Attitude Control Synthesis for Small Satellites Using Gradient Method. Part I - Nonlinear Equations

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Abstract: *The paper presents some aspects for synthesis of small satellites attitude control. The satellite nonlinear model presented here will be with six degrees of freedom. After movement equation linearization the stability and command matrixes will be established and the controller will be obtained using gradient and gradient method. Two attitude control cases will be analysed: the reaction wheels and the micro thrusters. The results will be used in the project European Space Moon Orbit - ESMO founded by European Space Agency in which the University POLITEHNICA of Bucharest is involved.*

Key Words: *nonlinear model, small satellites, gradient methods, mathematical model*

NOMENCLATURE

ξ - Rotation angle around body X_B axis

η - Rotation angle around body Y_B axis

ζ - Rotation angle around body Z_B axis

ψ - Attitude angle around z axis

θ - Attitude angle around y axis

ϕ - Attitude angle around x axis

$\boldsymbol{\omega}_{BI}$ -Angular velocity of the body frame relative to the inertial frame expressed in body frame;

$\boldsymbol{\omega}_{RI}$ -Angular velocity of the reference frame relative to the inertial frame expressed in relative frame;

$\boldsymbol{\omega}_{RIB}$ -Angular velocity of the reference frame relative to the inertial frame expressed in body frame;

$\boldsymbol{\omega}_{BR}$ -Angular velocity of the body frame relative to the reference frame expressed in body frame;

A, B, C, E - Satellite inertia moments;

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m - Satellite Mass;
a - Major semi axis of elliptical satellite orbit;
e - Eccentricity of elliptical satellite orbit;
t - Time;
$\mathbf{r}$ - Position vector of the satellite relative to the origin of the inertial frame – centre of the Earth;
T -Orbital period;
$\mathbf{v}$ - Velocity;
V_{Ix}, V_{Iy}, V_{Iz} - Velocity components expressed in inertial frame;
X_B, Y_B, Z_B - Body frame;
X_R, Y_R, Z_R - Reference frame;
X_I, Y_I, Z_I -Inertial frame
x_I, y_I, z_I - Satellite coordinate expressed in inertial frame.

1. INTRODUCTION

It is indisputable that today, the use of satellites is the spatial programs main goal, due to their importance in terms of telecommunications, remote sensing and navigation that they provide. During a satellite mission, the quality of Attitude Control System (ACS) is a basic element for achieving a good functioning condition. Building a suitable mathematical model that allows both the synthesis of the control system and its simulation is required for the ACS completion.

Starting from this requirement the paper will introduce three novelties:

- The use of the rotation angles for describing the kinematical equations;

- The linearization of *Trigger Schmidt* element used for applying the command moment, thru the Fourier Transformation

- The use of the adjoint analysis methods for the synthesis of optimal controller based on uncoupled state.

Unlike regular paper, which covers the regular aircraft cases, where the kinematical equations use Euler angles 0, in our case, when the satellite has a complex evolution, the papers 0,0 and 0 recommends the quaternion vector or the rotation angles. Using kinematical equations written with Euler angles, in addition to benefits related to the significance of physical measurable sizes, the following drawback is involved: the use of trigonometric functions in program algorithms and singularity in connection matrix for some specific angles. Despite the complications related to solving the kinematic equations, the rotation angles can be used for the attitude control, as it will be shown bellow.

2. GENERAL MOVEMENT EQUATIONS

2.1 Used frames

First of all, we must define a frame, called reference frame (RF), in which the satellite will be stabilized with respect to three axes 0. As we can see in Figure 1 the origin of reference frame is in the mass centre of the satellite and moves with it. Axis $\mathbf{Z}_R$ is orientated towards the Earth mass centre, axis $\mathbf{X}_R$ is in the orbit plane, the normal of $\mathbf{Z}_R$ axis and orientated towards the velocity direction. Axis $\mathbf{Y}_R$ is normal to the orbit plane, and completes an orthogonal right-hand system. In the same time we define an angular velocity $\boldsymbol{\omega}_{RI}$ which means the angular velocity of the reference frame related to inertial frame. Inertial frame X_I, Y_I, Z_I (IF) has its origin in the Earth centre being use for the description of the satellite orbital movement. The body frame $\mathbf{X}_B\mathbf{Y}_B\mathbf{Z}_B$ (BF) is an orthogonal frame heaving the axis, if possible, along the principal inertial axis. The attitude of the body frame related to the reference frame is defined using attitude angles of Euler type, or quaternion vector or, as we will present later on, the rotation angles.

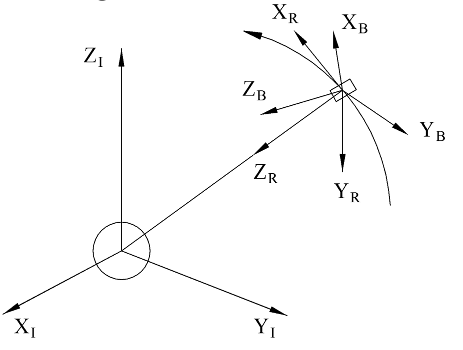

Fig. 1 definition of the used frames

2.2 The angular velocity of the body frame

The two main terms of the kinematical orientation of the satellite are:

- The angular velocity of the BF related to the RF:

$$\boldsymbol{\omega}_{BR} = p_B\mathbf{i} + q_B\mathbf{j} + r_B\mathbf{k} \tag{1}$$

- The angular velocity of the RF related to IF:

$$\boldsymbol{\omega}_{RI} = \omega_i\mathbf{i}_R + \omega_j\mathbf{j}_R + \omega_k\mathbf{k}_R \tag{2}$$

The angular velocity of the RF related to IF $\boldsymbol{\omega}_{RI}$: expressed in BF is:

$$\boldsymbol{\omega}_{RIB} = \omega_{iB}\mathbf{i} + \omega_{jB}\mathbf{j} + \omega_{kB}\mathbf{k} \tag{3}$$

The link between this two expressions being on the rotation matrix:

$$\boldsymbol{\omega}_{RIB} = \mathbf{A}_e\boldsymbol{\omega}_{RI}\,, \tag{4}$$

where, the rotation matrix $\mathbf{A}_e$ will be further defined

Finally we define the angular velocity of the BF related to the IF :

$$\boldsymbol{\omega}_{BI} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k} \tag{5}$$

