Unmanned aero-amphibious vehicle: preliminary study on conceptual design

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Abstract: *This research paper deals with the methods of designing of an unmanned aero-amphibious vehicle finally allowing its manufacture. The design does not includes only the structure and electronics circuit but also the control design that mainly consists of mathematical modeling of the control system. Aero-amphibious vehicle is a vehicle that has the potential to fly as well as to swim inside the water. So, the mathematical model would be more complex than the general quad copters or drones that fly these days. This mathematical model of aero-amphibious vehicle helps develop the autopilot control codes by converting them into transfer function and by applying the control laws. Structural design and electronic circuit design of this unmanned aerial vehicle were also presented in brief in this paper so that it would be easy to frame equations for the mathematical modeling. In order to justify the structural and electronic design, their constraints and limitations were also explained in this research paper. In the mathematical modeling part, different equations were solved using different methods and those methods were also demonstrated in this research paper.*

Key Words: *Unmanned Aerial-Aquatic Vehicle, Underwater Robotics, Mathematical Modeling*

1. INTRODUCTION

Unmanned aerial vehicle has become a familiar notion nowadays as UAVs are used not only in aviation but also in many other industrial areas. The combination of aviation and electronics with little bit of artificial intelligence has made most of the engineering work simple. People throughout the world trying to invent things that could make their work faster and with better efficiency.

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Unmanned Aerial Vehicle is such an invention that could be used in various applications. Unmanned Aero-amphibious Vehicle is next level of Unmanned Aerial Vehicle.

This Aero-amphibious Vehicle is an Unmanned Vehicle that can be operated in air as well as in water. The level of challenges in this field has attracted many enthusiasts to work on this research.

The typical vehicle in the market has undeveloped performance. The undeveloped performances of the vehicle are mainly due to lack of technology and complexities of design. Another important issue faced by the researchers is the cost of the product.

But still researchers are working to perfect the design with lowest cost possible. [1],[2], [3], [4]. Figure (a) shows an aero-amphibious vehicle which can fly in air and travel on the water surface.

This research work deals with mathematical modeling, so we focus more on its dynamics rather on electronics and communication systems of the vehicle.

2. APPLICATIONS

This research work has a wide range of application. One of the most important is to inspect bridges which has both aerial as well as aquatic environment. In general, the section of the bridge submerged in water is inspected separately from the portion above the water surface. But this type of aero-amphibious vehicle will solve this type of issues. This vehicle flies in the air and performs inspection and continues to swim inside the water to inspect the portion of the bridge submerged in water.

Another major application of this research work is to check oil spills inside the pipes. In Arabian countries, lot of oil is transported underground to many places. It can be fully filled flow or halfly filled flow, and in both cases checking for oil leakage is necessary. So in order to travel in the environment with oil and in the environment above oil, such aero-amphibious vehicle can be used. These aero-amphibious vehicles can also be used to search black box and other avionics parts in oceans. Few years ago, a Malaysian aircraft got crashed. Different nations searched its parts through helicopters.

Once they located the parts, underwater vehicles are sent in search of the aircraft's black box under the ocean. So in this case the helicopter and vehicle that can travel underwater can be combined to save time, and this research work is one as such[5], [6].

3. BASIC CONCEPTUAL DESIGN

Different methodologies were adopted based on the applications and constraints of each methodology. Therefore, there is more than one method for this research to work successfully.

The first idea implemented is to use the waterproof electronics and test it rigorously in both air and water. In case this idea fails then the back-up idea kept to make this vehicle possible is to apply corrosion-X sprays to electronic parts exposed to the aquatic environment and make it waterproof. Alternate backup solution kept was to use Li-Fi technology for underwater communication. For under water communication the frequency is different from that of land and air, so idea of using variable frequency transmitter receiver could be kept as one of the options. Though the above mentioned ideas were good, they all have their own limitations and constraints. Waterproof electronics parts can be used to fabricate this vehicle, but it will increase greatly the cost; also programming the flight controller board to make the unmanned aerial system and unmanned aquatic system stable together is a difficult job. Applying sprays like corrosion-X on electronic parts seemed to be a better idea, but it will last for 2-3 minutes only. But this research work can be useful only if the vehicle is at least 5-10 minutes. But to make this idea work, the corrosion-X sprays can be mixed with adhesives, but mixing them with adhesives will increase the weight of the unmanned vehicle. So, this idea could not be implemented. Newly emerged technology Li-Fi technology, could be used for the underwater communication, but it has range limitation of 1m. Also, the light penetrated inside the water experience refraction and bend, so the vehicle will be controlled with certain displacement always. So, this idea could not be implemented. After understanding all these constraints, various techniques have been implemented on modifying a quad copter that is used for taking photographs in rainy season. After observing and testing a Chinese submarine toy, the transmitter and receiver were embedded with underwater system controller board. Also, mini DC motors were embedded with that unmanned controller board and soldered together [7], [8], [9], [10], [11], [12].

