Comparative study between random vibration and linear static analysis using Miles method for thruster brackets in space structures

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Abstract: *Random vibrations occur during the launch at the fastening interface between thruster brackets and basic support of satellite. These are generated in the launcher by the motion of some mechanical parts, combustion phenomena or structural elements excited by the acoustic environment. The goal of this comparative study is to find a simplified and efficient validation method using FEM PATRAN-NASTRAN software for thruster brackets in the random vibrations environment for space applications. The random vibration analysis requests complex pre/ post processing efforts and large hardware resources for various geometrical shapes. The PATRAN-NASTRAN random vibration analysis consists of frequency response analysis (111 solver) and Acceleration Spectral Density (ASD) diagram, taking into account the natural frequencies of the bracket. The Miles method computes the root mean square acceleration (a_{RMS}) using the natural frequencies and the ASD diagram as input. As a conservative hypothesis in the random analysis, the three sigma standard deviation criteria in normal Gaussian distribution is applied at these RMS acceleration values, which means to multiply the a_{RMS} by a load factor of three. Simplified method consists of using linear static PATRAN-NASTRAN analysis (101 solver) where the a_{RMS} are introduced as loads. For validation of the simplified method, a comparative study was made between the random vibration and the linear static analysis. The final results are presented in detail in this article.*

Key Works*: Random vibrations, linear static analysis, 3 sigma standard deviation, Acceleration Spectral Density (ASD), natural frequencies, von Mises stress.*

1. INTRODUCTION

The various mechanical loads are not all equally important and depend on the type of the mechanical structure: i.e. does it concern a primary structure, the spacecraft structure or other secondary structures (such as solar panels, antennas, instruments and electronic boxes). Requirements are specified to cover loads encountered by handling, testing, during the launch phase and operations in transfer and final orbit, such as:

- natural frequencies
- steady-state (quasi-static) acceleration
- sine excitation
- random excitation
- acoustic noise
- transient loads
- shock loads
- temperatures

Launch of a spacecraft consists in a series of events, each of which has several independent sources of load for the launch vehicle and payload. One of them is the high-frequency random vibration environment, which typically has significant energy in the frequency range from 20 Hz to 2000 Hz, transmitted from the launch vehicle to the payload at the launch vehicle/payload interfaces 0.

Some load environments are treated as random phenomena, when the forces involved are controlled by non-deterministic parameters. Examples include high frequency engine thrust oscillation, aerodynamic buffeting of fairing, and sound pressure on the surfaces of the payload. Turbulent boundary layers will introduce also random loads.

1.1 Random Vibration Theory Overview

Random vibration analysis describes the forcing functions and the corresponding structural response statistically 0, 0. It is generally assumed that the phasing of vibration at different frequencies is statistically uncorrelated.

The amplitude of motion at each frequency is described by an Acceleration Spectral Density (ASD) function. In contrast to transient analysis which predicts time histories of response quantities, random vibration analysis generates the Acceleration Spectral Density of these response quantities.

From the Acceleration Spectral Density, the Root Mean Square (RMS) amplitude of the response quantity is calculated. The root-mean-square acceleration is the square root of the integral of the acceleration ASD over frequency. Random vibration limit loads are typically taken as the “3-sigma load”, obtained by multiplying the RMS load by load factor, j=3, based on the probability presented in the table below, for the normal Gaussian distribution criteria, in the conservative hypothesis.

Table 1. Probability for a random signal with normal distribution, 0

Value taken	Percent probability
$\mu - \sigma < x < \mu + \sigma$	68.27%
$\mu - 2\sigma < x < \mu + 2\sigma$	95.45%
$\mu - 3\sigma < x < \mu + 3\sigma$	99.73%
$\mu - 4\sigma < x < \mu + 4\sigma$	99.994%
$\mu - 5\sigma < x < \mu + 5\sigma$	99.99994%

In NASTRAN 0, 0, the calculation of frequency response and its use in random response analysis are performed by separate modules. The equations of motion are assumed to be linear and the statistical properties of the random excitation are assumed to be stationary with respect to time.

It is calculated the Acceleration Spectral Density of the response, $S_j(\omega)$ which is the direct Fourier transform of auto-correlation or cross-correlation function $R_j(\tau)$ - see formulas (1) and (2).

$$R_j(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T u_j(t)\, u_j(t-\tau)dt \tag{1}$$

$$S_j(\omega) = \lim_{T\to\infty} \frac{2}{T}\left|\int_0^T e^{-i\omega t} u_j(t)dt\right|^2 \tag{2}$$

Starting from the formula:

$$U_j(\omega) = H_{ja}(\omega)\cdot Q_a(\omega) \tag{3}$$

Where this formula represents the structure response, $U_j(\omega)$ to an excitation $Q_a(\omega)$, knowing the answer function $H_{ja}(\omega)$. In the formula (3), the spectral components are written, applying Fourier transform, i.e. $U_j(\omega)$ is Fourier transform applied to $u_j(t)$ function, $H_{ja}(\omega)$ is frequency response function and it is Fourier transform applied to $h_{ja}(t)$ and $Q_a(\omega)$ - excitation source, which may be a point force, is Fourier transform applied to $q_a(t)$. From Parseval theorem the signal energy is defined as [7]:

$$E = \int_{-\infty}^{+\infty} |x(t)|^2\, dt = \int_{-\infty}^{+\infty} |X(\omega)|^2\, d\omega \tag{4}$$

where $X(\omega)$ is the direct Fourier transform of the signal $x(t)$ and $|X(\omega)|^2$ represents the energy distribution of the signal as a function of frequency called energy spectral density.

Taking into account the Parseval theorem and using the notations presented below for squaring left-right terms, from relation (3), $U_j(\omega)$ and $Q_a(\omega)$:

$$S_j(\omega) = |U_j(\omega)|^2 \tag{5}$$

$$S_a(\omega) = |Q_q(\omega)|^2 \tag{6}$$

It is obtained relation (7):

$$S_j(\omega) = |H_{ja}(\omega)|^2 * S_a(\omega) \tag{7}$$

Knowing $S_j(\omega)$, can be calculated the RMS values of the response $\bar{u}_j$:

$$\bar{u}_j = [\frac{1}{4\pi}\sum_{i+1}^{N-1}[S_j(\omega_{i+1}) + S_j(\omega_i)](\omega_{i+1} - \omega_i)]^{1/2} \tag{8}$$

Figure 1 presents a simplified Flow Diagram for Random Analysis Module. The inputs to the module are the frequency responses, of quantities to loading conditions {Pa} at frequencies and the auto and cross-spectral densities of the loading conditions S_a and S_{ab}.

