Wave-wave regular interactions of a gasdynamic type

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Abstract: *Two gas dynamic analytic approaches [of a Burnat type / Martin type] are respectively used in order to construct two analogous and significant pairs of classes of solutions [isentropic pair / anisentropic (of a particular type) pair]. Each mentioned pair puts together a class of "wave" elements and a class of "wave-wave regular interaction" elements. A classifying parallel is finally constructed between the two analogous pairs of classes -- making evidence of some consonances and, concurrently, of some nontrivial contrasts.*

Key Words: *geometrical approach; two-dimensional solutions; quantifiable "amount" of genuine nonlinearity; regular interaction vs. irregular interaction: heuristic details*

1. INTRODUCTION

Finding a solution to a quasilinear system of a gas dynamic type

$$\sum_{j=1}^{n}\sum_{k=0}^{m} a_{ijk}(u)\frac{\partial u_j}{\partial x_k} = 0, \quad 1 \le i \le n \tag{1}$$

[ex. Euler isentropic /anisentropic; possibly multidimensional] is a hard task generally. A recent talk of these authors has considered constructively, in presence of certain integrability restrictions, some highly nontrivial and *significant* classes of solutions to such type of systems.

Two analytic approaches have been considered in our talk: a Burnat type "algebraic" and genuinely nonlinear approach [structured by a duality connection between the hodograph character and the physical character] and a Martin type two-dimensional "differential" approach [associated with a Monge−Ampère type representation].

A pair of significant classes of solutions has been associated in our talk to *each* of the two mentioned approaches.

In the *isentropic* case a Burnat type approach has been used to constructively structure: ● some *simple waves solutions* − here called *waves* [a first significant class], ● some *wave-wave regular interaction solutions* [a second significant class]; and, ● a *multidimensional extension* of the two classes mentioned above − with a *classifying potential*; a *regular* character of the wave-wave interaction described appeared to essentially reflect facts of a multidimensional and skew construction.

In the *anisentropic* case − and in two independent variables − a Martin type approach has been used, as associated with a *particular* gas dynamic example, to constructively structure an *anisentropic* **analogue** of the isentropic pair of classes mentioned above: the *anisentropic pair* which puts together ● some *pseudo simple waves solutions* [a first significant class] and ● some *pseudo wave-wave regular interaction solutions* [a second significant class]. Details concerning the nature of the mentioned analogous character have been presented.

A *classifying parallel* has been presented concurrently in our talk between the two analogous pairs of classes [isentropic, anisentropic] − making evidence of some *consonances* and, respectively, of some *nontrivial contrasts* of the two mentioned constructions [Burnat type, Martin type].

The regular passage [which uses the two analogous pairs of classes] from an isentropic description to an anisentropic description appeared to be **fragile**. Our talk also presented some essential details of this fragility.

The present paper [a small fragment of our recent talk] includes two selfsimilar isentropic examples − significant and highly nontrivial − of wave-wave regular interaction solutions.

We use these examples to suggest that a certain structure could be associated with each "amount" of genuine nonlinearity eventually available and that there is a *hierarch*y of such structures.

The two types of wave-wave regular interactions constructed in our talk [isentropic /anisentropic] appeared to parallel, from an *analytic, local* and *regular* prospect, some details [interactions of simple waves solutions] of the Zhang and Zheng two-dimensional *qualitative, global* and *irregular* construction. The two examples in the present paper suggest that a *regular* character of the wave-wave interaction described essentially reflect facts of a *multidimensional* and *skew* construction.

2. BURNAT TYPE "ALGEBRAIC" APPROACH. ISENTROPIC CONTEXT

For the multidimensional first order hyperbolic system of a gasdynamic type (1) the "algebraic" approach (Burnat [1]) starts with identifying *dual* pairs of directions $\vec{\beta}, \vec{\kappa}$ [we write $\vec{\kappa} \longleftrightarrow \vec{\beta}$] connecting [via their duality relation] the *hodograph* [= in the hodograph space H of the entities u] and *physical* [= in the physical space E of the independent variables] *characteristic details*.

The duality relation at $u^* \in H$ has the form:

$$\sum_{j=1}^{n} \sum_{k=0}^{m} a_{ijk}(u^*) \beta_k \kappa_j = 0, \quad 1 \le i \le n. \tag{2}$$

Here $\vec{\beta}$ is an *exceptional* direction [= *normal characteristic* direction (orthogonal in the physical space E to a characteristic character)].

A direction $\vec{\kappa}$ dual to an exceptional direction $\vec{\beta}$ is said to be a *hodograph characteristic* direction. The reality of exceptional/ hodograph characteristic directions implied in (2) is concurrent with the hyperbolicity of (1).

