Buckling of Flat Thin Plates under Combined Loading

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Abstract: *This article aims to provide a quick methodology to determine the critical values of the forces applied to the central plane of a flat isotropic plate at which a change to the stable configuration of equilibrium occurs. Considering the variety of shapes, boundary conditions and loading combinations, the article does not intend to make an exhaustive presentation of the plate buckling. As an alternative, there will be presented only the most used configurations such as: rectangular flat thin plates, boundary conditions with simply supported (hinged) or clamped (fixed) edges, combined loadings with single compression or single shear or combination between them, compression and shear, with or without transverse loading, encountered at wings and control surfaces shell of fin and rudder or stabilizer and elevator. The reserve factor and the critical stresses will be calculated using comparatively two methods, namely the methodology proposed by the present article and ASSIST 6.6.2.0 – AIRBUS France software, a dedicated software to local calculations, for a simply supported plate under combined loading, compression on the both sides and shear.*

Key Works*: buckling, thin plate, simply supported, hinged edge, clamped, fixed edge, combined loading, reserve factor, ASSIST*

1. INTRODUCTION

Shells and thin plates, in the variety of shapes of flat or curved panels of different configurations, reinforced by stiffeners, are widely found in structural elements of aerospace and aeronautical structures.

Because of the variety of shapes, boundary conditions and loading combinations, the article does not intend to make an exhaustive presentation of the plate buckling.

There will be presented only the most used configurations such as: rectangular flat thin plates, boundary conditions with simply supported (hinged) or clamped (fixed) edges, combined loadings with single compression or single shear or combination between them, compression and shear, with or without transverse loading, encountered at wings and control surfaces shell of fin and rudder or stabilizer and elevator.

To verify if the results are the same, a comparison will be made between this methodology and ASSIST 6.6.2.0 – AIRBUS France software, a dedicated software to local calculations, for a simply supported plate under combined loading.

2. METHOD USED TO DETERMINE CRITICAL LOADS

The forces are applied to the central plane of a flat isotropic plate at which a change to the stable configuration of equilibrium occurs.

The plate equation is written as follows - [1]:

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{1}{D}\left(N_x\frac{\partial^2 w}{\partial x^2}+N_y\frac{\partial^2 w}{\partial y^2}+2N_{xy}\frac{\partial^2 w}{\partial x\partial y}\right) \tag{1}$$

where:

- N_x, N_y, N_{xy} – the critical distributed forces (flow) – (N/mm);
- w – the displacement in the normal direction on the plate;
- $D=\dfrac{Et^3}{12(1-\nu^2)}$ - the bending stiffness of the plate;
- E – Young's modulus – (N/mm^2);
- ν - Poisson ‘s ratio;
- t – the thickness of the plate – (mm)

The strain energy of the internal forces is - [1]:

$$U=\frac{1}{2}D\iint\left\{\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)^2-2(1-\nu)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right]\right\}dxdy \tag{2}$$

The mechanical work of applied forces is - [1]:

$$W=-\frac{1}{2}\iint\left[N_x\left(\frac{\partial w}{\partial x}\right)^2+N_y\left(\frac{\partial w}{\partial y}\right)^2+2N_{xy}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right]dxdy \tag{3}$$

To determine the critical buckling load of plate an energy method is applied. Let's consider the strain energy and mechanical work variation, δU and δW, resulting from relationships (2) and (3), where w is virtual displacement.

The following situations are possible:

- a) $\delta U > \delta W$ – the steady state of plate is stable;
- b) $\delta U < \delta W$ – the steady state of plate is unstable;
- c) $\delta U = \delta W$ – the steady state of plate is neutral, at which a change to the stable configuration of equilibrium occurs.

The critical buckling load of plate is computed from the equality condition of relationships (2) and (3):

$$\begin{aligned}&\frac{1}{2}D\iint\left\{\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)^2-2(1-\nu)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right]\right\}dxdy=\\&=-\frac{1}{2}\iint\left[N_x\left(\frac{\partial w}{\partial x}\right)^2+N_y\left(\frac{\partial w}{\partial y}\right)^2+2N_{xy}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right]dxdy\end{aligned} \tag{4}$$

Solving the equation (4) for different configurations and fixed conditions of the plate allows the determination of critical loads, N_x, N_y and N_{xy}.

Median deformed surface of the plate can be described using double trigonometric series:

$$w=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}C_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{5}$$

where:

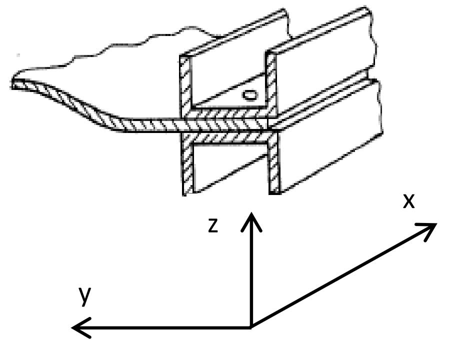
- *a* is the length and b is the width of plate;
- *m* and *n* are the numbers of half waves in the longitudinal and transverse direction of the plate;
- C_{mn} – coefficients.

