefficients.*

1. INTRODUCTION

The paper deals with the subsonic lifting surface theory, which is a well known tool in the analysis of the aerodynamic characteristics of the wings. The main difficulty encountered in the integration of the lifting surface equation is caused by the singularities of the kernel which is a function depending on two variables. One of the first papers dedicated to the lifting surface integral equation belongs to Multhopp [1]. Numerous extensions and improvements of Multhopp's method appeared later.

Among them we mention the doublet strip method considered by Ichikawa [2]. In the framework of this method, the integration domain is transformed into a rectangular domain which is divided into many chordwise strips. In the strip containing the control point, the proposed method properly accounts (by means of a series expansion) for Cauchy and logarithmic singularities.

In the framework of another method (the doublet lattice method) the lifting surface is represented by a grid of boxes of trapezoidal shape; at ¼ chord of each box is located a line of pressure doublets with constant strength.

The control points are located at box ¾ chord. Ueda and Dowell [3] utilize a single doublet on the box ¼ chord, midspan, for representing the lift force on the box (double point method). A hybrid scheme in which the best features of the doublet lattice and doublet point method are used is given by Eversman and Pitt [4].

L. Dragoş ([5] page 179) presented a new approach for discretizing the lifting surface integral equation. He treated the *2d* integral as an iterated integral and utilized adequate quadratures formulas with respect to the x and y variables.

The same procedure was utilized by Dragoş and Carabineanu [6] for solving the lifting surface equation in ground effects.

In order to satisfy the slipping condition on the ground they utilized the image method by placing in the fluid stream another wing symmetric to the original one with respect to the ground-plane.

In the present paper, in order to study the tunnel effects, we generalize this method by placing a grid of wings such that each wall of the tunnel should be a plane of symmetry for the wings from the cascade.

The idea was previously used by Dragoş and Carabineanu [7] for studying the lifting line equation including ground effects and by Dragoş, Carabineanu and Dumitrache [8] for studying the lifting line equation including tunnel effects.

Hence, for investigating the tunnel effects on the lifting surface one has to solve a *2d* integral equation whose kernel has a singular part and a regular one, the regular part representing the sum of a uniformly convergent series of smooth functions. We discretize the integral equation with the aid of Dragoş's quadrature formulas and we calculate the aerodynamic coefficients for the rectangular and elliptic wings.

2. THE STATEMENT OF THE PROBLEM

The aerodynamics problem is the following: an uniform fluid flow in a wind tunnel is perturbed by a thin wing.

One requires to determining the perturbed flow and the aerodynamic action of the fluid against the wing. We denote by (x, y, z) the dimensionless Cartesian coordinates, and by p the dimensionless perturbation of the pressure. The equations

$$\mathbf{V}=U_\infty(\mathbf{i}+\mathbf{v}),\quad P = p_\infty + \rho_\infty U_\infty^2 p \tag{1}$$

relate the dimensional velocity and pressure by the dimensionless velocity and pressure (**i** stands for the versor of the Ox –axis). Let

$$z = h(x, y), |h| << 1, |h'_x| << 1,\ (x, y) \in D, \tag{2}$$

be the dimensionless equation of the wing surface S (we consider that the wing is reduced to its skeleton).

We denote by D the projection of S on the Oxy – plane and by χ and χ_1 the distances from the Oxy – plane to the walls of the wind tunnel.

The lifting surface equation including tunnel effects is

$$\begin{aligned}&\frac{1}{4\pi}\int\int_D^* \frac{f(\xi,\eta)}{y_0^2}K^{(0)}(x_0, y_0)d\xi d\eta + \frac{1}{4\pi}\int\int_D f(\xi,\eta)N^{(1)}(x_0, y_0)d\xi d\eta - \\ &-\frac{1}{4\pi}\int\int_D f(\xi,\eta)N^{(2)}(x_0, y_0)d\xi d\eta = -h'_x(x, y), (x, y) \in D,\end{aligned} \tag{3}$$

where the star (*) indicates the two-dimensional hyper-singular integral in Hadamard's sense.