4. STRUCTURAL DESIGN

This phase consists of applying the concepts of dynamics both in air and under water and comprises of conceptual, preliminary and detail design of the vehicle. The design phases are divided into structural design and electronic design. The main portion of structural design is to provide a place for both unmanned aerial system controller board and unmanned aquatic system controller board. The structural design also includes the distribution of the forces and carrying those loads produced by the thrusters through the arms, equally. Also, the thrust produced by the thrusters in aerial system should perform the lift-off of the whole vehicle. The structure should withstand the pressure and forces developed during the underwater dynamics. While building structure the important thing that needs to be kept in mind is that it has to be light in weight, suitable to fly in sky as well as it has to be waterproof, suitable to swim under water. The electronics design plays the major role in this research work, as it includes the communication and control of the vehicle both in aerial and aquatic conditions. Soldering the controller board used for the aquatic system is also an important part of electronic design, as it should not disturb the adhesives and sprays used to make it waterproof.

This Aero-amphibious Vehicle could be used and divided into 3 parts, one exclusive system for applications of Aerial conditions, one exclusive system for applications of Aquatic conditions and another exclusive system for applications of Terrestrial conditions. This research work is unique as it has sandwiched all three applications in one single device. The design of application system for Aerial conditions will not be the same as that of application system for Aquatic conditions. The Design of application system for Terrestrial conditions is much more different than the application system of both Aerial and Aquatic conditions. All the three applications have their unique design. So trying to fit all three applications of different environments in a single system is a complex task.

The above discussed design constraints are the common ones which could be noticed with the naked eye, but there are several constraints that could be known only after testing the system in all the environmental conditions. Weight issues are also one of the major constraints while considering the Aerial condition. Reducing the weight of the vehicle is a priority in the case of Aerial condition. Also this reduction is preferable in the case of aquatic and terrestrial conditions. Designing the vehicle in such a way that the whole system can fully submerge in inside water is quite an important task. The Aerial system in this vehicle has to be so light in weight and durable as it has to carry the other systems as a payload. As it is an unmanned vehicle, it has complex electronic circuit and radio wave signal transmission to operate in air. But the electronics and communication channel won’t work inside water as the frequency of the communication channel being operated doesn't penetrate through water. This problem has made the research work more challenging and interesting. Making the electronics work under water is a possible task though it appears difficult, but making the radio waves pass through the underwater is an impossible task. So finding out a solution for that indulge one in rigorous research.

Figure (a): Unmanned Aero-amphibious Vehicle

5. DESIGN OF ELECTRONIC COMPONENTS

This vehicle is nothing but the combination of an unmanned aerial system with an unmanned aquatic system. Unmanned aerial system is so common these days, but the same unmanned aerial system needs to be modified to make it swim under water without disturbing its electronics needed to work in air. Similar system was JJRC H31 systems [13].These systems have electronics that are water resistant and perform motion in air. So, the main parts of JJRC H31like motors, propellers, flight controller boards and receiver could be used in this design to make the unmanned aerial part of the project. The unmanned aerial system can be controlled by 6 channel 2.4GHz transmitter-receiver which has 80m range in air. The lists of various parts used to build this aero-amphibious vehicle are in the table below:

Table 1: List of components used in Aero-amphibious Vehicle

1.	Unmanned aerial system control board
2.	Motors in an enclosed gear box- 20g Thrust
3.	Propeller gaurds on the sides of the motors- ABS
4.	Micro LiPO battery- 3.7V 400mAh
5.	6 Ch Transmitter- 2.4Ghz
6.	Propellers are kept after the testing of motors- ABS 136*29*40mm
7.	Unmanned aquatic system control board
8.	Ni-MH battery for aquatic system- 3.6V 120mAh
9.	Suction motor- 20g Thrust