3. BOUNDARY CONDITIONS OF THE PLATE

A rectangular plate has four edges; each of them can be restrained or loaded in a different way. One of the following possibilities exist for each edge:

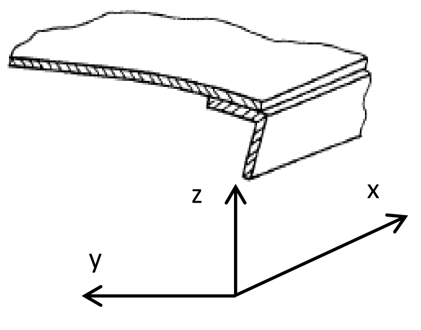
- clamped: $(w)_{y=0} = 0, \left(\frac{\partial w}{\partial y}\right)_{y=0} = 0$

The edge of the plate is prevented from rotation and deflection in a perpendicular direction to the plane of the plate [11].



- simply supported: $(w)_{y=0} = 0, \left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right)_{y=0} = 0$

The edge of the plate is not prevented from rotation, but is only prevented from deflection in a perpendicular direction to the plane of the plate [11].



- free:

The edge of the plate is not prevented from rotation and deflection in a perpendicular direction to the plane of the plate. In a structure it may be difficult to distinguish between clamped or simply supported edges.

Therefore, an intermediate form is used in the literature, namely the elastic or rotational restraint (an average between clamped and simply supported condition). In cases of doubt between the clamped and simply supported edges, it is suggested to use the latter one, which gives safe values for the initial buckling load.

4. SINGLE LOADING

Three single in-plane loads are possible:

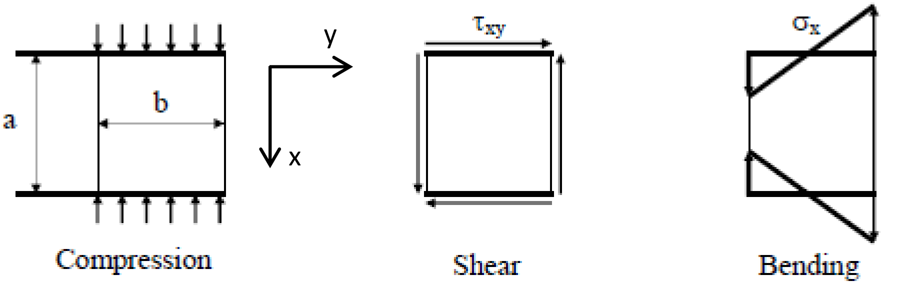


Figure 1 – Single loading of plate

The following formulas for the corresponding buckling stresses will be obtained from equation (4) with $\sigma=\frac{N_x(N_y)}{t};\ \tau=\frac{N_{xy}}{t}$:

$$\sigma_{cc,0}=\eta K_c E\left(\frac{t}{b}\right)^2;\ \tau_{cr,0}=\eta K_s E\left(\frac{t}{\min(a,b)}\right)^2;\ \sigma_{cf,0}=\eta K_f E\left(\frac{t}{b}\right)^2 \quad (6)$$

where:

- $\sigma_{cc,0}$ – critical single compression stress;
- $\tau_{cr,0}$ – critical single shear stress;
- $\sigma_{cf,0}$ – critical single bending stress;
- η – plasticity factor;
- $K=\frac{\pi^2}{12\left(1-\nu_e^2\right)}k$ - buckling factor;
- k is given in the diagrams, as defined [2], [3], [4], [5], [6], depending on the a/b ratio

If a plate is loaded with a transverse compression stress (as per y), b should be replaced by a in formula (6).

5. PLASTICITY CORRECTION FACTOR

The plastic correction factor η depends on E, E_t, E_s and η. These latter values depend on the stress value to be calculated.

The equation of the buckling stress can be formulated as follows: $\sigma_{cr}=\eta\sigma_{cr,e}$, where $\sigma_{cr,e}$ is the linear elastic buckling stress. In practice, for as long as $\sigma_{cr,e}$ is less than the proportionality limit σ_e of the material; the plastic correction factor may be considered as being 1.

For standard aluminum alloys: $\sigma_e=\frac{\sigma_{0.2}}{2}$ - AIRBUS hypothesis, where $\sigma_{0.2}$ is conventional allowable compressive yield stress.

Therefore, this calculation being iterative, the Ramberg and Osgood model is used:

$\varepsilon = \frac{\sigma}{E} + 0.002\left(\frac{\sigma}{\sigma_{0.2}}\right)^{n_c}$, $E_s = \frac{\sigma}{\varepsilon}$, $\frac{1}{E_t} = \frac{n_c}{E_s} + \frac{1-n_c}{E}$, where E_s and E_t are secant and tangent modulus and n_c is stress-strain shape factor for compression.