The response quantities $S_j(\omega)$ may be displacements, velocities, accelerations, internal forces, or stresses. The Acceleration Spectral Densities of the response quantities are calculated by different procedures depending on whether the loading conditions are correlated or uncorrelated.

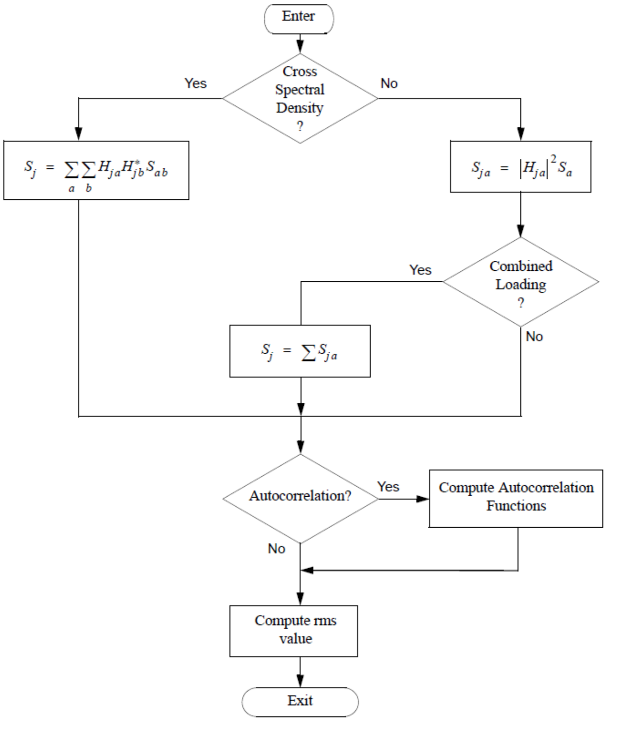

Figure 1. Flow Diagram for Random Analysis Module, 0

1.2 Miles' Method Overview

In 1954 John W. Miles 0, 0 developed the a_{RMS} (Root Mean Square acceleration) equation during the research for fatigue failure of aircraft structural components caused by the jet engine vibration and gust loading. He simplified his work modeling only one degree of freedom system. Despite that he was analyzing the stress of a component, the method can be used for another quantities as displacement, force or acceleration.

$$a_{RMS} = \sqrt{\frac{\pi}{2} \cdot f_n \cdot Q \cdot \left[ASD_n \right]} \tag{9}$$

where:

$$Q = \frac{1}{2\delta} \text{ – transmissibility at } f_n \text{ (}\delta\text{ is the critical damping ratio);} \tag{10}$$

$$ASD_n = ASD_1 \left(\frac{f_n}{f_1} \right)^{0.3322 \cdot s} \text{ - input Acceleration Spectral Density at } f_n, \left[\frac{g^2}{Hz} \right]; \tag{11}$$

f_n – natural frequency;

f_1 – lowest frequency given for the portion of the ASD diagram being interpolated;

s – slope of the ASD function, $[\text{dB/octave}]$;

Miles' equation is developed using a single degree of freedom (SDOF) system consisting of a mass, spring and damper, which is forced by the random vibration function F in the y direction like in the Figure 2.

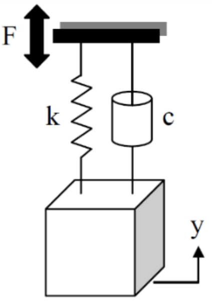

Figure 2. SDOF system, 0

An advantage of the Miles' equation is that during the design of a structure, if the modal analysis has been made to evaluate the resonant frequency of the structure, the Miles' equation can be used to evaluate the loads due to random vibrations.

Another advantage is that testing a multiple degree of freedom structure, the accelerations due to random vibration at resonant frequency can be evaluated accurately using Miles' equation, indicating the proportion of the overall RMS acceleration is occurring at a resonant peak from the entire frequency spectrum.

Despite the advantages of the Miles' equation, there are some disadvantages and one of them is that the accelerations cannot be evaluated during random vibration testing using Miles' equation. It means that a structure designed to 'three sigma' equivalent load will not fail under random vibration test, for the loads less than 3σ level.

Another disadvantage is that Miles' equation is developed on the response of a SDOF system for a flat random input ASD diagram. If the ASD input diagram has a different shape, results of the Miles' equation are not quite accurate and will not predict the rigid body response below the resonant frequency. This study claim to verify the results from a random vibrations problem on a thruster bracket structure in a space application, presented in figures below, using Miles's method and linear static analysis.

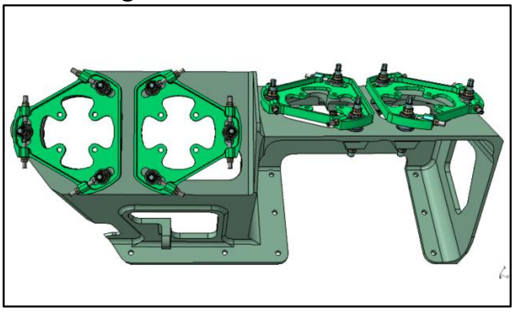

Figure 3. Spacecraft thruster bracket type 1

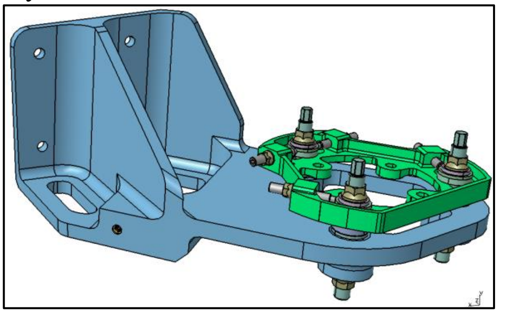

Figure 4. Spacecraft thruster bracket type 2

2. RANDOM VIBRATION ANALISYS

In this chapter the random vibration analysis is presented using the commercial software PATRAN-NASTRAN 0, 0. The NASTRAN random analysis uses the ".xdb" file from Nastran frequency response analysis (SOL 111). The analysis was made with Patran option: Tools/Random Analysis/Freq. Response/Enforced Motion. A frequency response analysis was made on X, Y and Z directions for each structure. The frequency range of ASD diagram is between 20 to 2000 Hz and the modal damping coefficient was supposed 3% from 0.1 Hz to 2000 Hz 0, see figure below.