The unmanned aerial system is made inside a liquid proof container and sealed it with hot glue. On the container walls, 4 holes are made to allow the wires of 4 motors to enter and embedded with the control board. The 4 motors and the container are linked by PVC pipes to transfer the thrust generated by the motors as well as it carries the wires from motors to control board. The motors are attached with gears and it is kept in in an enclosed box in such a way that propellers can be mounted easily. The motors used are actually high torque low weight DC motors manufactured in China. These motors are generally used in vibrator in cellphones, but it was modified and meshed with set of gears to increase the torque produced by it. We need to increase the torque with mininmum possible RPM to run the propellers in the shaft with no loss of thrust in total motor propeller assembly.

The propeller which was driven by the motor needs protection, so the side of the casing is attached with propeller guards. This guard is made of ABS plastic which is lightweight, having enough strength to protect the propeller from damaging and is also flexible. So installing the propeller guards at the sides will be a greater advantage for this project.

To power the unmanned aerial system, a 400mAh micro LiPo battery is used. The battery is charged using USB port for the inverter charger. The Figure (b) below shows the picture of the battery and the charger. The charger used is actually a Samsung charger from which the USB connector is removed and the charger for battery is attached.

Now the whole system is made compact and all the motors and propeller guards are mounted and connections are soldered. The unmanned aerial system need to be tested; first we check wether all four motors are working and simultaneously we will be able to check the R_x-T_x communication also.

We check the connection thorughly. Then the whole system need to be checked inside water. However the motors wont work inside water as R_x-T_x communication we use in conventional drones and the one used here for aerial flying purpose was of 2.4GHz frequency, which will not penetrate through water. We immerse the whole unmanned aerial system in water then we check wether it flies in the air. This makes sure that the unmanned aerial system is waterproof.

After the system is checked thoroughly, propellers are needed to be mounted. The propellers are not supposed to be mounted before or during checking. Fasten the propellers on the motors and check for the clockwise or counterclockwise directions. After the flying test, the unmanned aquatic system needs to be combined with the unmanned aerial system mechanically as well as electrically.

The unmanned aquatic system consists of a controller board which is waterproof and the R_x-T_x communication of the control board can penetrate through water. The communication under water used operates in 40 MHz frequency which is low enough to penetrate through the water. The system consists of a suction motor with a propeller to help the system perform up and down motion.

It has two micro motors for the forward, backward, right and left motions under water. Those motors are embedded with the controller board [13].

In the controller board there are six terminals are embedded which allows to connect 3 motors. It is programmed in such a way that 2 terminals connect one motor that rotates both clockwise and counterclockwise alone, so we can connect the suction motor which is used only for up and down motions.

Other 4 terminals are programmed in such a way that two motors can be embedded both operating in the forward, backward, left and right directions. Therefore, they are suitable for the other two motors.

The unmanned aquatic system has a different on and off mechanism such that the system can be waterproof while it is powered on. It has a plug which completes the circuit while it is inserted in and causes the system to remain watertight. It has LEDs for the power indication.

The unmanned aquatic system is powered by 120mAh Ni-MH 3.6V battery. It lasts for 10minutes and it is charged from the remote control of the unmanned aquatic system itself. The remote control is AA battery operated.

The battery with the control board is placed in the front of the unmanned aquatic system to balance the vehicle's center of gravity.

The suction motor is soldered with the unmanned aquatic system control board in the provided terminal.

The suction motor has two terminals, positive and negative. Reversing the polarity allows it to rotate both clockwise and counterclockwise directions.It is attached with a hydrofoil propeller to make it swim inside water.

The suction motor system is placed in the middle of the unmanned aquatic system as the suction needs to be effective.

The motors at the back, with two terminals each, were embedded with the controller board in the provision given for it. The board is programmed in such a way that when the motors run clockwise, the device moves forward.

When the motors run counterclockwise, the device moves backward. When the right motor rotates clockwise and the left motor rotates counterclockwise, the device turns left. When the left motor rotates clockwise and the right motor rotates counterclockwise, the device turns right.

This system is kept at the aft part of the unmanned aquatic system so that the moment created by the thrust of the motors balance the net moment of the vehicle. Also, the pusher configuration is better in generating thrust.