In the table below the expressions of the plasticity correction factor are given:

Table 1 – Expressions of plasticity correction factors

Loading	Boundary conditions	Equation
Compression and bending	Plate with unloaded hinged edges	$\eta_3 = \eta_1\left(0.5 + 0.25\sqrt{1+3\frac{E_t}{E_s}}\right)$
	Plate with unloaded fixed edges	$\eta_4 = \eta_1\left(0.352 + 0.324\sqrt{1+3\frac{E_t}{E_s}}\right)$
Shear	All conditions	$\eta_6 = \frac{G_s}{G}$ (in the ε formula use the equivalent normal stress $\sigma_{eq} = \tau\sqrt{3}$)

where:

- $\eta_1 = \left(\frac{1-\nu_e^2}{1-\nu^2}\right)\frac{E_s}{E}$, $G = \frac{E}{2(1+\nu_e)}$, $G_s = \frac{E_s}{2(1+\nu)}$
- $\nu = \frac{E_s}{E}\nu_e + \left(1 - \frac{E_s}{E}\right)\nu_p$, $\nu_p = 0.5$, ν_e *– Poisson's ratio – elastic*

Remarks:

- $\eta_1 > \eta_3 > \eta_4$;
- ν and ν_p are elastic - plastic and plastic Poisson's ratio;
- $\tau_{cr} = \eta_6 \tau_{cr,e}$, where $\tau_{cr,e}$ is the linear elastic buckling stress. As long as $\tau_{cr,e}$ is less than the proportionality limit τ_e of the material, the plastic correction factor may be considered as being 1. For standard aluminum alloys: $\tau_e = \frac{\sigma_{0.2}}{2\sqrt{3}}$ - von Mises hypothesis, where $\sigma_{0.2}$ is conventional allowable compressive yield stress.

6. RESERVE FACTORS FROM INTERACTION CURVES FOR COMBINED LOADING

For combined loadings the general conditions for failure are expressed by Shanley as follows:

$$R_1^x + R_2^y + R_3^z + \ldots = 1.0 \quad (7)$$

In this above expression, R_1, R_2, and R_3 could refer to compression, bending and shear and the exponents x, y and z give the relationship for combined stresses. The exponents are determined either theoretically or experimentally and the R_i coefficients are defined as:

$$R_i = \frac{\sigma_i}{\sigma_{adm,i}} \quad (8)$$

where:

- σ_i is effective stress, (N/mm^2);
- $\sigma_{adm,i}$ is allowable stress, (N/mm^2).

For the single load, the reserve factor, *RF*, is equal with: $RF = \frac{1}{R}$. For the biaxial case the reserve factor is determined as follows, see figure below:

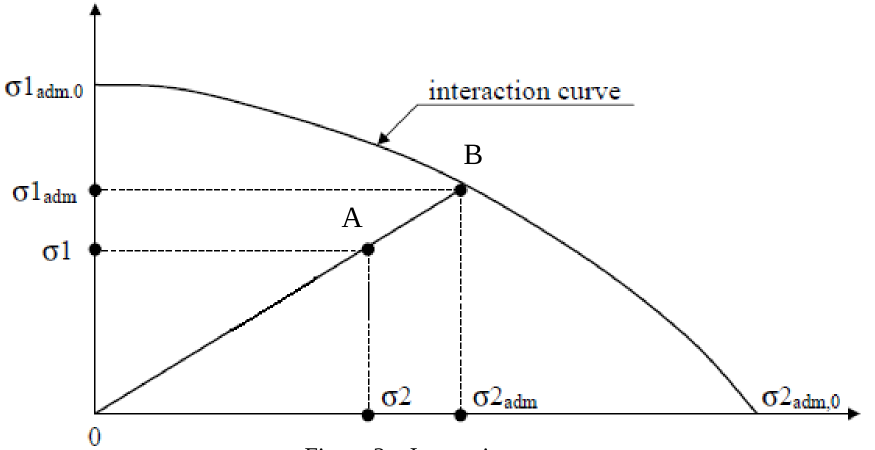


Figure 2 – Interaction curve - σ

$$R = \frac{\sigma_1}{\sigma_{1adm}} = \frac{\sigma_2}{\sigma_{2adm}} = \frac{\overline{OA}}{\overline{OB}},\ RF = \frac{1}{R} \tag{9}$$

where:

- σ_1 and σ_2 are effective stresses, (N/mm^2);
- $\sigma_{1adm,0}$ and $\sigma_{2adm,0}$ are critical stress under simple load, (N/mm^2);
- σ_{1adm} and σ_{2adm} are critical stress under multiple load, (N/mm^2).