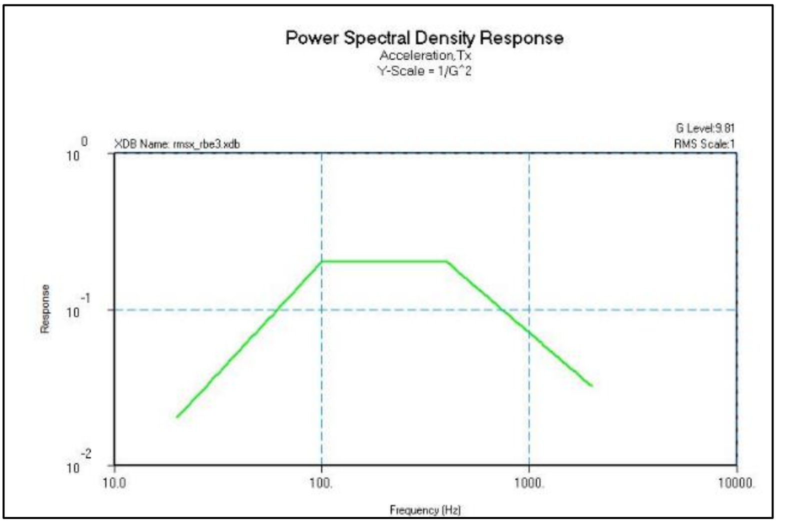

Figure 5. ASD diagram

2.1 FEM Description

For the idealization of the proposed structures, the basic element types are CQUAD4 and CTRIA3 shell elements for parts and CONM2 elements for thrusters. The position of the center of gravity and the mass inertial moments were constraint the same as in the real structure. The parts were joined together with RBE3 elements. The model units are: meters [m], kilograms [kg], seconds [s], Pascal [Pa] and Hertz [Hz].

For the attachment of the part with the satellite structure, the model was simply supported at the bottom edges with SPC (Single Point Constraints) as seen in the figure below.

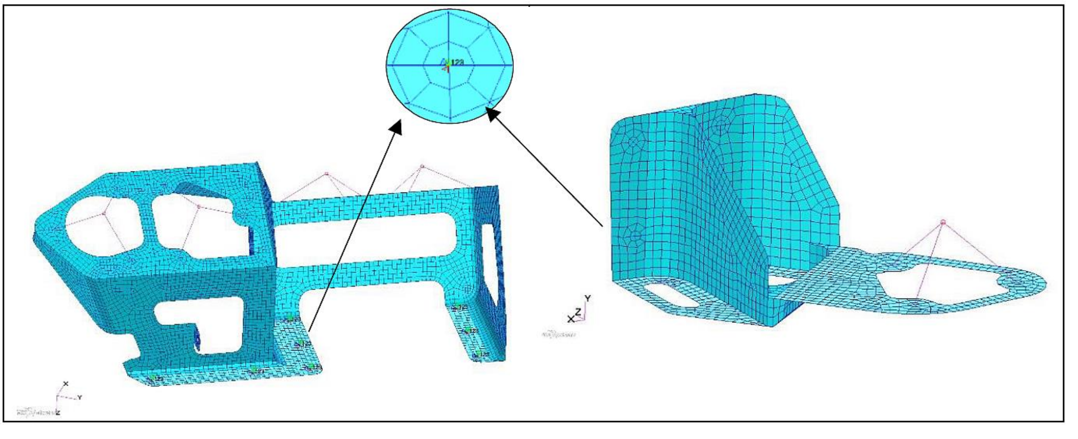

Figure 6. FEM and Boundary Conditions of the structures

2.2 RMS Von Mises Stress

For the representation of the results, the following PATRAN Plot Options were selected:
Domain: None
Method: Derive/Average
Extrapolation: Centroid
The maximum stress for each RMS analysis with the input specified is shown below.

Table 2. von Mises stress summary

Structure	Analysis direction	RMS max $\sigma_{\text{von Mises}}$ [MPa]	Location
Bracket 1	X	40.72	see Figure 7
	Y	59.05	see Figure 8
	Z	33.03	see Figure 9
Bracket 2	X	25.37	see Figure 10
	Y	29.81	see Figure 11
	Z	18.11	see Figure 12