In equation (3) the unknown function

$$f(x, y) = p(x, y, -0) - p(x, y, +0), \tag{4}$$

is the dimensionless jump of the pressure over the wing (pressure coefficient) and the kernels $K^{(0)}$, $N^{(1)}$ and $N^{(2)}$ are

$$K^{(0)}(x_0, y_0) = 1 + \frac{x_0}{R_0}, \tag{5}$$

$$\begin{aligned} N^{(1)}(x_0, y_0) = \sum_{n=-\infty}^{\infty} \frac{4\varsigma^2 [\chi_1 + n(\chi + \chi_1)]^2}{4[\chi_1 + n(\chi + \chi_1)]^2 + y_0^2} I^{(1,n)}(x_0, y_0) + \\ + \sum_{n=-\infty}^{\infty} \frac{4[\chi_1 + n(\chi + \chi_1)]^2 - y_0^2}{\left\{4[\chi_1 + n(\chi + \chi_1)]^2 + y_0^2\right\}^2} K^{(1,n)}(x_0, y_0), \end{aligned} \tag{6}$$

$$\begin{aligned} N^{(2)}(x_0, y_0) = \sum_{n=-\infty, n\neq 0}^{\infty} \frac{4\varsigma^2 n^2 (\chi + \chi_1)^2}{4n^2(\chi + \chi_1)^2 + y_0^2} I^{(2,n)}(x_0, y_0) + \\ + \sum_{n=-\infty, n\neq 0}^{\infty} \frac{4n^2(\chi + \chi_1)^2 - y_0^2}{\left\{4n^2(\chi + \chi_1)^2 + y_0^2\right\}^2} K^{(2,n)}(x_0, y_0). \end{aligned} \tag{7}$$

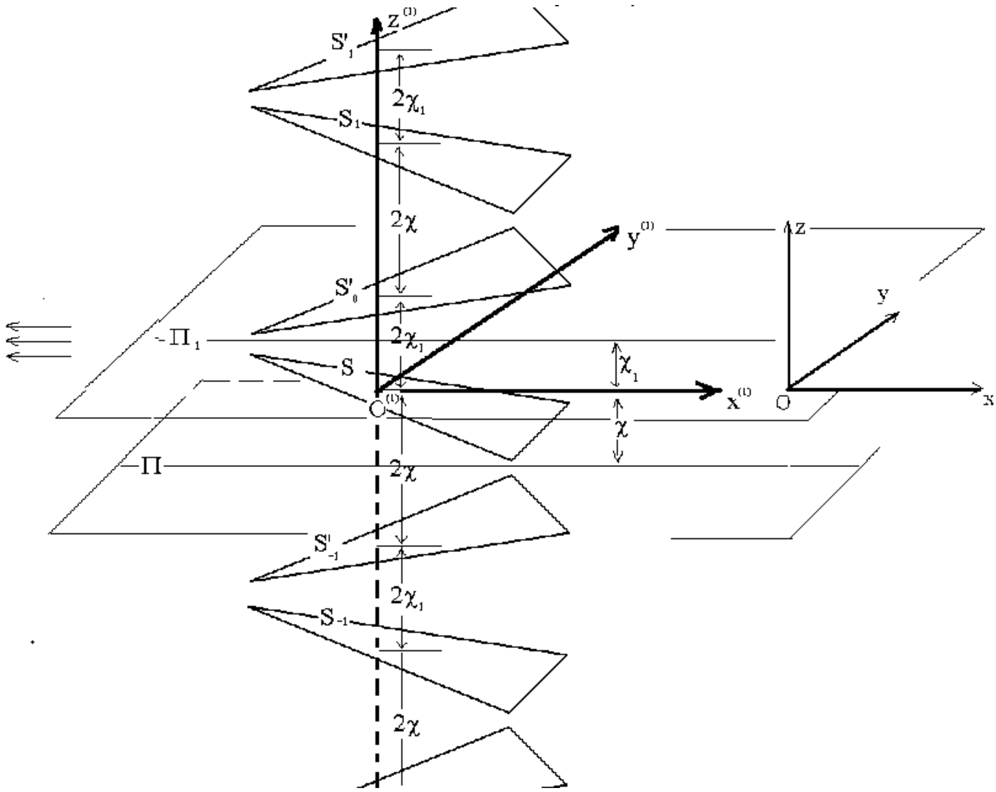

Fig. 1 – Method of images. Infinite grid of symmetrical profiles

We use the notations $x_0 = x - \xi, y_0 = y - \eta$, $K^{(1,n)}(x_0, y_0) = 1 + x_0 / R^{(1,n)}$, $K^{(2,n)}(x_0, y_0) = 1 + x_0 / R^{(2,n)}$, $I^{(1,n)}(x_0, y_0) = x_0 / \left[R^{(1,n)}\right]^3$, $I^{(2,n)}(x_0, y_0) = x_0 / \left[R^{(2,n)}\right]^3$,