Sometimes the diagram from figure 2 is given in the following form:

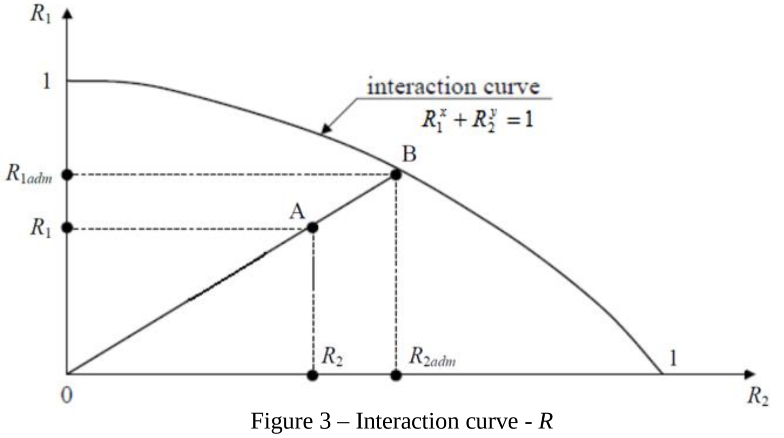


Figure 3 – Interaction curve - *R*

The failure of plate does not occur if the following condition is fulfilled: $R_1^x + R_2^y < 1$.

The coefficients, *R*, in the above figure are defined as:

$$R_1 = \frac{\sigma_1}{\sigma_{1adm,0}},\ R_2 = \frac{\sigma_2}{\sigma_{2adm,0}}$$

$$R_{1adm} = \frac{\sigma_{1adm}}{\sigma_{1adm,0}},\ R_{2adm} = \frac{\sigma_{2adm}}{\sigma_{2adm,0}} \tag{10}$$

$$R = \frac{R_1}{R_{1adm}} = \frac{R_2}{R_{2adm}} = \frac{\overline{OA}}{\overline{OB}},\ RF = \frac{1}{R}$$

The coefficients R_{1adm} and R_{2adm} have to satisfy the conditions: $R_{1adm}^{x} + R_{2adm}^{y} = 1$ and (10).

7. COMBINED LOADING WITHOUT LONGITUDINAL COMPRESSION

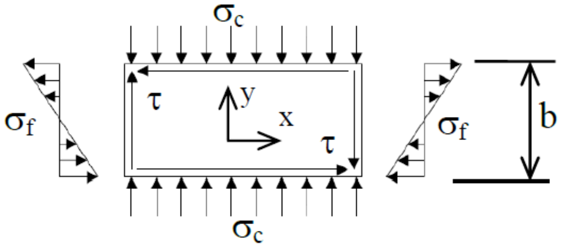


Figure 4 – Combined loading without longitudinal compression

a) Compression and shear:

The interaction equation is $R_c + R_s^2 = 1$ and the reserve factor is defined as $RF = \frac{2}{R_c + \sqrt{R_c^2 + 4R_s^2}}$, where: $R_c = \frac{\sigma_c}{\sigma_{cc,0}}$, $R_s = \frac{\tau}{\tau_{cr,0}}$.

b) Bending and shear:

The interaction equation is $R_s^2 + R_f^2 = 1$ and the reserve factor is defined as $RF = \frac{1}{\sqrt{R_s^2 + R_f^2}}$,

where: $R_f = \frac{\sigma_f}{\sigma_{cf,0}}$, $R_s = \frac{\tau}{\tau_{cr,0}}$.

c) Bending and compression:

The interaction equation is $R_c + R_f^{1.75} = 1$ and the reserve factor may be determined either graphically, using the interaction curves from [2], [3], [4], [5], [6] or numerically:

$$\begin{cases} R_{c,adm} + R_{f,adm}^{1.75} = 1 \\ \frac{R_c}{R_{c,adm}} = \frac{R_f}{R_{f,adm}} = R,\ RF = \frac{1}{R} \end{cases} \tag{11}$$

From the second equation of the system of equations (11) $R_{c,adm} = \frac{R_c}{R}$, $R_{f,adm} = \frac{R_f}{R}$ and introduced in the first equation, the reserve factor could be solved from the equation:

$$R_f^{1.75} * RF^{1.75} + R_c * RF - 1 = 0 \quad (12)$$

d) Bending and compression and shear:

The interaction equation is $R_c + R_s^2 + R_f^{\frac{1.75R_c+2R_s}{R_c+R_s}} = 1$ and the reserve factor may be determined either graphically using the interaction curves from [2], [3], [4], [5], [6] or numerically.

8. COMBINED LOADING WITH LONGITUDINAL COMPRESSION

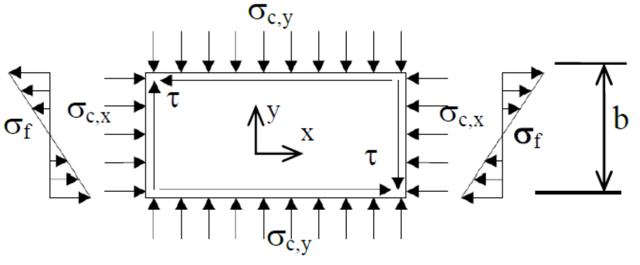


Figure 5 – Combined loading with longitudinal compression

For the calculus of the reserve factor, the following assumption is made:

- the coefficient R_c is computed for compression on the both directions;
- in presence of other loads such as shear or bending, the interaction curves of the previous chapter are used.