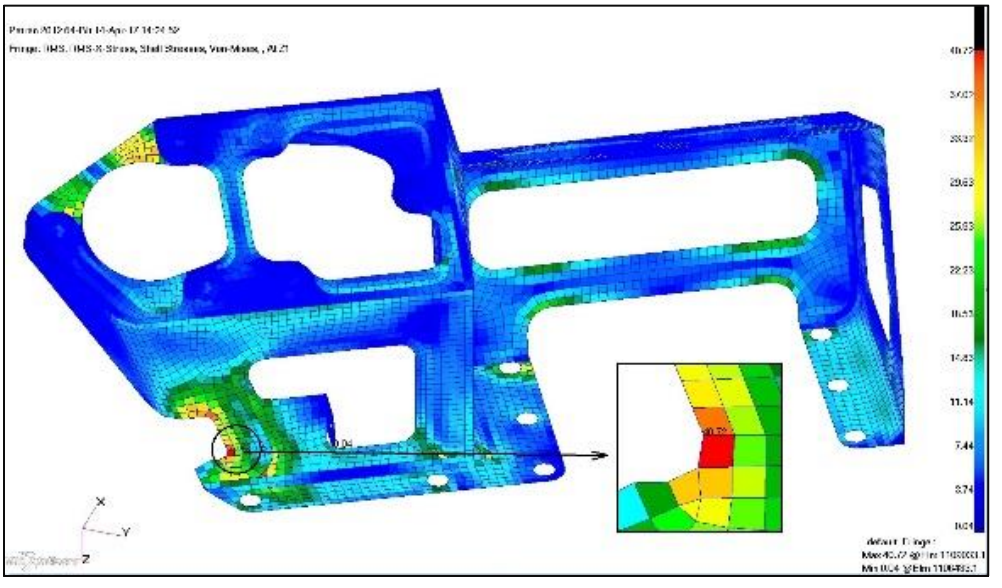

Figure 7. RMS von Mises stress in X direction for Bracket 1

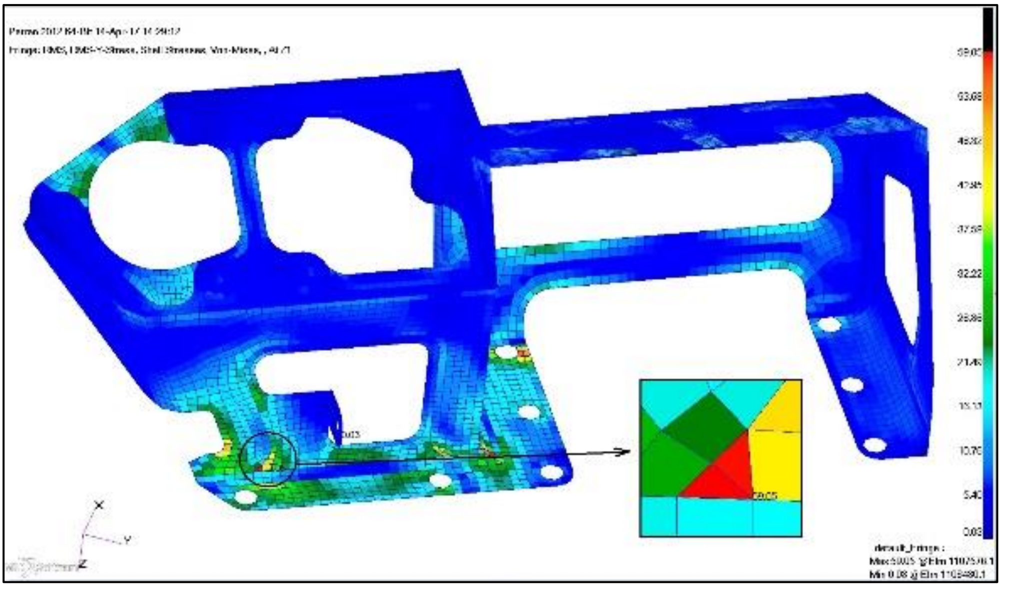

Figure 8. RMS von Mises stress in Y direction for Bracket 1

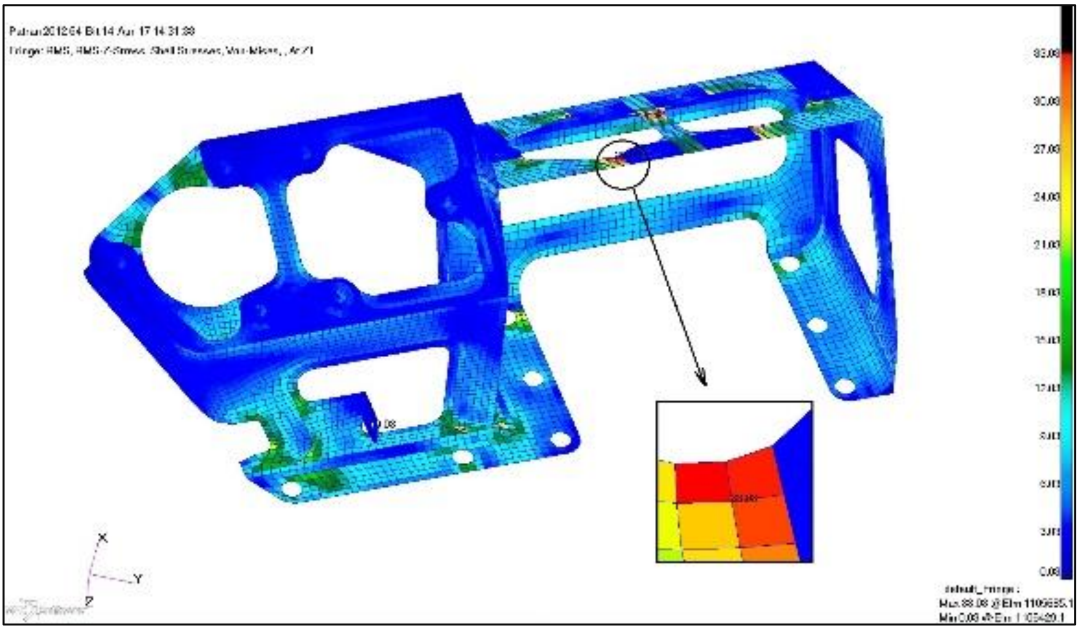

Figure 9. RMS von Mises stress in Z direction for Bracket 1

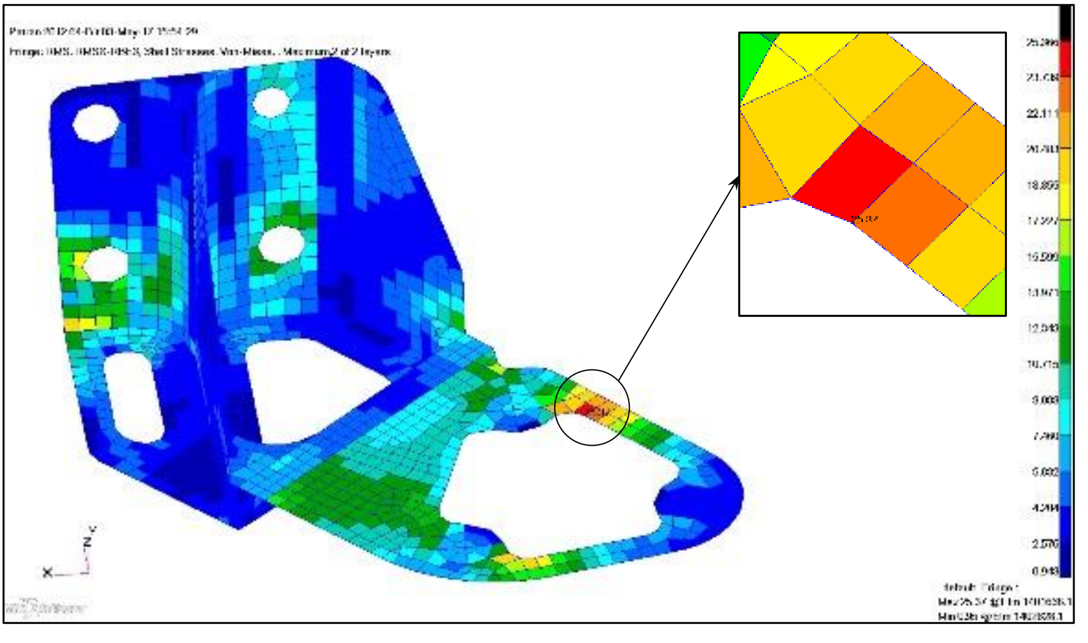

Figure 10. RMS von Mises stress in X direction for Bracket 2

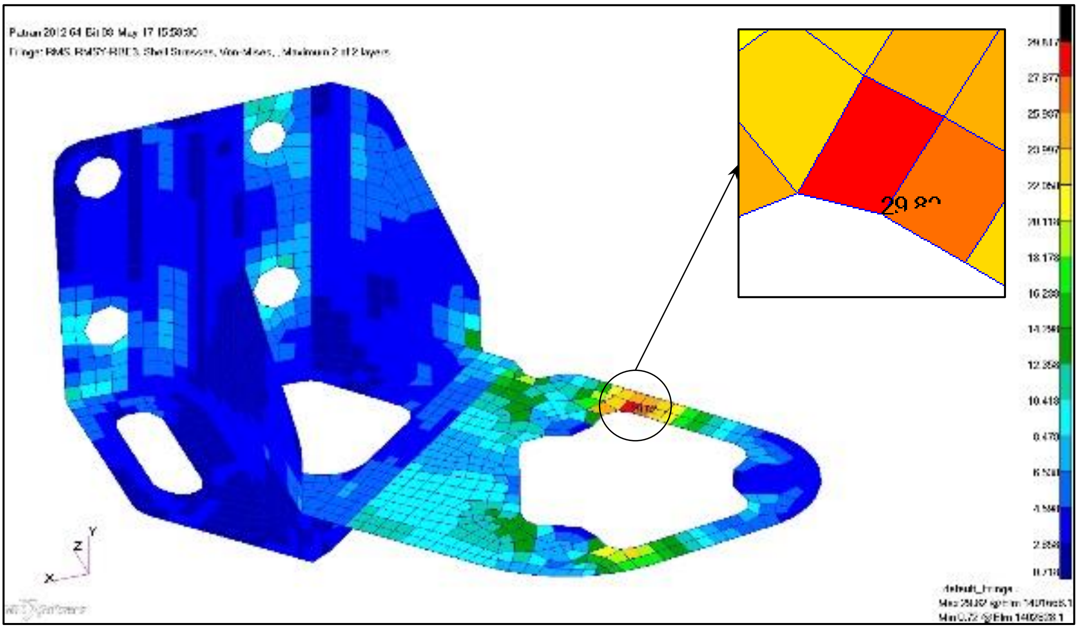

Figure 11. RMS von Mises stress in Y direction for Bracket 2

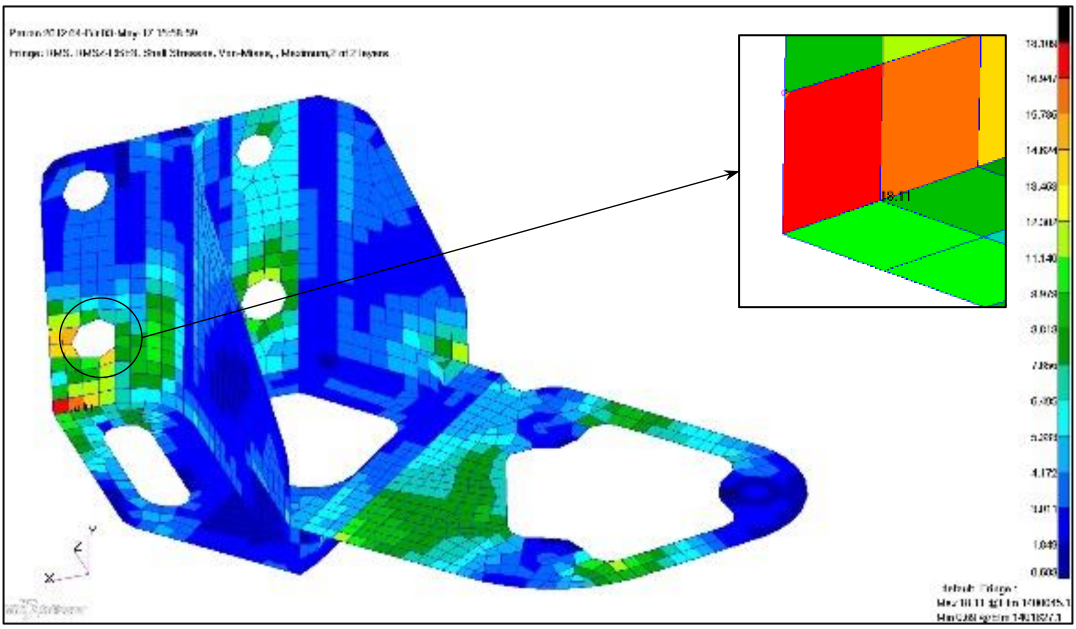

Figure 12. RMS von Mises stress in Z direction for Bracket 2

3. MILES' METHOD RESULTS

In this chapter the von Mises stress was evaluated using the Miles' relation for the proposed structures. As input, ASD diagram was used to excite only the first mode of the first natural frequency f_n of the structures.

3.1 Modal Analysis

Miles’ method assumes single-DOF behavior of a structure. An additional constraint on the application of Miles’ relation to elastic structures is that the shape of the single excited mode must approximate the profile of the structure under a static acceleration, in units of gravitational acceleration [g].

Considering this the first mode of the studied structures has to assume the approximate shape of the same structures under the applied acceleration.

The “equivalent static acceleration” approximation of the RMS acceleration response was computed for each structure using formula (9) and the a_{RMS} results are presented in table 3.

For a giving value of the natural frequency f_n the ASD level was determined directly from the ASD diagram and verified using the formula (11).

Table 3. Input for static analysis with g-load factor

Structure	Natural frequency f_n	ASD Level	a_{RMS}
	[Hz]	[g^2/Hz]	[g]
Bracket 1	84.942	0.159	18.803
Bracket 2	115.89	0.2	24.633

For each structure, the applied acceleration was imposed on X, Y and Z direction.