$R^{(1,n)} = \sqrt{x_0^2 + \varsigma^2\left\{y_0^2 + 4\left[\chi_1 + n(\chi + \chi_1)\right]^2\right\}}$, $R^{(2,n)} = \sqrt{x_0^2 + \varsigma^2\left[y_0^2 + 4n^2(\chi + \chi_1)^2\right]}$, $R_0 = \sqrt{x_0^2 + \varsigma^2 y_0^2}$. We denote by $M = U_\infty / c_\infty$ the Mach number, by $c_\infty = \sqrt{\gamma p_\infty / \rho_\infty}$ the sound velocity and by $\varsigma = \sqrt{1 - M^2}$.

We shall deal with the subsonic flow *(M<1)*. An equation similar to equation (3) was obtained by L. Dragoş in [9], in the case of the wing in ground effects. For obtaining equation (3) we generalized Dragoş's equation by considering an infinite grid of wings such that the walls of the tunnel constitute planes of symmetry for the wings cascade (figure 1).

3. A NEW FORM OF THE INTEGRAL EQUATION

In the sequel we shall consider

$$x = x_-(y),\ x = x_+(y), \tag{8}$$

The equations of the leading, respectively trailing edges and

$$y = b,\ y = -b, \tag{9}$$

The equations of the lateral edges which are straight lines (figure 2).

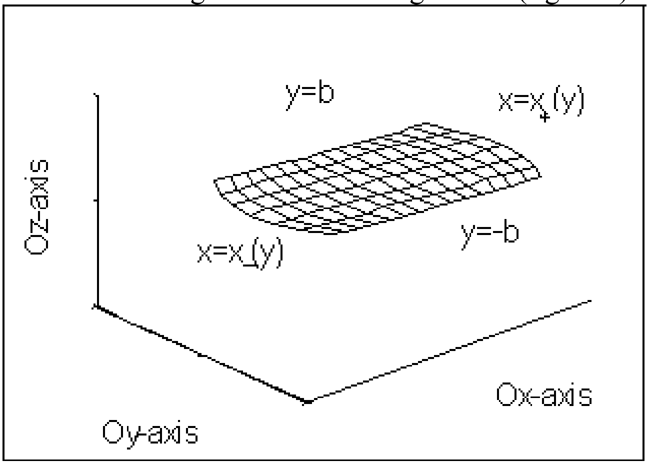

Fig. 2 – Leading, trailing and lateral edges of the wing

The quadrature formulae that we are going to introduce are valid for a class of wings satisfying on the lateral edges the relations

$$f(\xi, b) = f(\xi, -b) = 0,\ x_-(\pm b) < \xi < x_+(\pm b). \tag{10}$$

For integrating equation (3) we perform the changes of variables

$$(x, y) \to (u, v),\ (\xi, \eta) \to (\alpha, \beta), \tag{11}$$

by means of the equations

$$x = a(v)u + c(v),\ \xi = a(\beta)\alpha + c(\beta), \tag{12}$$

$$y = bv,\ \eta = b\beta, \tag{13}$$

where

$$a(v)=\frac{x_+(bv)-x_-(bv)}{2},\ c(v)=\frac{x_+(bv)+x_-(bv)}{2}, \tag{14}$$

and similarly for $a(\beta)$ and $c(\beta)$.

Hence the chordwise variables u,α and the span wise variables v,β will be defined on the interval $[-1,1]$. Since