9. CALCULUS OF RESERVE FACTOR WITH COMPRESSION IN BOTH DIRECTIONS

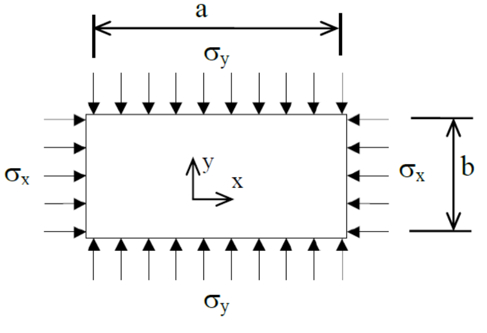


Figure 6 – Combined loading with compression on the both directions

a) the plate has all edges simply supported:

From chapter 2, applying the energy method, the equation (4) will become:

$$N_x \frac{m^2\pi^2}{a^2} + N_y \frac{n^2\pi^2}{b^2} = D\left(\frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{b^2}\right)^2 \quad (13)$$

Here m and n signify the number of half waves in the buckled plate in the *x* and *y* directions, respectively. Dividing by thickness *t* of plate in equation (13), it is obtained:

$$\sigma_{x,adm}\frac{m^2}{a^2}+\sigma_{y,adm}\frac{n^2}{b^2}=0.823\frac{Et^2}{1-\nu^2}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2 \tag{14}$$

To find $\sigma_{y,adm}$ for a given $\sigma_{x,adm}$, take m=1 and n=1, if:

$$C\left(1-4\frac{a^4}{b^4}\right)<\sigma_{x,adm}<C\left(5+2\frac{a^2}{b^2}\right),\ \text{where}\ C=\frac{0.823Et^2}{(1-\nu^2)a^2}. \tag{15}$$

If σ_x is too large to satisfy this inequality, take n=1 and m to satisfy:

$$C\left(2m^2-2m+1+2\frac{a^2}{b^2}\right)<\sigma_{x,adm}<C\left(2m^2+2m+1+2\frac{a^2}{b^2}\right) \tag{16}$$

If $\sigma_{x,adm}$ is too small to satisfy the first inequality, take m=1 and n to satisfy:

$$C\left[1-n^2(n-1)^2\frac{a^4}{b^4}\right]>\sigma_{x,adm}>C\left[1-n^2(n+1)^2\frac{a^4}{b^4}\right] \tag{17}$$

b) the plate has all edges clamped:

$$\sigma_{x,adm}+\frac{a^2}{b^2}\sigma_{y,adm}=1.1\frac{Et^2a^2}{1-\nu^2}\left(\frac{3}{a^4}+\frac{3}{b^4}+\frac{2}{a^2b^2}\right) \tag{18}$$

This equation is approximate and is most accurate when the plate is nearly square a σ_x and σ_y nearly equal.

The calculus of the reserve factor is made from the following relationships:

$$R_c=\frac{\sigma_x}{\sigma_{x,adm}}=\frac{\sigma_y}{\sigma_{y,adm}},\ RF=\frac{1}{R_c}\ \text{and (14) or (18)} \tag{19}$$

10. EXAMPLE – COMBINED LOADING WITH COMPRESSION ON THE BOTH DIRECTIONS AND SHEAR

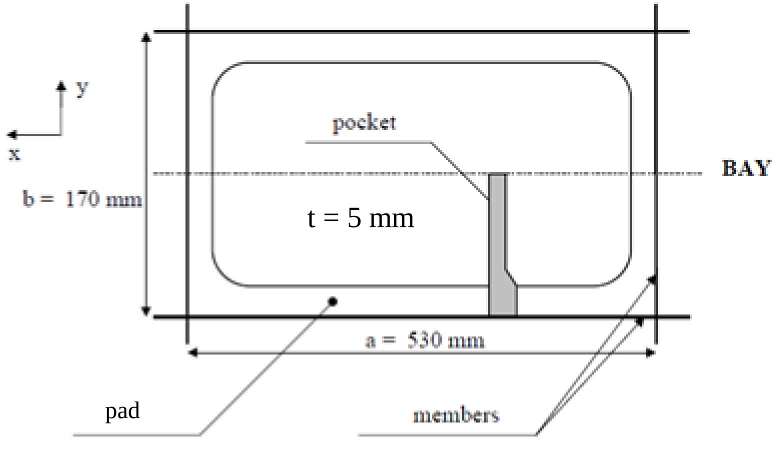


Figure 7 – Combined loading with compression in both directions and shear

- plate geometry: a = 530 mm, b = 170 mm, t = 5 mm;
- plate loading:
  σ_x = 18 N/mm^2
  σ_y = 9 N/mm^2
  τ_{xy} = 27 N/mm^2
- plate material: 2024 - T3:
  Ec = 70300 N/mm^2
  $\sigma_{0.2}$ = 270 N/mm^2
  σ_R = 440 N/mm^2
  n_c = 7.05
  ν_e = 0.33
- plate boundary conditions: all edges simply supported

a) the calculus of the coefficient R_c for compression in both x and y directions of the plate.