EXAMPLE 1. For the *one-dimensional* strictly hyperbolic version of system (1) a *finite* number n of dual pairs $\vec{\kappa}_i \longleftrightarrow \vec{\beta}_i$ consisting in $\vec{\kappa}_i = \vec{R}_i$ and $\vec{\beta}_i = \Theta_i(u)[-\lambda_i(u), 1]$, where $\vec{R}_i$

is a right eigenvector of the $n \times n$ matrix a and λ_i is an eigenvalue of a, are available ($i = 1, ..., n$). Each dual pair associates in this case, at each $u^* \in \mathcal{R}$ [for a suitable region $\mathcal{R} \subset H$], to a vector $\vec{\kappa}$ a *single* dual vector $\vec{\beta}$. □

EXAMPLE 2 (Peradzyński [8]). For the *two-dimensional* isentropic version of (1) an *infinite* number of dual pairs are available at each $u^* \in H$. Each dual pair associates, at the mentioned u^*, to a vector $\vec{\kappa}$ a *single* dual vector $\vec{\beta}$. □

EXAMPLE 3 (Peradzyński [9]). For the isentropic description corresponding to the *three-dimensional* version of (1) an *infinite* number of dual pairs are available at each $u^* \in H$. Each dual pair associates, at the mentioned u^* to a vector $\vec{\kappa}$ a *finite* [constant, $\neq 1$] number of k independent exceptional dual vectors $\vec{\beta}_j$, $1 \le j \le k$, and therefore has the structure $\vec{\kappa} \longleftrightarrow (\vec{\beta}_1, \ldots, \vec{\beta}_k)$. □

DEFINITION 4 (Burnat [1]). A curve $\mathcal{C} \subset H$ is said to be *characteristic* if it is tangent at each point of it to a characteristic direction $\vec{\kappa}$. A hypersurface $\mathcal{S} \subset H$ is said to be *characteristic* if it possesses at least a characteristic system of coordinates. □

3. GENUINE NONLINEARITY. SIMPLE WAVES SOLUTIONS

REMARK 5. As it is well-known (Lax [7]), in case of an one-dimensional strictly hyperbolic version of (1) a hodograph characteristic curve $\mathcal{C} \subset \mathcal{R} \subset H$, of index i, is said to be *genuinely nonlinear (gnl)* if the dual constructive pair $\vec{\kappa}_i \longleftrightarrow \vec{\beta}_i$ is restricted [the restriction is on the *pair*!] by $\vec{\kappa}_i(u) \diamond \vec{\beta}_i(u) \equiv \vec{R}_i(u) \cdot \mathrm{grad}_u \lambda_i(u) \neq 0$ in $\mathcal{R}$; see Example 1.

This condition transcribes the requirement $\frac{\mathrm{d}\vec{\beta}}{\mathrm{d}\alpha} \neq 0$ along *each* hodograph characteristic curve $\mathcal{C}$. □

DEFINITION 6. We naturally extend the *gnl* character of a hodograph characteristic curve $\mathcal{C}$ to the cases corresponding to Examples 2 and 3, by requiring along $\mathcal{C}$: $\left|\frac{\mathrm{d}\vec{\beta}}{\mathrm{d}\alpha}\right| \neq 0$ and, respectively, $\sum_{\mu=1}^{k}\left|\frac{\mathrm{d}\vec{\beta}_\mu}{\mathrm{d}\alpha}\right| \neq 0.$ □

DEFINITION 7a. A solution of (1) whose hodograph is laid along a *gnl* characteristic curve is said to be a *simple waves solution* (here below also called ***wave***). The *gnl* character implies a *nondegeneracy* [in the sense of a "funning out"] of such a solution. □

Here are three types of simple waves solutions, respectively associated, in presence of a *gnl* character, to the Examples 1−3 above, presented in an implicit form − a first application of the duality relation (2):

$$u(x,t)=U[\alpha(x,t)],\quad \alpha=\theta(\xi),\quad \xi=x-\varsigma_i(\alpha)t,$$

$$u=U[\alpha(x,t)],\quad \alpha=\theta(\xi),\quad \xi=\sum_{\nu=0}^{m}\beta_\nu[U(\alpha)]x_\nu=\sum_{\nu=0}^{m}\beta_\nu\{U[\theta(\xi)]\}x_\nu$$

$$u=U[\alpha(x,t)],\quad \alpha=\theta(\xi_{1,\ldots,}\xi_k),\quad \xi_j=\sum_{\nu=0}^{m}\beta_{j\nu}\{U[\theta(\xi_{1,\ldots,}\xi_k)]\}x_\nu;\quad 1\le j\le k.$$

4. GENUINE NONLINEARITY: A CONSTRUCTIVE EXTENSION. WAVE-WAVE REGULAR INTERACTIONS. RIEMANN−BURNAT INVARIANTS

Let $R_1,\ldots,R_p$ be *gnl* characteristic coordinates on a given characteristic region $\mathcal{R}$ of a hodograph surface $\mathcal{S}$ with the normal $\vec{n}$.