$$\det\frac{\partial(\xi,\eta)}{\partial(\alpha,\beta)}=a(\beta)b,$$

equation (3) is equivalent to

$$\begin{aligned}
&\frac{1}{4\pi}\int_{-1}^{*1}\frac{a(\beta)}{(v-\beta)^2}\left[\int_{-1}^{1}f(\alpha,\beta)K^{(0)}(u,v,\alpha,\beta)d\alpha\right]d\beta+\\
&+\frac{b^2}{\pi}\sum_{n=-\infty}^{\infty}\int_{-1}^{1}\frac{\varsigma^2[\chi_1+n(\chi+\chi_1)]^2a(\beta)}{4[\chi_1+n(\chi+\chi_1)]^2+b^2(v-\beta)^2}\left[\int_{-1}^{1}f(\alpha,\beta)I^{(1,n)}(u,v,\alpha,\beta)d\alpha\right]d\beta+\\
&+\frac{b^2}{4\pi}\sum_{n=-\infty}^{\infty}\int_{-1}^{1}\frac{a(\beta)\left\{4[\chi_1+n(\chi+\chi_1)]^2-b^2(v-\beta)^2\right\}}{\left\{4[\chi_1+n(\chi+\chi_1)]^2+b^2(v-\beta)^2\right\}^2}\left[\int_{-1}^{1}f(\alpha,\beta)K^{(1,n)}(u,v,\alpha,\beta)d\alpha\right]d\beta-\\
&-\frac{b^2}{\pi}\sum_{n=-\infty}^{\infty}\int_{-1}^{1}\frac{\varsigma^2n^2(\chi+\chi_1)^2a(\beta)}{4n^2(\chi+\chi_1)^2+b^2(v-\beta)^2}\left[\int_{-1}^{1}f(\alpha,\beta)I^{(2,n)}(u,v,\alpha,\beta)d\alpha\right]d\beta-\\
&-\frac{b^2}{4\pi}\sum_{n=-\infty,n\neq 0}^{\infty}\int_{-1}^{1}\frac{a(\beta)\left[4n^2(\chi+\chi_1)^2-b^2(v-\beta)^2\right]}{\left[4n^2(\chi+\chi_1)^2+b^2(v-\beta)^2\right]^2}\left[\int_{-1}^{1}f(\alpha,\beta)K^{(2,n)}(u,v,\alpha,\beta)d\alpha\right]d\beta=\\
&=-bh'_x(u,v),
\end{aligned} \tag{15}$$

where

$$K^{(0)}(u,v,\alpha,\beta)=1+\frac{\bar{x}_0}{\bar{R}_0},\ K^{(1,n)}(u,v,\alpha,\beta)=1+\bar{x}_0/\bar{R}^{(1,n)},$$

$$K^{(2,n)}(u,v,\alpha,\beta)=1+\bar{x}_0/\bar{R}^{(2,n)},\ I^{(1,n)}(u,v,\alpha,\beta)=\bar{x}_0/\left[\bar{R}^{(1,n)}\right]^3,$$

$$I^{(2,n)}(u,v,\alpha,\beta)=\bar{x}_0/\left[\bar{R}^{(2,n)}\right]^3,\ \bar{x}_0=a(v)u+c(v)-a(\beta)\alpha-c(\beta),$$

$$R^{(1,n)}=\sqrt{\bar{x}_0^2+\varsigma^2\left\{b^2(v-\beta)+4[\chi_1+n(\chi+\chi_1)]^2\right\}},$$

$$R^{(2,n)}=\sqrt{\bar{x}_0^2+\varsigma^2\left[b^2(v-\beta)+4n^2(\chi+\chi_1)^2\right]},\ R_0=\sqrt{\bar{x}_0^2+\varsigma^2b^2(v-\beta)^2},$$

$$h'_x(u,v)=h'_x(a(v)u+c(v),bv),\quad f(\alpha,\beta)=f(a(\beta)\alpha+c(\beta),b\beta).$$

In the new variables relations (10) become

$$f(\alpha,\pm 1)=0,\ -1<\alpha<1. \tag{16}$$

4. THE INTEGRATION OF THE LIFTING SURFACE EQUATION

Let

$$\alpha_i = \frac{2i-m}{m}, i = 0,1,...,m,$$

be equidistant nodes on the interval $[-1,1]$.

Replacing the integrals with Riemann sums we get

$$\int_{-1}^{1} f(\alpha,\beta)K^{(0)}(u,v,\alpha,\beta)d\alpha \approx \frac{2}{m}\sum_{i=1}^{m} f(\alpha_i,\beta)K^{(0)}(u,v,\alpha_i,\beta), \tag{17}$$

$$\int_{-1}^{1} f(\alpha,\beta)K^{(1,n)}(u,v,\alpha,\beta)d\alpha \approx \frac{2}{m}\sum_{i=1}^{m} f(\alpha_i,\beta)K^{(1,n)}(u,v,\alpha_i,\beta), \tag{18}$$

$$\int_{-1}^{1} f(\alpha,\beta)K^{(2,n)}(u,v,\alpha,\beta)d\alpha \approx \frac{2}{m}\sum_{i=1}^{m} f(\alpha_i,\beta)K^{(2,n)}(u,v,\alpha_i,\beta), \tag{19}$$