From equation (14), for m=1 and n=1, it is obtained:

$$\frac{\sigma_{x,adm}}{a^2}+\frac{\sigma_{y,adm}}{b^2}=0.823\frac{Et^2}{1-\nu^2}\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2 \tag{20}$$

and introducing the input data in the eqautions (19) and (20), the following system of equations is obtained:

$$\begin{cases}\dfrac{\sigma_{x,adm}}{530^2}+\dfrac{\sigma_{y,adm}}{170^2}=0.823\dfrac{70300*5^2}{1-0.33^2}\left(\dfrac{1}{530^2}+\dfrac{1}{170^2}\right)^2\\ \dfrac{18}{\sigma_{x,adm}}=\dfrac{9}{\sigma_{y,adm}}\end{cases} \tag{21}$$

From system (21), it is obtained:

$\sigma_{x,adm}$ = 113.32 N/mm^2 and $\sigma_{y,adm}$ = 56.66 N/mm^2, where $\sigma_{x,adm}$ satisfies inequality (15).

It is not necessary to apply the plasticity correction factor, because $\sigma_e = \sigma_{0.2}$ / 2 = 135 N/mm^2. From relationship (19), R_c = 0.159 is obtained.

b) the calculus of the coefficient R_s for shear

From relationship (6), it is obtained:

$$\tau_{cr,0}=\eta_6\frac{\pi^2}{12*\left(1-0.33^2\right)}*k_s*70300*\left(\frac{5}{170}\right)^2=\eta_6*322.18\ \text{N/mm}^2.$$

k_s = 5.74, it is given in the diagrams, as defined [2], [3], [4], [5], [6] depending on the minimum value of (a/b ; b/a) ratios.

Because $\tau_{cr,e}$ = 322.18 N/mm^2 is greater than $\tau_e = \dfrac{\sigma_{0.2}}{2*\sqrt{3}}$ = 77.94 N/mm^2, a plasticity correction factor is aplied, η_6.

A schematic method to calculate the plasticity correction factor is presented below: $\sigma_{cr,corrected}=\dfrac{\sigma_{0.2}}{2}=\dfrac{270}{2}=135\,\text{N/mm}^2$ is assumed as initial value.

$$\varepsilon=\frac{\sigma}{E}+0.002\left(\frac{\sigma}{\sigma_{0.2}}\right)^{n_c}=\frac{135}{70300}+0.002*\left(\frac{135}{270}\right)^{7.05}=0.00194$$

$$E_s = \frac{\sigma}{\varepsilon} = \frac{135}{0.00194} = 69752 \text{ N/mm}^2.$$

$$E_t = \left(\frac{n_c}{E_s} + \frac{1-n_c}{E}\right)^{-1} = \left(\frac{7.05}{69752} + \frac{1-7.05}{70300}\right)^{-1} = 66609 \text{ N/mm}^2.$$

$$\nu = \frac{E_s}{E}\nu_e + \left(1 - \frac{E_s}{E}\right)\nu_p = \frac{69752}{70300} * 0.33 + \left(1 - \frac{69752}{70300}\right) * 0.5 = 0.331$$

$$\eta_1 = \left(\frac{1-\nu_e^2}{1-\nu^2}\right)\frac{E_s}{E} = \left(\frac{1-0.33^2}{1-0.331^2}\right) * \frac{69752}{70300} = 0.993$$

$$\eta_3 = \eta_1\left(0.5 + 0.25\sqrt{1+3\frac{E_t}{E_s}}\right) = 0.993 * \left(0.5 + 0.25 * \sqrt{1+3*\frac{66609}{69752}}\right) = 0.9845$$

$$\sigma_{cr,elastic} = \frac{\sigma_{cr,corrected}}{\eta_3} = \frac{135}{0.9845} = 137.13 \text{ N/mm}^2.$$

Thus, for $\sigma_{cr,corrected}$ = 135 N/mm^2 corresponds $\sigma_{cr,elastic}$ = 137.13 N/mm^2.

By varying the $\sigma_{cr,corrected}$ (using an average between the value found and the initial value of the previous step), the corresponding $\sigma_{cr,elastic}$ can be calculated. With these values the curve σ_{cr} can be plotted as shown in the figure bellow.