Figure (b): Unmanned Aero-amphibious Vehicle

The combination of the unmanned aerial system and the unmanned aquatic system becomes an unmanned aero-amphibious system as shown in Figure (b). The device could be fitted with movable landing gear to make it suitable to work in land as well.

6. VEHICLE DYNAMIC MODELLING

The modeling of the vehicle in air must be framed separately and modeling of the vehicle under water must be framed separately, in order to frame the modeling of such aero amphibious vehicle.

Modeling of this vehicle includes complex aerodynamic calculations and different control methods.

In order to apply control methods, it is necessary to use the accelerometer, gyroscope and GPS to obtain position and attitude measurements.

6.1 Modeling of Force and Torque Matrices

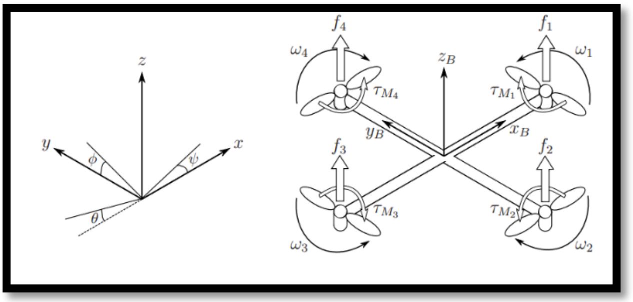

Figure (c): Inertial Frame and Body frame [14]

By considering the Figure (c), the linear position of the unmanned aerial system can be defined in inertial frame- x axes, y axes and z axes as ξ. Also the angular position of the unmanned aerial system can be defined in inertial frame as η. The rotation of unmanned aerial system in y-axis is given by the pitch angle θ. The rotation of unmanned aerial system in x-axis is given by roll angle Φ. The rotation of unmanned aerial system in z-axis is given by yaw angle ψ. The linear and angular position of the unmanned aerial system will be inserted in vector q [14]

$$\xi = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{, Linear position of the inertial frame} \tag{1}$$

$$\eta = \begin{bmatrix} \Phi \\ \theta \\ \psi \end{bmatrix} \text{, Angular position of the inertial frame} \tag{2}$$

$$q = \begin{bmatrix} \xi \\ \eta \end{bmatrix} \text{ Vector containing linear position and angular position of the inertial frame} \tag{3}$$

$$\xi = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}, \ \eta = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \ q = \begin{bmatrix} \xi \\ \eta \end{bmatrix}$$

$$\dot{\xi} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{, Linear Velocity of the inertial frame}$$

Let us align the body frame along with the center of mass of the unmanned system. The linear velocities in body frame are given by V_B and the angular velocities in body frame are given by ν.

$$V_B = \begin{bmatrix} \upsilon_{x,}B \\ \upsilon_{y,}B \\ \upsilon_{z,}B \end{bmatrix} \text{, Linear Velocity of the body frame} \tag{4}$$

$$\upsilon = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \text{, Angular Velocity of the body frame} \tag{5}$$

Rotation matrix from body frame to inertial frame is given by

$$R = \begin{bmatrix} C_\psi C_\theta & C_\psi S_\theta S_\Phi - S_\psi C_\Phi & C_\psi S_\theta C_\Phi + S_\psi S_\Phi \\ S_\psi C_\theta & S_\psi S_\theta S_\Phi + C_\psi C_\Phi & S_\psi S_\theta C_\Phi - C_\psi S_\Phi \\ -S_\theta & C_\theta S_\Phi & C_\theta C_\Phi \end{bmatrix} \quad (6)$$

where: sin(x) is given by S_x, cos(x) is given by C_x.

But rotation matrix R is orthogonal so

$R^{-1}= R^T$ is the inertial frame to body frame rotation matrix.

W_η is the inertial frame to body frame angular velocity transformation matrix.

W_η^{-1} is the body frame to inertial frame angular velocity transformation matrix.