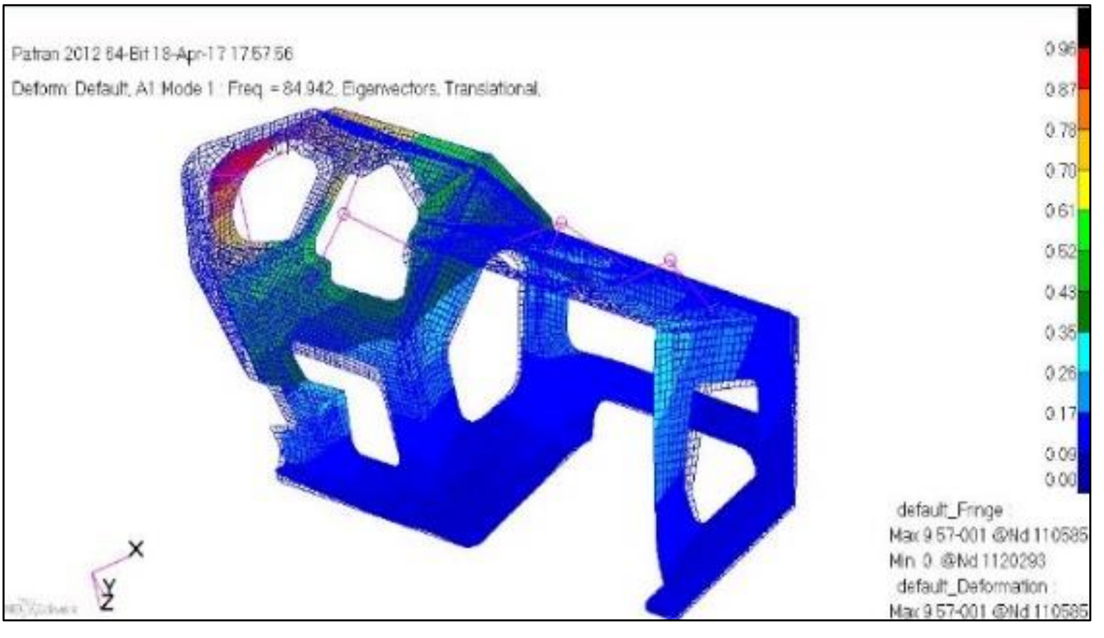

Figure 13. First frequency mode for Bracket 1, in combined bending and torsion, f=89.94 Hz

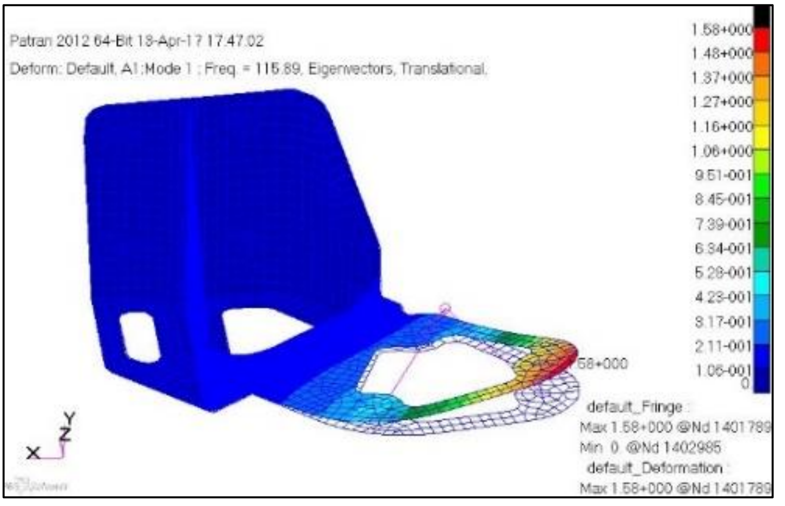

Figure 14. First frequency mode for Bracket 2, in bending, f=115.89 Hz

3.2 Static Analysis

The obtained results of the static analysis (SOL 101 of MSC Patran/Nastran) with the applied acceleration input are summarized in the table below which includes the maximum and local von Mises stresses. In the table below the local von Mises stresses corresponds to the location of the maximum value of RMS von Mises stresses.

Table 4. von Mises stress summary

Structure	Analysis direction	RMS applied acceleration [g]	Max $\sigma_{\text{von Mises}}$ [MPa]	Local $\sigma_{\text{von Mises}}$ [MPa]	Location
Bracket 1	X	18.803	43.88	15.16	see Figure 15
	Y	18.803	63.54	63.54	see Figure 16
	Z	18.803	35.08	10.84	see Figure 17
Bracket 2	X	24.633	22.06	20.52	see Figure 18
	Y	24.633	19.95	19.95	see Figure 19
	Z	24.633	12.89	12.21	see Figure 20