Between these three sizes there is the link:

$$\boldsymbol{\omega}_{BI} = \boldsymbol{\omega}_{BR} + \boldsymbol{\omega}_{RIB} \tag{6}$$

Because all components are expressed in the body frame, the scalar link can be written immediately:

$$\begin{bmatrix}\omega_x & \omega_y & \omega_z\end{bmatrix}^T = \begin{bmatrix}p_B & q_B & r_B\end{bmatrix}^T + \mathbf{A}_e\begin{bmatrix}\omega_i & \omega_j & \omega_k\end{bmatrix}^T \tag{7}$$

Because the RF are defined by unitary vectors: $\mathbf{i}_R, \mathbf{j}_R, \mathbf{k}_R$, their definition based on the position vector $\mathbf{r}$ and velocity $\mathbf{v}$:

$$\mathbf{i}_R = \mathbf{j}_R \times \mathbf{k}_R = \frac{\mathbf{r}\times(\mathbf{v}\times\mathbf{r})}{r|\mathbf{v}\times\mathbf{r}|} = \frac{r^2\mathbf{v}-\mathbf{r}\cdot(\mathbf{v}\cdot\mathbf{r})}{r|\mathbf{v}\times\mathbf{r}|} \tag{8}$$

Deriving in reference frame we obtain successively

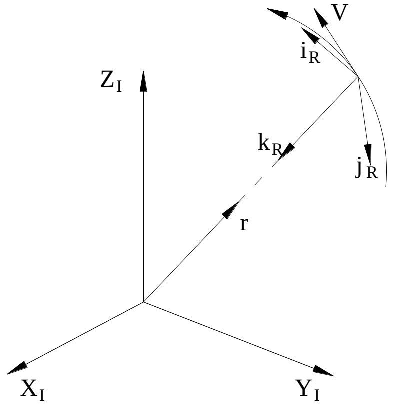

Fig. 2 definition of the unitary vectors for the reference frame

$$\begin{aligned}
\frac{d\mathbf{i}_R}{dt} &= (\omega_i\mathbf{i}_R + \omega_j\mathbf{j}_R + \omega_k\mathbf{k}_R)\times\mathbf{i}_R = \omega_k\mathbf{j}_R - \omega_j\mathbf{k}_R;\\
\frac{d\mathbf{j}_R}{dt} &= (\omega_i\mathbf{i}_R + \omega_j\mathbf{j}_R + \omega_k\mathbf{k}_R)\times j_R = \omega_i\mathbf{k}_R - \omega_k\mathbf{i}_R;\\
\frac{d\mathbf{k}_R}{dt} &= (\omega_i\mathbf{i}_R + \omega_j\mathbf{j}_R + \omega_k\mathbf{k}_R)\times\mathbf{k}_R = \omega_j\mathbf{i}_R - \omega_i\mathbf{j}_R.
\end{aligned} \tag{9}$$

If we perform the scalar product of each relation with unitary vectors $\mathbf{i}_R, \mathbf{j}_R, \mathbf{k}_R$, we find that:

$$\omega_i = \frac{d\mathbf{j}_R}{dt}\mathbf{k}_R = -\frac{d\mathbf{k}_R}{dt}\mathbf{j}_R;\ \omega_j = \frac{d\mathbf{k}_R}{dt}\mathbf{i}_R = -\frac{d\mathbf{i}_R}{dt}\mathbf{k}_R;\ \omega_k = \frac{d\mathbf{i}_R}{dt}\mathbf{j}_R = -\frac{d\mathbf{j}_R}{dt}\mathbf{i}_R \tag{10}$$

Taking into account by defining relations (8) and the features of mixed product we obtain successively:

$$\begin{aligned}
\omega_i &= -\frac{d\mathbf{k}_R}{dt}\mathbf{j}_R = \left(\frac{1}{r}\frac{d\mathbf{r}}{dt} - \frac{\dot{r}}{r^2}\mathbf{r}\right)\frac{\mathbf{v}\times\mathbf{r}}{|\mathbf{v}\times\mathbf{r}|} = \frac{1}{r|\mathbf{v}\times\mathbf{r}|}\mathbf{v}(\mathbf{v}\times\mathbf{r}) - \frac{\dot{r}}{r^2|\mathbf{v}\times\mathbf{r}|}\mathbf{r}(\mathbf{v}\times\mathbf{r})\\
&= \frac{1}{r|\mathbf{v}\times\mathbf{r}|}\mathbf{r}(\mathbf{v}\times\mathbf{v}) - \frac{\dot{r}}{r^2|\mathbf{v}\times\mathbf{r}|}\mathbf{v}(\mathbf{r}\times\mathbf{r}) = 0
\end{aligned} \tag{11}$$

Similarly, we obtain:

$$\omega_j = \frac{d\mathbf{k}_R}{dt}\mathbf{i}_R = \left(\frac{\dot{r}}{r^2}\mathbf{r} - \frac{1}{r}\mathbf{v}\right)\frac{r^2\mathbf{v} - \mathbf{r}\cdot(\mathbf{v}\cdot\mathbf{r})}{r|\mathbf{v}\times\mathbf{r}|} = \frac{1}{r^2|\mathbf{v}\times\mathbf{r}|}\left[(\mathbf{r}\cdot\mathbf{v})^2 - r^2v^2\right] \tag{12}$$

Taking into account that:

$$|\mathbf{v}\times\mathbf{r}| = rv\cos\beta\,;\mathbf{r}\cdot\mathbf{v} = rv\sin\beta\,, \tag{13}$$

finally we obtain:

$$\omega_j = -\frac{v}{r}\cos\beta = -\frac{h}{r^2} \tag{14}$$

If the orbit is circular, obviously we have:

$$\omega_j = -\frac{v}{r} \tag{15}$$

Finally to obtain ω_k, we write as follows:

$$\omega_k = -\frac{dj_R}{dt}\mathbf{i}_R = -\frac{1}{|\mathbf{v}\times\mathbf{r}|}\frac{d}{dt}(\mathbf{v}\times\mathbf{r})\mathbf{i}_R - \frac{1}{|\mathbf{v}\times\mathbf{r}|}[\ddot{\mathbf{r}}\times\mathbf{r} + \mathbf{v}\times\dot{\mathbf{r}}]\mathbf{i}_R = -\frac{1}{|\mathbf{v}\times\mathbf{r}|}(\ddot{\mathbf{r}}\times\mathbf{r})\mathbf{i}_R \tag{16}$$