$\dot{\eta} = W_\eta^{-1} \upsilon$,

$$\begin{bmatrix} \dot{\Phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & S_\Phi T_\theta & C_\Phi T_\theta \\ 0 & C_\Phi & -S_\Phi \\ 0 & S_\Phi / C_\theta & C_\Phi / C_\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}, \text{ Angular velocity in inertial frame} \quad (7)$$

$$\dot{\eta} = \begin{bmatrix} 1.0986 \\ 1.4165 \\ -1.3021 \end{bmatrix}$$

$$\upsilon = W_\eta \dot{\eta},$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -S_\theta \\ 0 & C_\Phi & C_\theta S_\Phi \\ 0 & -S_\Phi & C_\theta C_\Phi \end{bmatrix} \begin{bmatrix} \dot{\Phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}, \text{ Angular velocity in body frame} \quad (8)$$

The matrix W_η^{-1} is invertible if $\theta \neq (2k-1)\phi/2, (k \epsilon \mathbb{Z})$. Since the unmanned aerial system is having symmetric structure along x-axes and y-axes, the inertial matrix which is diagonal matrix **I** will have

$$I_{xx}=I_{yy}, \qquad I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}.$$

$$I_{xx} = \frac{2mr^2}{5} + 2ml^2 \text{ Moment of inertia along x-x axes} \quad (9)$$

$$I_{yy} = \frac{2mr^2}{5} + 2ml^2 \text{ Moment of inertia along y-y axes} \quad (10)$$

$$I_{zz} = \frac{2mr^2}{5} + 4ml^2 \text{ Moment of inertia along z-z axes} \quad (11)$$

$$I_{xx} = 11.52 \times 10^{-3} kgm^2$$

$$I_{yy} = 11.52 \times 10^{-3} kgm^2$$

$$I_{zz} = 23.04 \times 10^{-3} kgm^2$$

$$I = \begin{bmatrix} 11.52 \times 10^{-3} & 0 & 0 \\ 0 & 11.52 \times 10^{-3} & 0 \\ 0 & 0 & 23.04 \times 10^{-3} \end{bmatrix}$$

Let rotor numbers be denoted as i. Let angular velocity be denoted as ω_i. Let the force created be f_i. This force generated in the direction of rotating axis. There will be also torque generated by the acceleration as well as the angular velocity of the rotor. Let the torque be denoted asτ_{M_i}.The force and torque given by

$$F_i = k\omega_i^2, \tag{12}$$

$$\tau_{M_i} = b\ \omega_i^2 + I_M\dot{\omega}_i\ , \tag{13}$$

where k is lift constant and b is drag constant and I_M is the moment of inertia of the rotor.
All the forces together create thrust T in the z-axis direction of the body frame. Torque Matrix $\tau_B$has torques in their corresponding body frame angles.

$$T = \sum_{i=1}^{4} F_i \tag{14}$$

$$T = k\sum_{i=1}^{4} \omega^2{}_i \tag{15}$$

$$T^B = \begin{bmatrix} 0 \\ 0 \\ T \end{bmatrix}, T = 0.1192\ \text{N},\ T^B = \begin{bmatrix} 0 \\ 0 \\ 0.1192 \end{bmatrix}$$

Angular velocities of the motors are:$\omega_1 = 100, \omega_2 = -100, \omega_3 = 100, \omega_4 = -100$

$$T_B = \begin{bmatrix} \tau_\Phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} lk\ (-\omega^2{}_2 + \omega^2{}_4) \\ lk\ (-\omega^2{}_1 + \omega^2{}_3) \\ \sum_{i=1}^{4} \tau M_i \end{bmatrix}, \tag{16}$$

where l is the distance from center of the unmanned vehicle to the rotor

$$lk(-\omega^2{}_2 + \omega^2{}_4) = 2lk\omega^2 = 2 \times 0.11 \times 2.980 \times 10^{-6} \times 100^2$$

$$lk(-\omega^2{}_2 + \omega^2{}_4) = 6.556 \times 10^{-3}$$

$$T_B = \begin{bmatrix} 6.556 \times 10^{-3} \\ 6.556 \times 10^{-3} \\ 4.56 \times 10^{-3} \end{bmatrix}$$

Thrust and torque of the vehicle in body frame are found.