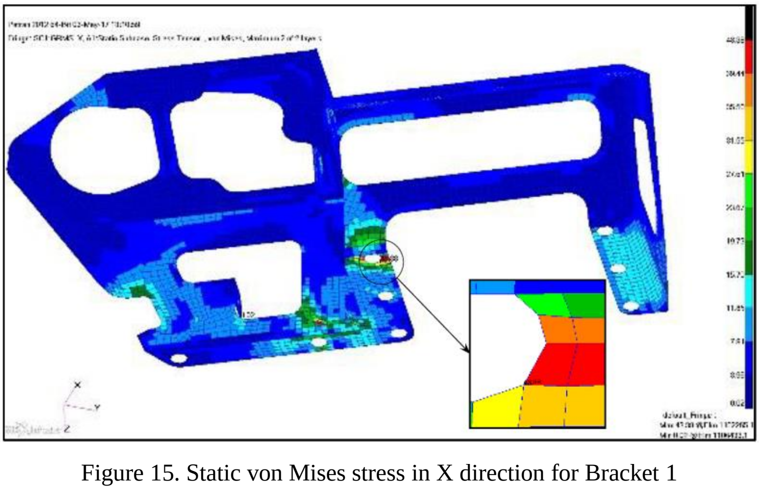

Figure 15. Static von Mises stress in X direction for Bracket 1

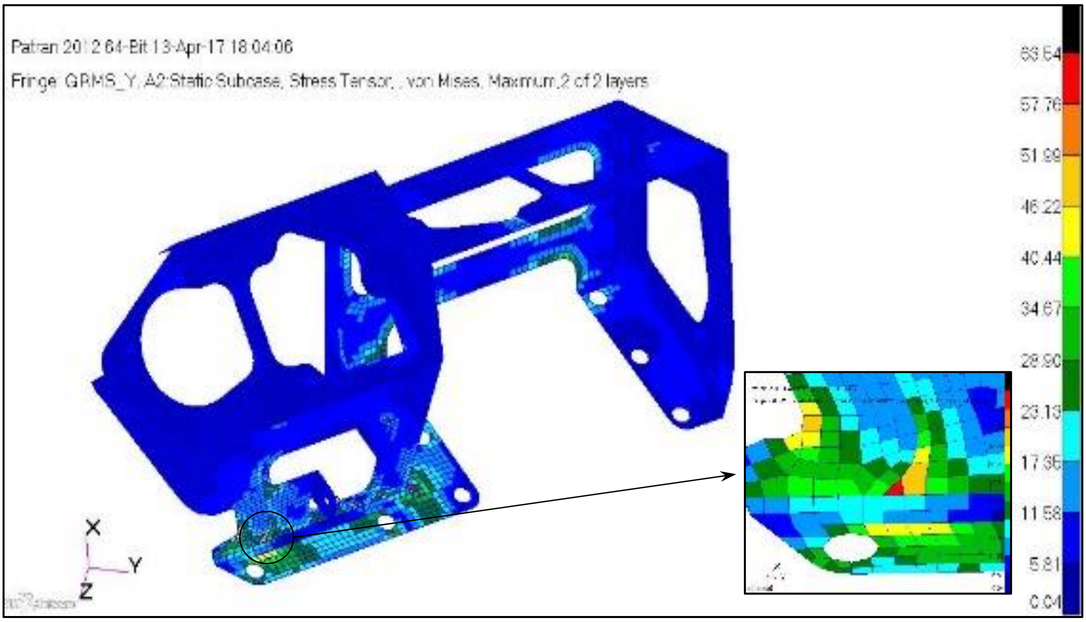

Figure 16. Static von Mises stress in Y direction for Bracket 1

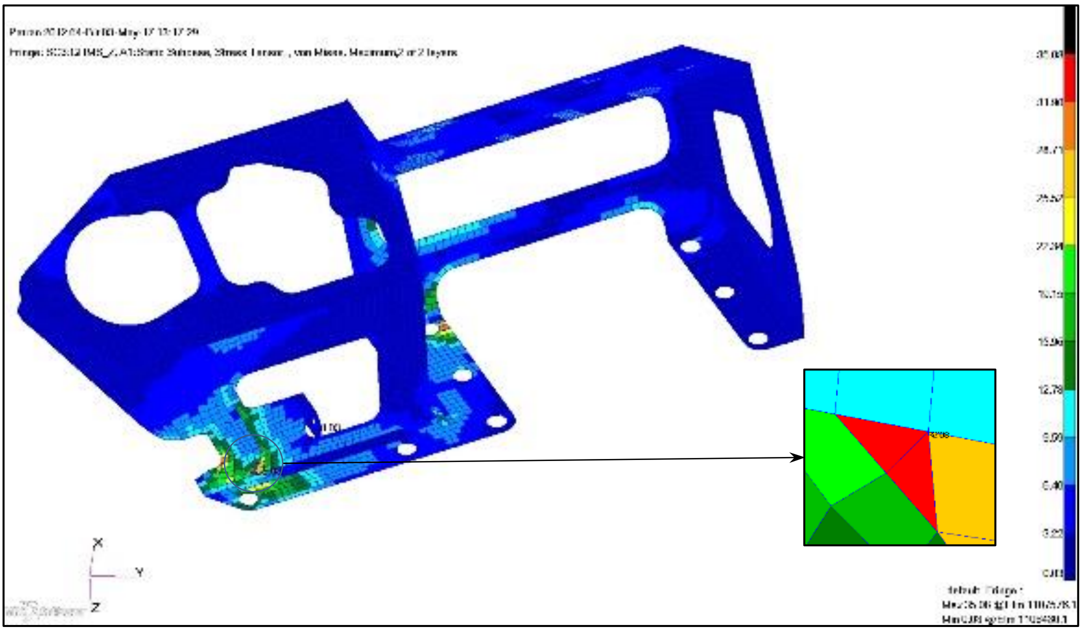

Figure 17. Static von Mises stress in Z direction for Bracket 1

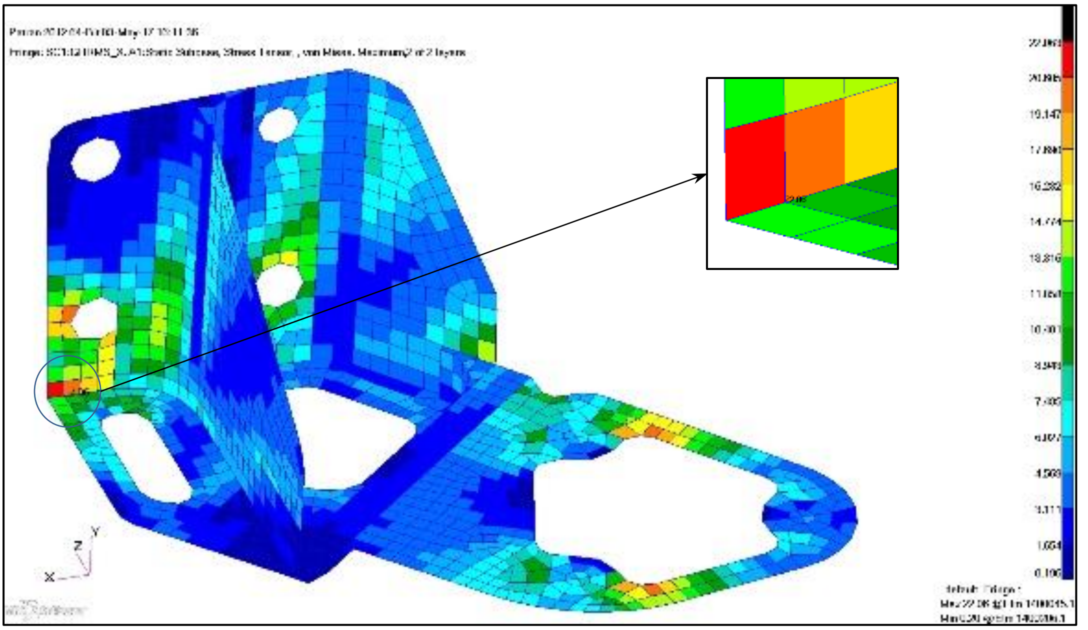

Figure 18. Static von Mises stress in X direction for Bracket 2

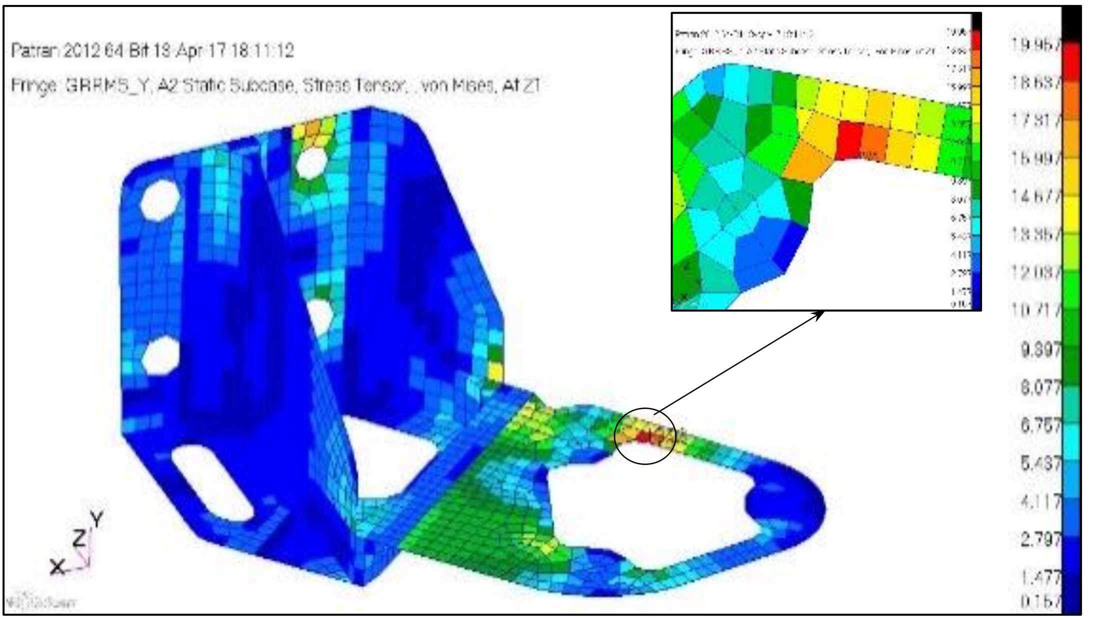

Figure 19. Static von Mises stress in Y direction for Bracket 2

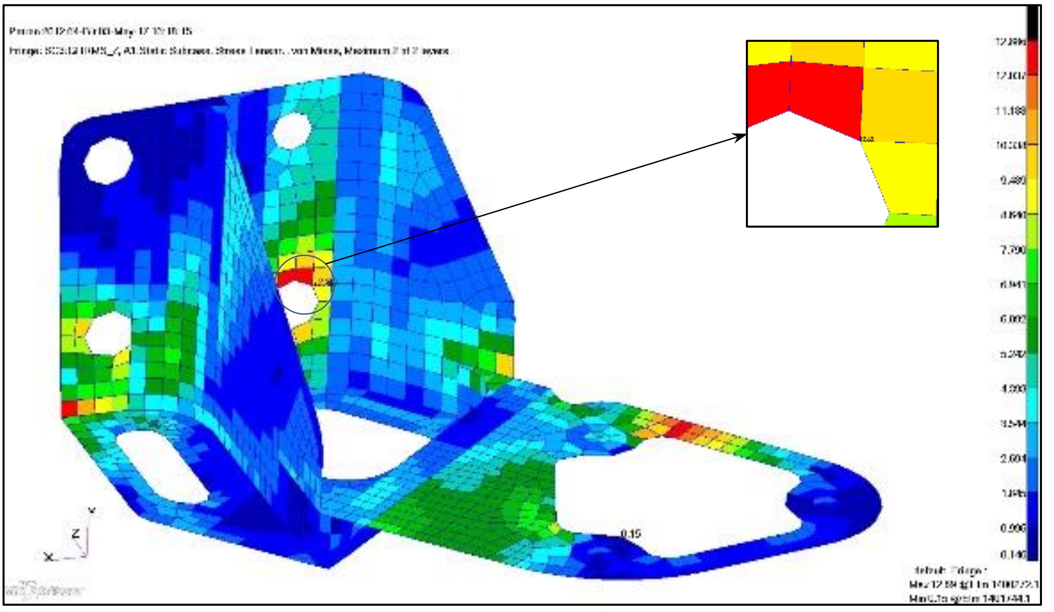

Figure 20. Static von Mises stress in Z direction for Bracket 2

4. CONCLUSIONS

In this article, a detailed comparison between linear static analysis using Miles' equation and random vibration analysis is done in PATRAN/NASTRAN commercial software.

The obtained results are presented in table below.

Table 5. Analysis results comparison

Structure	Direction	Local von Mises Stress [MPa]		Relative error (%)
		Random vibration analysis	Miles' equation Static analysis	
Bracket 1	X	40.72	15.16	62.7
	Y	59.05	63.54	7.6
	Z	33.03	10.84	67.2
Bracket 2	X	25.37	20.52	19.1
	Y	29.81	19.95	33.1
	Z	18.11	12.21	32.6

A first conclusion of this study is that the static analysis using Miles' equation, which is the simplest method, can be used to predict a preliminary structure behavior in stand of laborious random analysis with software PATRAN-NASTRAN. This is the major out coming contribution of the authors.

A second conclusion and authors contribution is that the predictability of the structure's behavior was made applying Miles' equation in the linear variation of ASD diagram.

Another conclusion is that, from the table above it seems that Miles' equation gives the best prediction for simple structures which have a combined bending-torsion first mode of vibration, but it also gives good results for structures with a simple mode of vibration.

Finally, the last conclusion is the short runtime of the static analysis using Miles' equation against random vibration analysis, as presented in Table 6.

Table 6. Analysis results runtime comparison

Structure	CPU [s]	
	Random vibration analysis	Miles equation static analysis
Bracket 1	894.5	6.17
Bracket 2	246.6	2.38

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