$$\int_{-1}^{1} f(\alpha,\beta)I^{(1,n)}(u,v,\alpha,\beta)d\alpha \approx \frac{2}{m}\sum_{i=1}^{m} f(\alpha_i,\beta)I^{(1,n)}(u,v,\alpha_i,\beta), \tag{20}$$

$$\int_{-1}^{1} f(\alpha,\beta)I^{(2,n)}(u,v,\alpha,\beta)d\alpha \approx \frac{2}{m}\sum_{i=1}^{m} f(\alpha_i,\beta)I^{(2,n)}(u,v,\alpha_i,\beta). \tag{21}$$

Taking into account (16) we shall assume for f the following behaviour

$$f(\alpha,\beta) = \sqrt{1-\beta^2}\,\overline{f}(\alpha,\beta), \tag{22}$$

where $\overline{f}(\alpha,\beta)$ is finite for $\beta = \pm 1$.

Dragoş and Popescu [10] gave the following Gauss-type quadrature formula for computing hypersingular *1d* integrals with generalized Cauchy kernels

$$\int_{-1}^{*1} \sqrt{1-\beta^2}\,\frac{F(\beta)d\beta}{(\beta-\beta_k)^2} \approx \frac{\pi}{p+1}\sum_{j\neq k, j=1}^{p}\left[1-(-1)^{j+k}\right]\frac{1-\beta_j^2}{(\beta_j-\beta_k)}F(\beta_j) - \pi\frac{p+1}{2}F(\beta_k) \tag{23}$$

where

$$\beta_j = \cos\frac{j\pi}{p+1}, j = 1,...,p. \tag{24}$$

We also take into account the Gauss quadrature formula for smooth functions

$$\int_{-1}^{1} \sqrt{1-\beta^2}\,F(\beta)d\beta \approx \frac{\pi}{p+1}\sum_{j=1}^{p}\left(1-\beta_j^2\right)F(\beta_j). \tag{25}$$

Using the quadrature formulas (17)-(25) we obtain the following discretized form of equation (15):

$$\sum_{i=1}^{m}\sum_{j=1}^{p}\overline{f}\left(\alpha_i,\beta_j\right)K_{lkij} = -bh'_x\left(\alpha_l,\beta_k\right),\quad l=1,...,m,\ \ k=1,...,p, \tag{26}$$

where

$$K_{lkij} = K_{lkij}^{(0)} + K_{lkij}^{(1)} + I_{lkij}^{(1)} - K_{lkij}^{(2)} - I_{lkij}^{(2)}, \tag{27}$$

$$K_{lkij}^{(0)} = \frac{1}{2m(p+1)}\left[1-(-1)^{j+k}\right]\frac{1-\beta_j^2}{\left(\beta_j-\beta_k\right)^2}a\left(\beta_j\right)K^{(0)}\left(\alpha_l,\beta_k,\alpha_i,\beta_j\right),\ j\neq k, \tag{28}$$

$$K_{lkik}^{(0)} = -\frac{p+1}{4m}a\left(\beta_j\right)K^{(0)}\left(\alpha_l,\beta_k,\alpha_i,\beta_k\right), \tag{29}$$

$$K_{lkij}^{(1)} = \frac{b^2\left(1-\beta_j^2\right)a\left(\beta_j\right)}{2m(p+1)}\sum_{n=-\infty}^{\infty}\frac{4\left[\chi_1+n(\chi+\chi_1)\right]^2-b^2\left(\beta_k-\beta_j\right)^2}{\left\{4\left[\chi_1+n(\chi+\chi_1)\right]^2+b^2\left(\beta_k-\beta_j\right)^2\right\}^2}K^{(1,n)}\left(\alpha_l,\beta_k,\alpha_i,\beta_j\right), \tag{30}$$

$$I_{lkij}^{(1)} = \frac{2b^2\left(1-\beta_j^2\right)a\left(\beta_j\right)}{m(p+1)}\sum_{n=-\infty}^{\infty}\frac{\varsigma^2\left[\chi_1+n(\chi+\chi_1)\right]^2}{4\left[\chi_1+n(\chi+\chi_1)\right]^2+b^2\left(\beta_k-\beta_j\right)^2}I^{(1,n)}\left(\alpha_l,\beta_k,\alpha_i,\beta_j\right), \tag{31}$$