6.2 Modeling through Newton-Euler Equations for angular accelerations

By assuming the unmanned aerial system as a rigid body, the dynamics of the system is described by the Newton-Euler Equations. While considering the body frame, required force for the mass $m\dot{V}_B$ to accelerate are summed up with centrifugal force $\upsilon \times (m\ V_B)$ to equal gravity R^TG and total rotor thrust T_B.

$$m\dot{V}_B + \upsilon \times (m\ V_B) = R^TG + T_B. \tag{17}$$

Centrifugal force becomes zero in inertial frame, so only thrust and gravitational force are responsible for acceleration in the UAV.

$$m\ddot{\xi} = G + RT_B \tag{18}$$

$$\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} = -g\begin{bmatrix}0\\0\\1\end{bmatrix} + \frac{T}{m}\begin{bmatrix}C_\psi S_\theta C_\Phi + S_\psi S_\Phi\\ S_\psi S_\theta C_\Phi - C_\psi S_\Phi\\ C_\theta C_\Phi\end{bmatrix} \text{ Linear acceleration in inertial frame} \tag{19}$$

$$\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} = \begin{bmatrix}0\\0\\-9.81\end{bmatrix} + 0.25\begin{bmatrix}0\\0\\1\end{bmatrix}, \qquad \begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} = \begin{bmatrix}0\\0\\-9.56\end{bmatrix}$$

While considering the body frame, the sum of inertial angular acceleration $I\dot{\upsilon}$, centripetal force $\upsilon\times(I\upsilon)$ and gyroscopic forces Γ will be equal to the torque τ.

$$I\dot{\upsilon} + \upsilon \times (I\upsilon) + \Gamma = \tau \tag{20}$$

$$\dot{\upsilon} = I^{-1}\left(-\begin{bmatrix}p\\q\\r\end{bmatrix} \times \begin{bmatrix}I_{xx}p\\ I_{yy}q\\ I_{zz}r\end{bmatrix} - I_r\begin{bmatrix}p\\q\\r\end{bmatrix} \times \begin{bmatrix}0\\0\\1\end{bmatrix}\omega_\Gamma + \tau\right) \tag{21}$$

$$\begin{bmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} = \begin{bmatrix}(I_{yy}-I_{zz})qr/I_{xx}\\ (I_{zz}-I_{xx})pr/I_{yy}\\ (I_{xx}-I_{yy})pq/I_{zz}\end{bmatrix} - I_r\begin{bmatrix}q/I_{xx}\\ -p/I_{yy}\\ 0\end{bmatrix}\omega_\Gamma + \begin{bmatrix}\tau_\Phi/I_{xx}\\ \tau_\theta/I_{yy}\\ \tau_\psi/I_{zz}\end{bmatrix} \text{ Angular acceleration in body frame}$$

where $\omega_\Gamma = \omega_1 - \omega_2 + \omega_3 - \omega_4$

$$\begin{bmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} = \begin{bmatrix}(-0.01152 \times 100 \times 100)/0.01152\\ (0.01152 \times 100 \times 100)/0.01152\\ 0\end{bmatrix} - 4.4\times 10^{-16}\begin{bmatrix}8680.56\\ -8680.56\\ 0\end{bmatrix}400 + \begin{bmatrix}0.5691\\0.5691\\0.1979\end{bmatrix}$$

$$\begin{bmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} = \begin{bmatrix}-10000\\ 10000\\ 0\end{bmatrix} - \begin{bmatrix}1.53\times 10^{-3}\\ 1.53\times 10^{-3}\\ 0\end{bmatrix} + \begin{bmatrix}0.5691\\0.5691\\0.1979\end{bmatrix} = \begin{bmatrix}-0.5824\\ 1.5706\\ 0.1979\end{bmatrix}$$

The angular accelerations are given by

$$\ddot{\eta} = \frac{d}{dt}\left(W^{-1}{}_\eta \upsilon\right) = \frac{d}{dt}\left(W^{-1}{}_\eta\right)\upsilon + W^{-1}{}_\eta\dot{\upsilon} \tag{22}$$

$$\ddot{\eta} = \begin{bmatrix}0 & \dot{\Phi}C_\Phi T_\theta + \dot{\theta}S_\Phi/C^2{}_\theta & -\dot{\Phi}S_\Phi C_\theta + \dot{\theta}C_\Phi/C^2{}_\theta\\ 0 & -\dot{\Phi}S_\Phi & -\dot{\Phi}C_\Phi\\ 0 & \frac{\dot{\Phi}C_\Phi}{C_\theta} + \dot{\Phi}S_\Phi T_\theta/C_\theta & \frac{\dot{\Phi}S_\Phi}{C_\theta} + \dot{\Phi}C_\Phi T_\theta/C_\theta\end{bmatrix}\upsilon + W^{-1}{}_\eta\dot{\upsilon} \tag{23}$$