But, for Keplerian case we can write:

$$\ddot{\mathbf{r}} = -\mu\frac{\mathbf{r}}{r^3} \tag{17}$$

obtaining that:

$$\omega_k = \frac{\mu}{r^3|\mathbf{v}\times\mathbf{r}|}[\mathbf{r}\times\mathbf{r}]\mathbf{i}_R = 0 \tag{18}$$

Obviously, for Keplerian movement, there is no acceleration outside from the orbit plan, so we can write:

$$\omega_{RI} = \begin{bmatrix}0 & \omega_j & 0\end{bmatrix}^T \tag{19}$$

where:

$$\omega_j = -\frac{h}{r^2} \tag{20}$$

Taking into account the relation (4) we can write finally:

$$\begin{bmatrix}p_B & q_B & r_B\end{bmatrix}^T = \begin{bmatrix}\omega_x & \omega_y & \omega_z\end{bmatrix}^T - \mathbf{A}_e\begin{bmatrix}0 & \omega_j & 0\end{bmatrix}^T \tag{21}$$

This relationship is important because by measuring or calculating the angular velocity of the BF related to IF (i.e.) $\boldsymbol{\omega}_{BI} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$ and using then (21), we can_determine the angular velocity of the BF related with RF $\boldsymbol{\omega}_{BR} = \begin{bmatrix}p_B & q_B & r_B\end{bmatrix}^T$ that can be used to control the vehicle attitude related to RF.

2.3 Kinematic equations

Unlike paper 0, which covers the regular flight vehicles, where the kinematic equations use Euler angles, in our case, when we have unusual or unpredicted attitude, the papers 0, 0 recommend quaternion or rotation angles. Using kinematic equations written with Euler angles the following drawback is involved: singularity of the connection matrix and the use of trigonometric functions in program algorithms. In paper 0 a group of three angles, called the rotation angles, were first introduced. The sizes were used to describe the aircraft movement.

The angles of rotation have the advantage that they can be measured easily on board of the space vehicle. Furthermore they retain the advantages of quaternion, removing singularity from kinematical equations written with the attitude angles (32). Also, they allow the polynomial expression of the kinematical equations, an important advantage in building high-speed algorithms and easily implemented on hardware support. The angles of rotation retain the advantage of the Euler type angles, namely that of being quantities directly measurable with a concrete physical meaning.

It is well known that a sequence of rotations of a rigid body with a fixed point can be replaced by a single rotation σ around an axis through the fixed point.

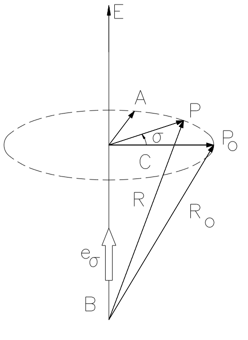

Fig. 3 Single rotation from fixed frame to mobile frame

In order to build kinematic equations we will use two frames:

$OX_0Y_0Z_0$ - The fixed frame with unitary vectors: $\mathbf{I},\mathbf{J},\mathbf{K}$;

$Oxyz$ – The mobile frame, linked by body with unitary vectors $\mathbf{i},\mathbf{j},\mathbf{k}$;

We suppose that the body has the angular velocity $\boldsymbol{\omega}_{BR}$ with the components (p_B,q_B,r_B) in mobile frame *Oxyz:*

$$\boldsymbol{\omega}_{BR}=\mathbf{i}p_B+\mathbf{j}q_B+\mathbf{k}r_B\,. \tag{22}$$

Axis *E* is the axis around which a single rotation σ is necessary to overlap the frame $OX_0Y_0Z_0$ over the frame *Oxyz* (fig. 3). The unitary vector for axis *E* is $\mathbf{e}_\sigma$:

$$\mathbf{e}_\sigma=\mathbf{I}l+\mathbf{J}m+\mathbf{K}n, \tag{23}$$

Considering the notations from figure 3, we can write:

$$\mathbf{A}=\mathbf{e}_\sigma\times\mathbf{R}_0;\quad \mathbf{B}=\mathbf{e}_\sigma\cdot(\mathbf{e}_\sigma\cdot\mathbf{R}_0);\quad \mathbf{C}=\mathbf{R}_0-\mathbf{B}\,. \tag{24}$$

In this case, the relation between the position vectors of the point *P* and the point P_0 become successively:

$$\mathbf{R} = \mathbf{B} + \mathbf{A}\sin\sigma + \mathbf{C}\cos\sigma;$$
$$\mathbf{R} = \mathbf{e}_\sigma \cdot (\mathbf{e}_\sigma \cdot \mathbf{R}_0) + (\mathbf{e}_\sigma \times \mathbf{R}_0)\sin\sigma + [\mathbf{R}_0 - \mathbf{e}_\sigma \cdot (\mathbf{e}_\sigma \cdot \mathbf{R}_0)]\cos\sigma; \tag{25}$$
$$\mathbf{R} = \mathbf{R}_0 \cos\sigma + \mathbf{e}_\sigma \cdot (\mathbf{e}_\sigma \cdot \mathbf{R}_0)(1-\cos\sigma) + (\mathbf{e}_\sigma \times \mathbf{R}_0)\sin\sigma.$$

If the point P_0 is located initially on the axis X_0, the point P will be finally on-axis x.