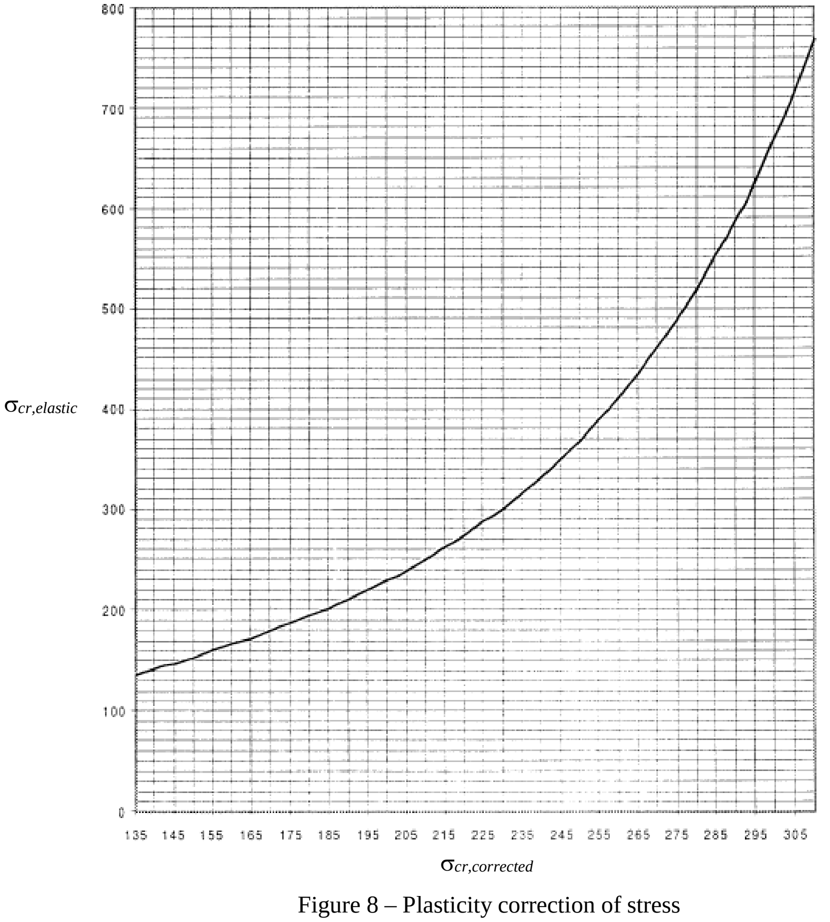


Figure 8 – Plasticity correction of stress

Hence, $\sigma_{cr,elastic,eq} = \tau_{cr,e}\sqrt{3} = 322.18*\sqrt{3} = 558.03$ N/mm^2. From the diagram above, it is obtained $\sigma_{cr,corrected,eq}$ = 291 N/mm^2 and $\eta_6 = \frac{\sigma_{cr,corrected,eq}}{\sigma_{cr,elastic,eq}} = \frac{291}{558.03} = 0.521$ and $\tau_{cr,0} = \eta_6 * \tau_{cr,e} = 0.521* 322.18 = 167.86$ N/mm^2. From relationship (8) we get $R_s = \frac{\tau_{xy}}{\tau_{cr,0}} = \frac{27}{167.86} = 0.161$. Using the interaction curve for compression and shear, it is obtained: $RF = \frac{2}{R_c + \sqrt{R_c^2 + 4R_s^2}} = \frac{2}{0.159 + \sqrt{0.159^2 + 4*0.161^2}} = 3.86$

This example was made with the AIRBUS software ASSIST 6.6.2.0 for Windows. The results are presented in the figure below:

Designation	"test"	
Plate length (mm)	(mm)	530
Plate width (mm)	(mm)	170
Thickness	(mm)	5
Radius of curvature	(mm)	

Material characteristics		
Name	"2024 PL T3"	
Young modulus (Ec)	(MPa)	70300
Poisson's ratio (mu)		0.33
Yield stress in Comp (Fcy)	(MPa)	270
Ultimate tensile stress (Ftu)	(MPa)	440
R-O ratio (nc)		7.05

Boundary conditions	
Edge1	Simply supported
Edge2	Simply supported
Edge 3_4	Simply supported

Edge1, Edge2, Edge3, Edge 4

Critical stress	Sigma (MPa)	Plasticity Correction (Eta)	Buckling Coefficient (k)
Critical normal stress along x-axis under multiple load	-69	0.995	2.02
Critical normal stress along y-axis under multiple load	-56	---	---
Critical shear stress under multiple load	104	0.521	5.74
Critical bending stress under multiple load	---	---	---

Stress margin — Case : "Computation case"		
Loading		
X_axis stress	(MPa)	-18
Shear stress	(MPa)	27
Bending stress	(MPa)	---
Y_axis stress	(MPa)	-9
Load ratio		
In compression		0.16
Under shear stress		0.161
Under bending stress		---
MARGIN OF SAFETY(MS)	(%)	285
Reserve factor (RF)		3.85