$$W^{-1}{}_\eta\dot{\upsilon} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}-0.5824\\ 1.5706\\ 0.1979\end{bmatrix} \qquad W^{-1}{}_\eta\dot{\upsilon} = \begin{bmatrix}-0.5824\\ 1.5706\\ 0.1979\end{bmatrix}.$$

$$\ddot{\eta} = \frac{d}{dt}\left(W^{-1}{}_\eta\right)\upsilon + W^{-1}{}_\eta\dot{\upsilon} \tag{24}$$

$$\ddot{\eta} = \begin{bmatrix} 0 & 0 & 1.4165 \\ 0 & 0 & 1.0986 \\ 0 & 1.0986 & 0 \end{bmatrix} \begin{bmatrix} 100 \\ 100 \\ 100 \end{bmatrix} + \begin{bmatrix} -0.5824 \\ 1.5706 \\ 0.1979 \end{bmatrix}$$

$$\ddot{\eta} = \begin{bmatrix} 141.65 \\ 109.86 \\ 109.86 \end{bmatrix} + \begin{bmatrix} -0.5824 \\ 1.5706 \\ 0.1979 \end{bmatrix} \qquad \ddot{\eta} = \begin{bmatrix} 141.06 \\ 111.43 \\ 110.05 \end{bmatrix}$$

Angular accelerations in body frame and inertial frame are found.

6.3 Modeling through Euler- Lagrange equations for Total Thrust and Torque

This equation defines that the Lagrangian is equal to the sum of the translational energy $E_{trans}$and rotational energy E_{rot}minus potential energy E_{pot}.

$$L(q, \dot{q}) = E_{trans} + E_{rot} - E_{pot}$$

$$L(q, \dot{q}) = (m/2)\, \dot{\xi}^T \dot{\xi} + (1/2)\, \upsilon^T I \upsilon - mgz \tag{25}$$

Linear forces are the total thrust of the rotors. Linear Euler-Lagrange equations are

$$F = RT_B = m\ddot{\xi} + mg \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{26}$$

$$F = 0.476 \begin{bmatrix} 0 \\ 0 \\ -9.56 \end{bmatrix} + 0.47 \times 9.81 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad F = \begin{bmatrix} 0 \\ 0 \\ -4.55 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 4.61 \end{bmatrix} \qquad F = \begin{bmatrix} 0 \\ 0 \\ 0.061 \end{bmatrix}$$

Jacobian matrix J(η) from ν to $\dot{\eta}$is

$$J(\eta) = J = W^T{}_{\eta}\, I\, W_{\eta} \tag{27}$$

$$J(\eta) = \begin{bmatrix} I_{xx} & 0 & -I_{xx}S_{\theta}) \\ 0 & I_{yy}C^2{}_{\Phi} + I_{zz}S^2{}_{\Phi} & (I_{yy} - I_{zz})C_{\Phi}S_{\Phi}C_{\theta} \\ -I_{xx}S_{\theta} & (I_{yy} - I_{zz})C_{\Phi}S_{\Phi}C_{\theta} & I_{xx}S^2{}_{\theta} + I_{yy}S^2{}_{\theta}C^2{}_{\theta} + I_{zz}C^2{}_{\Phi}C^2{}_{\theta} \end{bmatrix} \tag{28}$$

$$J(\eta) = \begin{bmatrix} 0.01152 & 0 & 0 \\ 0 & 0.01152 & 0 \\ 0 & 0 & 0.02304 \end{bmatrix}$$

In inertial frame, rotational energy E_{rot} can be written as

$$E_{rot} = (1/2)\, \upsilon^T I \upsilon = (1/2)\, \ddot{\eta}^T J \ddot{\eta} \tag{29}$$

Angular Euler-Lagrange equations are

$$\tau = \tau_B = J\ddot{\eta} + \frac{d}{dt}(J)\dot{\eta} - \frac{1}{2}\frac{\partial}{\partial \eta}(\dot{\eta}^T J \dot{\eta}) = J\ddot{\eta} + C(\eta, \dot{\eta})\, \dot{\eta} \tag{30}$$

Where matrix C (η,$\dot{\eta}$) is Coriolis term, consists of gyroscopic and centripetal force.