Because the vectors $\mathbf{R}$ and $\mathbf{R}_0$ are equal in module, we can substitute in relation (25):

$$\mathbf{R} \to \mathbf{i}; \quad \mathbf{R}_0 \to \mathbf{I}.$$

Similarly, if the point P is located on the axis y or z, we can substitute:

$$\mathbf{R} \to \mathbf{j}; \quad \mathbf{R}_0 \to \mathbf{J}; \quad \mathbf{R} \to \mathbf{k}; \quad \mathbf{R}_0 \to \mathbf{K}.$$

Finally we obtain the system:

$$\begin{aligned}
\mathbf{i} &= \mathbf{I}\cos\sigma + (\mathbf{I}l + \mathbf{J}m + \mathbf{K}n)l(1-\cos\sigma) + (\mathbf{I}l + \mathbf{J}m + \mathbf{K}n) \times \mathbf{I}\sin\sigma;\\
\mathbf{j} &= \mathbf{J}\cos\sigma + (\mathbf{I}l + \mathbf{J}m + \mathbf{K}n)m(1-\cos\sigma) + (\mathbf{I}l + \mathbf{J}m + \mathbf{K}n) \times \mathbf{J}\sin\sigma;\\
\mathbf{k} &= \mathbf{K}\cos\sigma + (\mathbf{I}l + \mathbf{J}m + \mathbf{K}n)n(1-\cos\sigma) + (\mathbf{I}l + \mathbf{J}m + \mathbf{K}n) \times \mathbf{K}\sin\sigma.
\end{aligned} \tag{26}$$

If we denote $c = \cos\sigma; \quad s = \sin\sigma$, we obtain the relation:

$$[\mathbf{i} \quad \mathbf{j} \quad \mathbf{k}]^T = \mathbf{A}_e [\mathbf{I} \quad \mathbf{J} \quad \mathbf{K}]^T,$$

where, $\mathbf{A}_e$ is the direct rotation matrix:

$$\mathbf{A}_e = \begin{bmatrix} c + l^2(1-c) & lm(1-c) + ns & ln(1-c) - ms \\ lm(1-c) - ns & c + m^2(1-c) & mn(1-c) + ls \\ ln(1-c) + ms & mn(1-c) - ls & c + n^2(1-c) \end{bmatrix} \tag{27}$$

Thus, the overall rotation angle can be expressed by the superposition of three simultaneous rotations along the mobile frame axes:

$$\xi = \sigma l; \quad \eta = \sigma m; \quad \zeta = \sigma n \tag{28}$$

The sizes are called the rotation angles:

ξ - Rotation angle around x axis;

η - Rotation angle around y axis;

ζ - Rotation angle around z axis.

The angles verify the relation:

$$\sigma^2 = \xi^2 + \eta^2 + \zeta^2 \tag{29}$$

Using rotation angles, from (27) the rotation matrix becomes:

$$\mathbf{A}_e = \begin{bmatrix} a\xi^2 + c & a\eta\xi + b\zeta & a\zeta\xi - b\eta \\ a\xi\eta - b\zeta & a\eta^2 + c & a\zeta\eta + b\xi \\ a\xi\zeta + b\eta & a\eta\zeta - b\xi & a\zeta^2 + c \end{bmatrix} \tag{30}$$

where:

$$a = \frac{1-c}{\sigma^2}; \quad b = \frac{s}{\sigma}; \quad c = \cos\sigma; \quad s = \sin\sigma \tag{31}$$

From (29) we obtain the derivatives:

$$\frac{\partial\sigma}{\partial\xi}=\frac{\xi}{\sigma};\quad \frac{\partial\sigma}{\partial\eta}=\frac{\eta}{\sigma};\quad \frac{\partial\sigma}{\partial\zeta}=\frac{\zeta}{\sigma} \tag{32}$$

with which we obtain the derivatives matrix $\mathbf{A}_e$:

$$\frac{\partial\mathbf{A}_e}{\partial\xi}=\begin{bmatrix}2a\xi & a\eta & a\zeta\\ a\eta & 0 & b\\ a\zeta & -b & 0\end{bmatrix}+\frac{\partial\mathbf{A}_e}{\partial\sigma}\frac{\xi}{\sigma};\quad \frac{\partial\mathbf{A}_e}{\partial\eta}=\begin{bmatrix}0 & a\xi & -b\\ a\xi & 2a\eta & a\zeta\\ b & a\zeta & 0\end{bmatrix}+\frac{\partial\mathbf{A}_e}{\partial\sigma}\frac{\eta}{\sigma}$$
$$\frac{\partial\mathbf{A}_e}{\partial\zeta}=\begin{bmatrix}0 & b & a\xi\\ -b & 0 & a\eta\\ a\xi & -a\eta & 2a\zeta\end{bmatrix}+\frac{\partial\mathbf{A}_e}{\partial\sigma}\frac{\zeta}{\sigma} \tag{33}$$

where:

$$\frac{\partial\mathbf{A}_e}{\partial\sigma}=\begin{bmatrix}a'\xi^2-s & a'\eta\xi+b'\zeta & a'\zeta\xi-b'\eta\\ a'\xi\eta-b'\zeta & a'\eta^2-s & a'\zeta\eta+b'\xi\\ a'\xi\zeta+b'\eta & a'\eta\zeta-b'\xi & a'\zeta^2-s\end{bmatrix} \tag{34}$$

where we denoted

$$a'=\frac{da}{d\sigma}=\frac{\sigma s+2c-2}{\sigma^3};\ b'=\frac{db}{d\sigma}=\frac{\sigma c-s}{\sigma^2}; \tag{35}$$

An interesting case is $\sigma\to 0$.
For this situation we have:

$$a'=\frac{da}{d\sigma}=\frac{\sigma s+2c-2}{\sigma^3};\ b'=\frac{db}{d\sigma}=\frac{\sigma c-s}{\sigma^2}; \tag{36}$$

and matrix derivatives are:

$$\mathbf{A}_e=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix};\ \frac{\partial\mathbf{A}_e}{\partial\sigma}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix};\ \frac{\partial\mathbf{A}_e}{\partial\xi}=\begin{bmatrix}0&0&0\\0&0&1\\0&-1&0\end{bmatrix};$$
$$\frac{\partial\mathbf{A}_e}{\partial\eta}=\begin{bmatrix}0&0&-1\\0&0&0\\1&0&0\end{bmatrix};\ \frac{\partial\mathbf{A}_e}{\partial\zeta}=\begin{bmatrix}0&1&0\\-1&0&0\\0&0&0\end{bmatrix} \tag{37}$$

Because the rotation matrix using Euler angles 0, 0, 0 or rotation angles (30) is the same regardless of the variables used, we obtain the following relationships between different variables (Euler angles, rotation angles)

The attitude angles from the rotation angles are given by:

$$\tan\phi=-\frac{a_{3,2}}{a_{2,2}}=\frac{b\xi-a\zeta\eta}{a\eta^2+c};\tan\theta=\frac{a_{1,3}}{a_{1,1}}=\frac{b\eta-a\zeta\xi}{a\xi^2+c}\ \ \sin\psi=-a_{1,2}=b\zeta+a\eta\xi; \tag{38}$$

Also, we can obtain the rotation angles from attitude angles using the relations:

$$\xi = \frac{a_{2,3} - a_{3,2}}{2b}\,;\ \eta = \frac{a_{3,1} - a_{1,3}}{2b}\quad \zeta = \frac{a_{1,2} - a_{2,1}}{2b} \tag{39}$$

where:

$$b = \frac{\sin\sigma}{\sigma}\,;\qquad \sigma = \arccos c \qquad c = \left(a_{1,1} + a_{2,2} + a_{3,3} - 1\right)/\,2\,. \tag{40}$$

Next, we will try to obtain the connection between the derivatives of rotation angles and the components of rotation velocity in the body frame.