$$K_{lkij}^{(2)} = \frac{b^2\left(1-\beta_j^2\right)a\left(\beta_j\right)}{2m(p+1)}\sum_{n=-\infty,n\neq 0}^{\infty}\frac{4n^2\left(\chi+\chi_1\right)^2-b^2\left(\beta_k-\beta_j\right)^2}{\left\{4n^2\left(\chi+\chi_1\right)^2+b^2\left(\beta_k-\beta_j\right)^2\right\}^2}K^{(2,n)}\left(\alpha_l,\beta_k,\alpha_i,\beta_j\right), \tag{32}$$

$$I_{lkij}^{(2)} = \frac{2b^2\left(1-\beta_j^2\right)a\left(\beta_j\right)}{m(p+1)}\sum_{n=-\infty}^{\infty}\frac{\varsigma^2 n^2\left(\chi+\chi_1\right)^2}{4n^2\left(\chi+\chi_1\right)^2+b^2\left(\beta_k-\beta_j\right)^2}I^{(2,n)}\left(\alpha_l,\beta_k,\alpha_i,\beta_j\right), \tag{33}$$

The algebraic linear system (26) consists of $p\times m$ equations for $p\times m$ unknowns $\overline{f}\left(\alpha_i,\beta_j\right)$.

5. CALCULUS OF THE AERODYNAMICAL COEFFICIENTS OF THE WING

The coefficients which are important from the point of view of aerodynamics are the lift coefficient

$$C_L = \frac{2}{A}\iint_D f(\xi,\eta)d\xi d\eta = \frac{2b}{A}\int_{-1}^{1}\int_{-1}^{1}a(\beta)f(\alpha,\beta)d\alpha d\beta, \tag{34}$$

and the rolling and pitching moment coefficients

$$C_x = \frac{2}{Aa_0}\iint_D \eta f(\xi,\eta)d\xi d\eta = \frac{2b^2}{Aa_0}\int_{-1}^{1}\int_{-1}^{1}\beta a(\beta)f(\alpha,\beta)d\alpha d\beta, \tag{35}$$

$$C_y = \frac{-2}{Aa_0}\iint_D \xi f(\xi,\eta)d\xi d\eta = \frac{-2b}{Aa_0}\int_{-1}^{1}\int_{-1}^{1}a(\beta)\left(a(\beta)\alpha+c(\beta)\right)f(\alpha,\beta)d\alpha d\beta, \tag{36}$$

where A is the area of D and a_0 is the dimensionless chord of the wing. For calculating numerically the lift and moment coefficients we use the quadrature formulas (25) and

$$\int_{-1}^{1} g(\alpha)F(\alpha,\beta) = \sum_{i=1}^{m} F(\alpha_i,\beta)\int_{\alpha_{i-1}}^{\alpha_i} g(\alpha)d\alpha. \tag{37}$$

Hence

$$C_L = \frac{4b\pi}{(p+1)mA}\sum_{j=1}^{p}\sum_{i=1}^{m}\left(1-\beta_j^{\,2}\right)a\left(\beta_j\right)F\left(\alpha_i,\beta_j\right), \tag{38}$$

$$C_x = \frac{4b^2\pi}{(p+1)mAa_0}\sum_{j=1}^{p}\sum_{i=1}^{m}\left(1-\beta_j^{\,2}\right)\beta_j a\left(\beta_j\right)F\left(\alpha_i,\beta_j\right), \tag{39}$$

$$C_y = \frac{-4b\pi}{(p+1)mA}\sum_{j=1}^{p}\sum_{i=1}^{m}\left(1-\beta_j^{\,2}\right)a\left(\beta_j\right)\left[\frac{2i-m-1}{m}a\left(\beta_j\right)+c\left(\beta_j\right)\right]F\left(\alpha_i,\beta_j\right). \tag{40}$$

6. NUMERICAL RESULTS AND CONCLUSION

The first example is the incompressible flow ($\varsigma = 1$) past the elliptical wing

$$z = -\varepsilon x, \quad x^2 + \frac{y^2}{b^2} \le 1, \quad b = 2. \tag{41}$$

In figure 3 we present the jump of the pressure (more precisely $f / b\varepsilon$) over the elliptical wing. We consider two cases, $\chi = 10, \chi_1 = 10$ and $\chi = 10, \chi_1 = 1$.