$$C(\eta, \dot{\eta}) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \tag{31}$$

$C_{11} = 0$

$C_{12} = (I_{yy} - I_{zz})(\dot{\theta} C_{\Phi} S_{\Phi} + \dot{\psi} S^2{}_{\Phi} C_{\theta}) + (I_{zz} - I_{yy})\dot{\psi} C^2{}_{\Phi} C_{\theta} - I_{xx}\dot{\psi} C_{\theta}$

$C_{13} = (I_{zz} - I_{yy})\dot{\psi} C_{\Phi} S_{\Phi} C^2{}_{\theta}$

$C_{21} = \left(I_{zz} - I_{yy}\right)\left(\dot{\theta} C_{\Phi} S_{\Phi} + \dot{\psi} S_{\Phi} C_{\theta}\right) + \left(I_{yy} - I_{zz}\right)\dot{\psi} C^2{}_{\Phi} C_{\theta} + I_{xx}\dot{\psi} C_{\theta}$

$C_{22} = (I_{zz} - I_{yy})\dot{\Phi} C_{\Phi} S_{\Phi}$

$C_{23} = -I_{xx}\dot{\psi} S_{\theta} C_{\theta} + I_{yy}\dot{\psi} S^2{}_{\Phi} S_{\theta} C_{\theta} + I_{zz}\dot{\psi} C^2{}_{\Phi} S_{\theta} C_{\theta}$

$C_{31} = \left(I_{yy} - I_{zz}\right)\dot{\psi} C_{\Phi} S_{\Phi} C^2{}_{\theta} - I_{xx}\dot{\theta} C_{\theta}$

$C_{32} = \left(I_{zz} - I_{yy}\right)\left(\dot{\theta} C_{\Phi} S_{\theta} S_{\Phi} + \dot{\Phi} S^2{}_{\Phi} C_{\theta}\right) + \left(I_{yy} - I_{zz}\right)\dot{\Phi} C^2{}_{\Phi} C_{\theta} + I_{xx}\dot{\psi} C_{\theta} C_{\theta} - I_{yy}\dot{\psi} S^2{}_{\Phi} S_{\theta} C_{\theta} - I_{zz}\dot{\psi} C^2{}_{\Phi} S_{\theta} C_{\theta}$

$C_{33} = \left(I_{yy} - I_{zz}\right)\dot{\Phi} C_{\Phi} S_{\Phi} C^2{}_{\theta} - I_{yy}\dot{\theta} S^2{}_{\Phi} S_{\theta} C_{\theta} + I_{xx}\dot{\theta} S_{\theta} C_{\theta} - I_{zz}\dot{\theta} C^2{}_{\Phi} S_{\theta} C_{\theta}$

$C_{11} = 0$ $C_{12} = 0$ $C_{13} = 0$ $C_{21} = 0$ $C_{22} = 0$ $C_{23} = 0$ $C_{31} = -0.0163$ $C_{32} = -0.0127$ $C_{33} = 0$

$\ddot{\eta} = J^{-1}(\tau_B - C(\eta, \dot{\eta})\dot{\eta}).$

$$C(\eta, \dot{\eta})\dot{\eta} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.0163 & -0.0127 & 0 \end{bmatrix} \begin{bmatrix} 1.0986 \\ 1.4165 \\ -1.3021 \end{bmatrix} \qquad C(\eta, \dot{\eta})\dot{\eta} = \begin{bmatrix} 1.607 \\ 1.266 \\ 2.536 \end{bmatrix}$$

From Euler-Lagrange Equations we found Total Thrust and Torque required for the Vehicle.

7. CONCLUSIONS

The mathematical modeling of unmanned aero-amphibious vehicle was framed successfully with the help of Newton-Euler equations and Euler-Lagrange equations considering different frame of references. Now these equations could be converted into transfer functions and by applying the classical control laws autopilot codes could be generated. The construction and fabrication of aero-amphibious vehicle was also successfully completed. The structural and electronic circuit designs have tackled their constraints and overcame the limitations that were set these days due the lacking of technology and unavailability of some electronic components. The trademark of this research project was the main idea implemented to overcome the challenges that existed in such vehicles. The design of the vehicle was unique and appropriate to tackle the constraints set by the nature. The unique design alone helped to frame the mathematical modeling equations irrespective of whichever medium the vehicle travels.

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