Thus, as rotation around axis E is an equivalent transformation in terms of the two systems, it follows that the vector $\mathbf{e}_\sigma$ projections are identical:

$$\mathbf{e}_\sigma = \mathbf{I}l + \mathbf{J}m + \mathbf{K}n = \mathbf{i}l + \mathbf{j}m + \mathbf{k}n\,. \tag{41}$$

If this relationship is derived with respect to time we obtain:

$$\mathbf{I}\dot{l} + \mathbf{J}\dot{m} + \mathbf{K}\dot{n} = \mathbf{i}\dot{l} + \mathbf{j}\dot{m} + \mathbf{k}\dot{n} + \boldsymbol{\omega}_{BR} \times \mathbf{e}_\sigma\,, \tag{42}$$

where:

$$\boldsymbol{\omega}_{BR} \times \mathbf{e}_\sigma = \mathbf{i}(q_B n - r_B m) + \mathbf{j}(r_B l - p_B n) + \mathbf{k}(p_B m - q_B l), \tag{43}$$

thus:

$$\mathbf{I}\dot{l} + \mathbf{J}\dot{m} + \mathbf{K}\dot{n} = \mathbf{i}(\dot{l} + q_B n - r_B m) + \mathbf{j}(\dot{m} + r_B l - p_B n) + \mathbf{k}(\dot{n} + p_B m - q_B l). \tag{44}$$

If we multiply successively by $\mathbf{i}, \mathbf{j}, \mathbf{k}$ it results:

$$\begin{bmatrix} \mathbf{I}\cdot\mathbf{i} & \mathbf{J}\cdot\mathbf{i} & \mathbf{K}\cdot\mathbf{i} \\ \mathbf{I}\cdot\mathbf{j} & \mathbf{J}\cdot\mathbf{j} & \mathbf{K}\cdot\mathbf{j} \\ \mathbf{I}\cdot\mathbf{k} & \mathbf{J}\cdot\mathbf{k} & \mathbf{K}\cdot\mathbf{k} \end{bmatrix} \begin{bmatrix} \dot{l} \\ \dot{m} \\ \dot{n} \end{bmatrix} = \begin{bmatrix} \dot{l} \\ \dot{m} \\ \dot{n} \end{bmatrix} + \begin{bmatrix} 0 & n & -m \\ -n & 0 & l \\ m & -l & 0 \end{bmatrix} \begin{bmatrix} p_B \\ q_B \\ r_B \end{bmatrix},$$

or otherwise:

$$\left[\mathbf{I} - \mathbf{A}_e\right] \begin{bmatrix} \dot{l} \\ \dot{m} \\ \dot{n} \end{bmatrix} = \begin{bmatrix} 0 & -n & m \\ n & 0 & -l \\ -m & l & 0 \end{bmatrix} \begin{bmatrix} p_B \\ q_B \\ r_B \end{bmatrix}. \tag{45}$$

Introducing the matrix $\mathbf{A}_e$ given by (46), the left member of the relationship becomes:

$$\begin{bmatrix} 1-c & -ns & ms \\ ns & 1-c & -ls \\ -ms & ls & 1-c \end{bmatrix} \begin{bmatrix} \dot{l} \\ \dot{m} \\ \dot{n} \end{bmatrix} - (1-c) \begin{bmatrix} l^2 & lm & \mathrm{nl} \\ lm & m^2 & \mathrm{mn} \\ nl & mn & n^2 \end{bmatrix} \begin{bmatrix} \dot{l} \\ \dot{m} \\ \dot{n} \end{bmatrix} \equiv \frac{2t}{1+t^2} \begin{bmatrix} t & -n & m \\ n & t & -l \\ -m & l & t \end{bmatrix} \begin{bmatrix} \dot{l} \\ \dot{m} \\ \dot{n} \end{bmatrix} - ,$$
$$(1-c)(l\dot{l} + m\dot{m} + n\dot{n}) \begin{bmatrix} l \\ m \\ n \end{bmatrix}, \tag{46}$$

where we noted $t = \mathrm{tg}(\sigma/2)$

Since the projections of unitary vector $\mathbf{e}_\sigma$ satisfying the relationship:

$$l^2+m^2+n^2=1, \tag{47}$$

It results from differentiation:

$$l\dot{l}+m\dot{m}+n\dot{n}=0 \tag{48}$$

making the last term of the previous development to be null.
On the other hand the reverse matrix of the first term of relation (46) is

$$\begin{bmatrix} t & -n & m \\ n & t & -l \\ -m & l & t \end{bmatrix}^{-1}=\frac{1}{t(1+t^2)}\begin{bmatrix} l^2+t^2 & lm+nt & nl-mt \\ lm-nt & m^2+t^2 & mn+lt \\ nl+mt & mn-lt & n^2+t^2 \end{bmatrix}. \tag{49}$$

Multiplying by inverse matrix thus defined, relation (46) becomes:

$$2t\begin{bmatrix} \dot{l} \\ \dot{m} \\ \dot{n} \end{bmatrix}=\begin{bmatrix} 1-l^2 & -lm-nt & -nl+mt \\ -lm+nt & 1-m^2 & -mn-lt \\ -nl-mt & -mn+lt & 1-n^2 \end{bmatrix}\begin{bmatrix} p_B \\ q_B \\ r_B \end{bmatrix}, \tag{50}$$

which leads to algebraic relations:

$$\begin{aligned} 2\dot{l} &= r_B m - q_B n + (p_B - l\dot{\sigma})/t; \\ 2\dot{m} &= p_B n - r_B l + (q_B - m\dot{\sigma})/t; \\ 2\dot{n} &= q_B l - p_B m + (r_B - n\dot{\sigma})/t, \end{aligned} \tag{51}$$

where:

$$\dot{\sigma}=\mathbf{\Omega}\cdot\mathbf{e}_\sigma=p_B l+q_B m+r_B n. \tag{52}$$