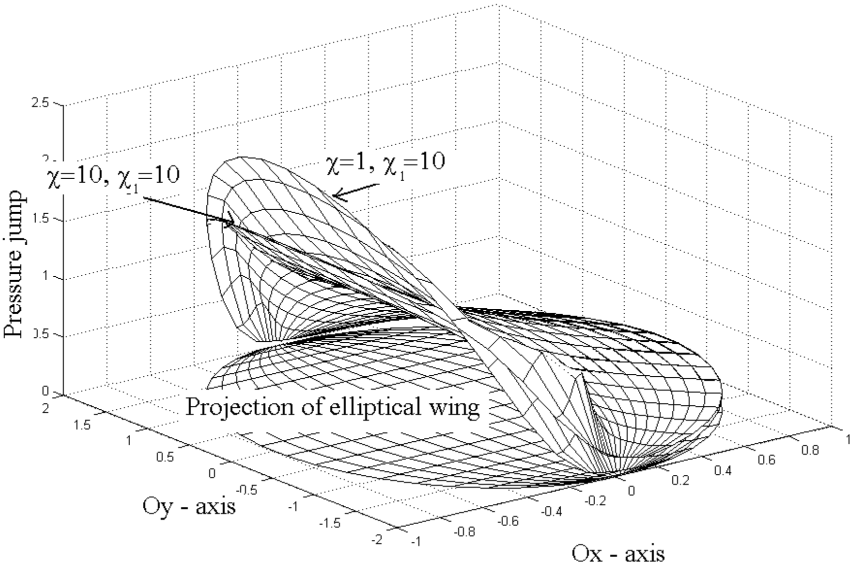

Fig. 3 – Pressure jump over the elliptical wing

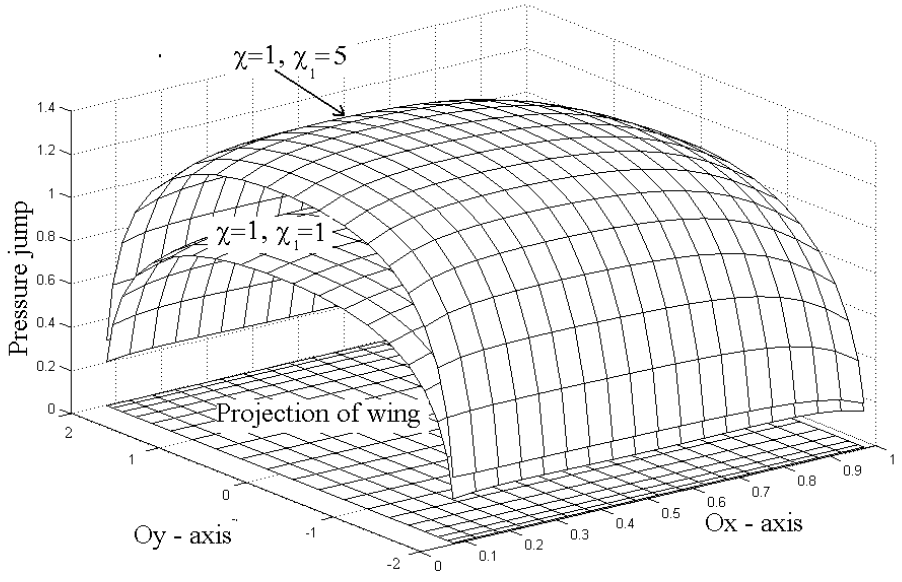

Fig. 4 – Pressure jump over the rectangular wing

We also studied the incompressible flow past a rectangular wing whose cross section is an arc of parabola

$$z = \varepsilon(1 - x^2),\ 0 \le x \le 1,\ -b \le y \le b,\ x_+(y) = 0, x_-(y) = 1. \tag{42}$$

In figure 4 we considered *b=2* and we calculated the jump of the pressure (more precisely $f/b\varepsilon$) over the wing for $\chi = 1, \chi_1 = 1$ and $\chi = 1, \chi_1 = 5$.

CONCLUSIONS

We presented a method to calculate the influence of the impermeable walls on the pressure field over the wing (ground and tunnel effects).

It is well known that there were been constructed aircrafts which take advantage of the ground effects for improving their aerodynamic performances – ekranoplanes.

The results presented herein may also be utilized for transferring the experimental results obtained for wings in wind tunnels to the wings moving in free space.

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