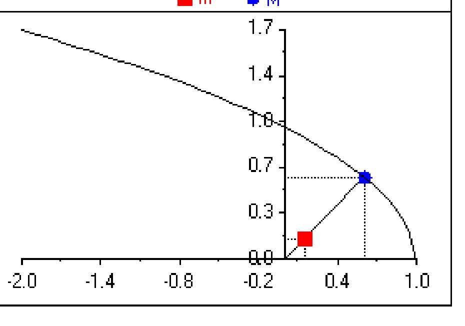


Figure 9 – Calculation of reserve factor with ASSIST

As it can be seen, the results are practically identical: the value of the reserve factor calculated with this methodology is RF = 3.86 while the value obtained using the dedicated software ASSIST is RF = 3.85.

11. CONCLUSIONS

This article establishes a quick methodology to determine the critical values of the forces applied to the central plane of a flat isotropic plate at which a change to the stable configuration of equilibrium occurs. Because there are a plenty of shapes, boundary conditions and loading combinations, it is not possible to make an exhaustive presentation of the plate buckling in this article.

For this reason, there were presented only the most used configurations, such as rectangular flat thin plates, boundary conditions with simply supported (hinged) or clamped (fixed) edges, combined loadings with single compression or single shear or combination between them, compression and shear, with or without transverse loading, encountered at wings and control surfaces shell of fin and rudder or stabilizer and elevator.

The reserve factor and the critical stresses will be calculated using comparatively two methods, namely the methodology proposed by the present article and ASSIST 6.6.2.0 – AIRBUS France software, a dedicated software to local calculations, for a simply supported plate under combined loading, compression in the both sides and shear.

In the table below it is presented comparatively the results of the calculation for the reserve factor and critical stresses using this methodology and ASSIST, using a proper mathematical model.

Table 2 – Critical stresses and reserve factor

Method of calculation	Effective stresses (N/mm²)			Critical stresses under multiple load (N/mm²)			Load ratio		Reserve factor
	σ_x	σ_y	τ_{xy}	$\sigma_{cr,x}$	$\sigma_{cr,y}$	τ_{cr}	R_c	R_s	RF
This methodology	-18	-9	27	-69.48	-56.66	104.22	0.159	0.161	3.86
ASSIST				-69	-56	104	0.16	0.161	3.85

Remarks:

- the results are practically identical for critical stresses under multiple load, load ratios and reserve factors;
- $\sigma_{cr,x} = RF*\sigma_x$. Due to the transverse load σ_y, only the longitudinal critical compression stress $\sigma_{cr,x}$ is affected;
- $\sigma_{cr,y} = \sigma_{y,adm}$, this critical stress is not penalized;
- $\tau_{cr} = RF*\tau_{xy}$;

The original contributions of the author are:

a) the analytical calculation of critical stresses $\sigma_{x,adm}$ and $\sigma_{y,adm}$ was made using the equations (14), (18) and (19) for the case with biaxial compression and all edges simply supported or clamped;
b) building the chart from figure 3 – Interaction curve – R, where the coefficients R_i are obtained by dividing with $\sigma_{adm,0}$ of the terms from figure 2– Interaction curve – σ.

This article also is intended to be a calculus guide for students and design and stress engineers, adapted to the INCAS needs enabling a correct understanding of the phenomenon of the plates stability.

REFERENCES

[1] S. P. Timoshenko and J. M. Gere, *Theory of Elastic Stability,* McGRAW-HILL BOOK COMPANY, INC. 1961.
[2] *** AIRBUS Static Stress Manual, Metallic Materials, MTS 004, Issue C, 1999.
[3] E. F. Bruhn, *Analysis and Design of Flight Vehicle Structures*, 1973.
[4] *** HSB 45111-01, Issue B, 1969, *Buckling of rectangular plates under various loading types and support conditions*.
[5] *** HSB 45111-04, Issue B, 1970, *Buckling of flat, rectangular plates under bending and compression*.
[6] *** HSB 45112-01, Issue C, 1970, *Shear buckling of flat, rectangular plates*.
[7] *** HSB 45113-01, Issue B, 1969, *Buckling under combined loading*.
[8] *** HSB 51200-01, Issue D, 2006, *Reserve factors from interaction curves for combined loading*.
[9] *** ASSIST 6.6.2.0 for Windows, *AIRBUS Software – FRANCE,* 2010.
[9] R. J. Roark, W. C. Young, *Formulas for Stress and Strain, Fifth Edition*, 1975
[10] G. V. Vasiliev and V. Giurgiutiu, *The stability of the aeronautical structures,* Bucharest, Tehnical Publishing House, 1990.
[11] * * * *FOKKER TECHNICAL HANDBOOK*, TH3 Strength Data, Issue 068, Issue data 951208.