By derivation of the definition relations (28) we obtain:

$$\begin{gathered} \dot{\xi}=(1-h)l\dot{\sigma}+h(p_B-ntq_B+mtr_B)\,;\,\dot{\eta}=(1-h)m\dot{\sigma}+h(ntp_B+q_B-ltr_B)\,; \\ \dot{\varsigma}=(1-h)n\dot{\sigma}+h(-mtp_B+ltq_B+r_B)\,, \end{gathered} \tag{53}$$

where derivatives of the angles of rotation can be put in the form:

$$\begin{bmatrix}\dot{\xi} & \dot{\eta} & \dot{\zeta}\end{bmatrix}^T=\mathbf{W}_R\begin{bmatrix}p_B & q_B & r_B\end{bmatrix}^T \tag{54}$$

in which, with local notations:

$$h=\frac{\sigma}{2t};\quad f=\frac{1-h}{\sigma^2}, \tag{55}$$

connection matrix $\mathbf{W}_R$ is given by:

$$\mathbf{W}_R=f\begin{bmatrix} \xi^2 & \eta\xi & \zeta\xi \\ \xi\eta & \eta^2 & \zeta\eta \\ \xi\zeta & \eta\zeta & \zeta^2 \end{bmatrix}+\frac{1}{2}\begin{bmatrix} 0 & -\zeta & \eta \\ \zeta & 0 & -\xi \\ -\eta & \xi & 0 \end{bmatrix}+h\mathbf{I}, \tag{56}$$

or, in compact form:

$$\mathbf{W}_R = \begin{bmatrix} f\xi^2 + h & f\eta\xi - \zeta/2 & f\zeta\xi + \eta/2 \\ f\xi\eta + \zeta/2 & f\eta^2 + h & f\zeta\eta - \xi/2 \\ f\xi\zeta - \eta/2 & f\eta\zeta + \xi/2 & f\zeta^2 + h \end{bmatrix} \tag{57}$$

If we denote:

$$\mathbf{a}_R = [\xi \quad \eta \quad \zeta]^T \tag{58}$$

we can write relation (54) in the following form:

$$\dot{\mathbf{a}}_R = \mathbf{W}_R \boldsymbol{\omega}_{BR} \tag{59}$$

Relation (59) represents kinematic equations written using the rotation angles, being equivalent with regular relation indicated in papers 0,0,0 which is written using the attitude angles.

Using the following notations:

$$h' = \frac{dh}{d\sigma} = \frac{1}{2}\frac{s-\sigma}{1-c}; \quad f' = \frac{df}{d\sigma} = \frac{\sigma^2 + \sigma s - 4(1-c)}{2\sigma^3(1-c)}, \tag{60}$$

we can determinate the derivatives of the matrix $\mathbf{W}_R$:

$$\frac{\partial \mathbf{W}_R}{\partial \xi} = \begin{bmatrix} 2f\xi & f\eta & f\zeta \\ f\eta & 0 & -1/2 \\ f\zeta & 1/2 & 0 \end{bmatrix} + \frac{\partial \mathbf{W}_R}{\partial \sigma}\frac{\xi}{\sigma}; \frac{\partial \mathbf{W}_R}{\partial \eta} = \begin{bmatrix} 0 & f\xi & 1/2 \\ f\xi & 2f\eta & f\zeta \\ -1/2 & f\zeta & 0 \end{bmatrix} + \frac{\partial \mathbf{W}_R}{\partial \sigma}\frac{\eta}{\sigma};$$

$$\frac{\partial \mathbf{W}_R}{\partial \zeta} = \begin{bmatrix} 0 & -1/2 & f\zeta \\ 1/2 & 0 & f\eta \\ f\xi & f\eta & 2f\zeta \end{bmatrix} + \frac{\partial \mathbf{W}_R}{\partial \sigma}\frac{\zeta}{\sigma}, \tag{61}$$

where:

$$\frac{\partial \mathbf{W}_R}{\partial \sigma} = \begin{bmatrix} f'\xi^2 + h' & f'\eta\xi & f'\zeta\xi \\ f'\xi\eta & f'\eta^2 + h' & f'\zeta\eta \\ f'\xi\zeta & f'\eta\zeta & f'\zeta^2 + h' \end{bmatrix}. \tag{62}$$

An interesting case is when $\sigma \to 0$.

For this situation we have:

$$\lim_{\sigma\to 0} h = 1 \ \lim_{\sigma\to 0} f = \frac{1}{12}; \ \lim_{\sigma\to 0} h' = 0 \ \lim_{\sigma\to 0} f' = 0 \tag{63}$$

and the derivatives are:

$$\mathbf{W}_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \ \frac{\partial \mathbf{W}_R}{\partial \sigma} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \ \frac{\partial \mathbf{W}_R}{\partial \xi} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1/2 \\ 0 & 1/2 & 0 \end{bmatrix};$$

$$\frac{\partial \mathbf{W}_R}{\partial \eta} = \begin{bmatrix} 0 & 0 & 1/2 \\ 0 & 0 & 0 \\ -1/2 & 0 & 0 \end{bmatrix}; \ \frac{\partial \mathbf{W}_R}{\partial \zeta} = \begin{bmatrix} 0 & -1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{64}$$

Beside equations (60) which describe the vehicle orientation there still are three equations which start from force equations in the Earth frame (66) and that describe the linear coordinates of the vehicle:

$$\dot{x}_I = V_{Ix}\,;\ \dot{y}_I = V_{Iy}\ \ \dot{z}_I = V_{Iz} \tag{65}$$

2.4 Dynamical equations

Developing Newton's ***inverse square law of force*** and moment equations presented in paper 0, and considering that the satellite has no moving parts and its weight is constant, we can write two groups of equations

- Force equations in the Earth frame from Newton's law:

$$\dot{V}_{Ix} = -\frac{\mu}{r^3}x_I\,;\ \dot{V}_{Iy} = -\frac{\mu}{r^3}y_I\,;\ \dot{V}_{Iz} = -\frac{\mu}{r^3}z_I \tag{66}$$

where:

$$r = \sqrt{x_I^2 + y_I^2 + z_I^2} \tag{67}$$

- Moment equations in the body frame, relations known as Euler dynamic equations:

$$\begin{aligned}\begin{bmatrix}\dot{\omega}_x\\ \dot{\omega}_y\\ \dot{\omega}_z\end{bmatrix} &= \mathbf{J}^{-1}\begin{bmatrix}L_G\\ M_G\\ N_G\end{bmatrix} + \mathbf{J}^{-1}\begin{bmatrix}L_C\\ M_C\\ N_C\end{bmatrix} - \mathbf{J}^{-1}\begin{bmatrix}\omega_y h_{wz} - \omega_z h_{wy}\\ \omega_z h_{wx} - \omega_x h_{wz}\\ \omega_x h_{wy} - \omega_y h_{wx}\end{bmatrix} - \mathbf{J}^{-1}\begin{bmatrix}\dot{h}_{wx}\\ \dot{h}_{wy}\\ \dot{h}_{wz}\end{bmatrix} +\\ &+ \mathbf{J}^{-1}\begin{bmatrix}(B-C)\omega_y\omega_z + E\omega_x\omega_y - F\omega_z\omega_x + D(\omega_y^2 - \omega_z^2)\\ (C-A)\omega_z\omega_x + F\omega_y\omega_z - D\omega_x\omega_y + E(\omega_z^2 - \omega_x^2)\\ (A-B)\omega_x\omega_y + D\omega_z\omega_x - E\omega_y\omega_z + F(\omega_x^2 - \omega_y^2)\end{bmatrix}\end{aligned} \tag{68}$$

where

$$\mathbf{h}_w = \begin{bmatrix}h_{wx} & h_{wy} & h_{wz}\end{bmatrix}^T, \tag{69}$$

is the momentum exchange device (reaction wheel).

The matrix for the inertia moment is given by:

$$\mathbf{J} = \begin{bmatrix}A & -F & -E\\ -F & B & -D\\ -E & -D & C\end{bmatrix}, \tag{70}$$

and the inertial moments are given by:

$$\begin{gathered}A = \int (y^2 + z^2)\,\mathrm{d}m\ ; B = \int (z^2 + x^2)\,\mathrm{d}m\ ; C = \int (x^2 + y^2)\,\mathrm{d}m\,;\\ E = \int zx\,\mathrm{d}m\ ; F = \int xy\,\mathrm{d}m\ ; D = \int yz\,\mathrm{d}m\,.\end{gathered} \tag{71}$$

The moment applied to vehicle has two terms:

- Gravitationally moment term:

$$\mathbf{M}_G = \begin{bmatrix}L_G & M_G & N_G\end{bmatrix}^T \tag{72}$$

- Command term which is performed using micro-jet:

$$\mathbf{M}_C = \begin{bmatrix}L_C & M_C & N_C\end{bmatrix}^T \tag{73}$$

State vector for these equations is:

$$\boldsymbol{\omega}_{BI} = \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}^T, \tag{74}$$

and means the rotation speed of the body frame related to the inertial frame, heaving components along the body frame. These nonlinear differential equations have no closed analytical solution.

2.5. Gravitational Moment

The space vehicle has an asymmetric body, situation where there is a tendency to align its principal axes of inertia according to the direction of the gravitational field.

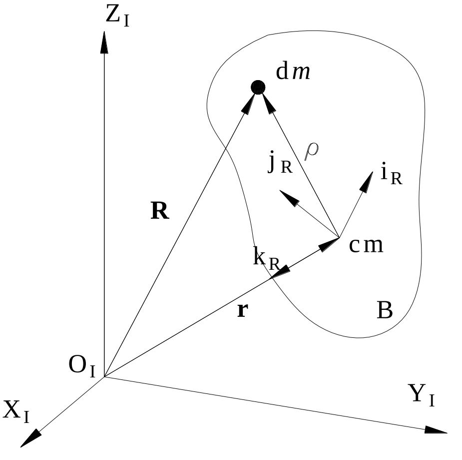

Fig. 4 gravitational moment of mass element

If we assume that we have a vehicle, whose centre of mass (cm) is positioned at a distance $\mathbf{r}$ from the Earth's centre, and a mass element $\mathrm{d}m$ belonging to the vehicle positioned at a distance $\boldsymbol{\rho}$ from the vehicle centre of mass and a distance $\mathbf{R}$ from the centre of the Earth we can write the link between them:

$$\mathbf{R} = \mathbf{r} + \boldsymbol{\rho} \tag{75}$$

In the reference frame, the position vector for the centre of mass has the form:

$$\mathbf{r} = -\mathbf{k}_R r \tag{76}$$

If we wish to express this vector in the body frame we have:

$$\begin{bmatrix} r_x & r_y & r_z \end{bmatrix}^T = \mathbf{A}_A \begin{bmatrix} 0 & 0 & -r \end{bmatrix}^T, \tag{77}$$

from which:

$$r_x = -ra_{1,3};\ r_y = -ra_{2,3};\ r_z = -ra_{3,3} \tag{78}$$

The position vector for the mass element is given by:

$$\boldsymbol{\rho} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \tag{79}$$

The mass element involves the following elementary gravitational moment:

$$\mathrm{d}\mathbf{M}_G = \boldsymbol{\rho} \times \mathrm{d}\mathbf{G} = -\frac{\mu \mathrm{d}m}{R^3}\boldsymbol{\rho} \times \mathbf{R} = -\frac{\mu \mathrm{d}m}{R^3}\boldsymbol{\rho} \times \mathbf{r} \tag{80}$$

For R^{-3}, taking into account that: $\rho < R$ and $\rho < r$, we can successively write:

$$R^2 = r^2 + \rho^2 + 2\boldsymbol{\rho}\mathbf{r} \cong r^2\left(1 + 2\frac{\boldsymbol{\rho}\mathbf{r}}{r^2}\right) \tag{81}$$

from where:

$$\frac{1}{R^3} \cong \frac{1}{r^3}\left(1 - 3\frac{\boldsymbol{\rho}\mathbf{r}}{r^2}\right) \tag{82}$$

In this case, the gravitational moment becomes:

$$\mathbf{M}_G = -\mu\int_{\mathbf{m}}\frac{\mathrm{d}m}{R^3}\boldsymbol{\rho}\times\mathbf{r} \cong -\frac{\mu}{r^3}\int_{\mathbf{m}}\left(1 - 3\frac{\boldsymbol{\rho}\mathbf{r}}{r^2}\right)\boldsymbol{\rho}\times\mathbf{r}dm \tag{83}$$

$$\mathbf{M}_G \cong \frac{3\mu}{r^5}\int_{\mathbf{m}}(\boldsymbol{\rho}\mathbf{r})(\boldsymbol{\rho}\times\mathbf{r})dm \tag{84}$$

Scalar product becomes:

$$\boldsymbol{\rho}\mathbf{r} = -r(a_{13}x + a_{23}y + a_{33}z) \tag{85}$$

and the vector product is:

$$\boldsymbol{\rho}\times\mathbf{r} = -r(a_{3,3}y - a_{2,3}z)\mathbf{i} - r(a_{1,3}z - a_{3,3}x)\mathbf{j} - r(a_{2,3}x - a_{1,3}y)\mathbf{k} \tag{86}$$

And the integral expression, neglecting the inertial products becomes:

$$\mathbf{M}_G \cong \frac{3\mu}{r^3}[(B-C)a_{2,3}a_{3,3}\mathbf{i} + (C-A)a_{1,3}a_{3,3}\mathbf{j} + (A-B)a_{1,3}a_{2,3}\mathbf{k}] \tag{87}$$

In order to evaluate the coefficient size we can express the coefficient $3\mu/r^3$ using the angular velocity ω_j:

$$3\frac{\mu}{r^3} = 3\frac{h^2 r}{(1-e^2)ar^4} = 3\frac{\omega_j^2 r}{(1-e^2)a} \tag{88}$$

In this case, the gravity gradient moment components become:

$$L_G = 3\frac{\omega_j^2 r}{(1-e^2)a}(B-C)a_{2,3}a_{3,3};\ M_G = 3\frac{\omega_j^2 r}{(1-e^2)a}(C-A)a_{1,3}a_{3,3}$$
$$N_G = 3\frac{\omega_j^2 r}{(1-e^2)a}(A-B)a_{1,3}a_{2,3} \tag{89}$$

If we consider rotation angles we will obtain:

$$L_G = 3\frac{\omega_j^2 r}{(1-e^2)a}(B-C)(a\zeta\eta + b\xi)(a\zeta^2 + c)$$
$$M_G = 3\frac{\omega_j^2 r}{(1-e^2)a}(C-A)(a\zeta\xi - b\eta)(a\zeta^2 + c) \tag{90}$$
$$N_G = 3\frac{\omega_j^2 r}{(1-e^2)a}(A-B)(a\zeta\xi - b\eta)(a\zeta\eta + b\xi)$$

3. AUXILIARY EQUATIONS

For guidance command we need the integrals term defined hereby, if we use rotation angles:

$$\begin{bmatrix}\dot{I}_\xi & \dot{I}_\eta & \dot{I}_\zeta\end{bmatrix}^T = \begin{bmatrix}\xi & \eta & \zeta\end{bmatrix}^T \tag{91}$$

In case of the satellite's orientation control by means of thrusters the guidance is done by their symmetrical arrangement about the rotation axis. Torque application in either direction can be done only by switching between two motors. For that it is necessary that the chain of command to contain a switching element to achieve a discrete output, constant amplitude, modulated in duration. As shown in 0, the control system can be described by a Schmidt trigger type element, whose functional diagram is given in Figure 5. It is noted that this element is composed of a nonlinear block, a relay with hysteresis and insensitivity zone and a linear integrator block that allows additional tuning of the system. To control the output, this is turned to the entry, forming a feedback loop.

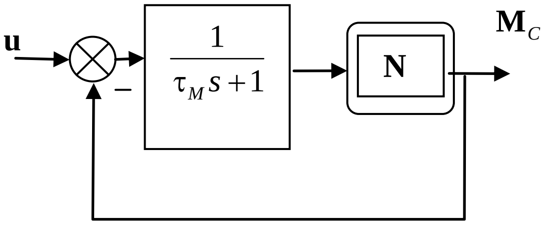

Fig. 5 command type Trigger Schmidt with nonlinear element

The nonlinear element is of relay type, with insensitivity zone and hysteresis, as we can see in figure 6

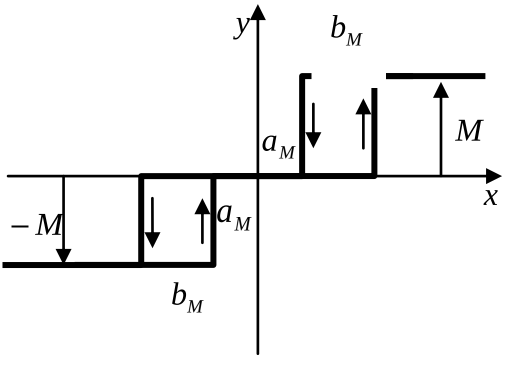

Fig. 6 nonlinear element operating schedule (N)

As we can see in figure 6 the size of insensitivity zone is $2a_M$, the size of hysteresis zone is b_M, saturation command is $\pm M$ where τ_M are time constants and k_M^u gain constants.

In case we use reaction wheels the operation of the electric motor shall be considerated which can be described as in work 0 like a linear differential equation of order 2.

$$\ddot{\mathbf{h}}_w + a_w\dot{\mathbf{h}}_w + b_w\mathbf{h}_w = -k_w^u\mathbf{u}_w \tag{92}$$

If we denote:

$$\mathbf{y}_w = -\dot{\mathbf{h}}_w \tag{93}$$

we can put the system in regular form:

$$\dot{\mathbf{y}}_w = -a_w \mathbf{y}_w + b_w \mathbf{h}_w + k_w^u \mathbf{u}_w \quad \dot{\mathbf{h}}_w = -\mathbf{y}_w, \tag{94}$$

4. CONCLUSIONS

The paper presents some synthesis aspects of the simulation model, developed for the calculation of the Attitude Control System- ACS of the small satellite which uses as command a micro jet engine or reaction wheels. As a conclusion we must underline the novelty introduced by the paper, namely the description of the model by using the rotation angles. This leads to polynomial forms for the rotation and connection matrix which eliminate the singularities of the connection matrix in case of Euler's angles. **O**n the other hand, these 3 values are independent and on the same time they have an angular dimension, and so they are measurable. This creates a great advantage on opposition to the usage of the Hamilton's quaternion.

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