Solutions of the *intermediate* system

$$\frac{\partial u_l}{\partial x_s}=\sum_{k=1}^{p}\eta_k\kappa_{kl}(u)\beta_{ks}(u), u\in\mathcal{R};\ 1\le l\le n, 0\le s\le m; \vec{\kappa}_k\perp\bar{n}, 1\le k\le p \tag{3}$$

appear to concurrently satisfy the system (1) [we carry (3) into (1) and use (2)]. This indicates an "algebraic" importance of the concept of dual pair (Burnat [1]).

DEFINITION 7b. A solution of (1) whose hodograph is laid on a characteristic surface is said to correspond to a ***wave-wave regular interaction*** if its hodograph possesses a *gnl* system of coordinates *and* there exists a set of *Riemann−Burnat invariants* $R(x)$, structuring the dependence on *x* of the solution *u* by a *regular* interaction representation

$$u_l=u_l[R_1(x_0,\ldots,x_m),\ldots,R_p(x_0,\ldots,x_m)],\ 1\le l\le n. \tag{4}$$

REMARK 8. It is easy to see that for a wave-wave regular interaction solution associated to (4) $R_i(x)$ must fulfill an (overdetermined and Pfaff) system

$$\frac{\partial R_k}{\partial x_s}=\eta_k\beta_{ks}[u(R)],\ 1\le k\le p, 0\le s\le m. \tag{5}$$

Sufficient restrictions for solving (5) are proposed in [5], [6], [8], [9]. Also see [2], [3]. □

- The *gnl* character of the contributing simple waves solutions results in an *ad hoc gnl* character of the wave-wave *regular* interaction solution constructed.

REMARK 9. Four circumstances appear to be significant for solutions with a characteristic hodograph: *(a)* the isentropic case of a characteristic hodograph surface for which *all* the coordinate systems are *gnl*; *(b)* the isentropic case of a characteristic hodograph surface for which *only a part* of the coordinate system are *gnl*; *(c)* the isentropic case of a characteristic hodograph surface for which all the coordinate systems are *linearly degenerate (ldg)* ("$\neq$ 0" is replaced by "= 0" in Definition 6); *(d)* the anisentropic case of a hodograph surface which is not Burnat characteristic (Definition 4); for such a circumstance a characteristic character of the hodograph surface may persist in an alternative sense (ex. in a Martin sense, see [4]). □

5. GENUINE NONLINEARITY / LINEAR DEGENERACY: SOME TWO-DIMENSIONAL DETAILS

In case of the two-dimensional system

$$\begin{cases} \frac{\partial c}{\partial t} + v_x \frac{\partial c}{\partial x} + v_y \frac{\partial c}{\partial y} + \frac{\gamma - 1}{2} c \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \right) = 0 \\ \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + \frac{2}{\gamma - 1} c \frac{\partial c}{\partial x} = 0 \\ \frac{\partial v_y}{\partial t} + \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + \frac{2}{\gamma - 1} c \frac{\partial c}{\partial y} = 0, \end{cases} \tag{6}$$

corresponding to an *isentropic* flow (in usual notations), condition (2) leads [as $n = m + 1$], at a point $u^* \in H$, to the form in $\vec{\kappa}$

$$c^2 \kappa_1 \left[\left(\frac{2}{\gamma - 1} \right)^2 \kappa_1^2 - (\kappa_2^2 + \kappa_3^2) \right] = 0 \tag{7}$$

The hodograph characteristic cone (7) will be connected with [as $n = m + 1$] the exceptional cone

$$(\beta_0 + \beta_1 v_x^* + \beta_2 v_y^*)[(\beta_0 + \beta_1 v_x^* + \beta_2 v_y^*)^2 - c^{*2}(\beta_1^2 + \beta_2^2)] = 0. \tag{7'}$$

- We have to notice in this respect that *not any* pair of directions $\vec{\kappa}$, $\vec{\beta}$ with $\vec{\kappa}$ from (7) and $\vec{\beta}$ from (7′) is dual. In fact, to each $\vec{\kappa}$ from (7) a *single* $\vec{\beta}$ from (7′) corresponds (cf. Example 2).

Precisely:

- given $\vec{\beta}$ we obtain from (2) [$\vec{\kappa}$ in terms of $\vec{\beta}$]

$$\vec{\beta} = (v_x \beta_1 + v_y \beta_2, -\beta_1, -\beta_2) \longrightarrow \vec{\kappa} = (0, -\beta_2, \beta_1),$$

$$\vec{\beta} = [-(v_x \beta_1 + v_y \beta_2) - \varepsilon c, \beta_1, \beta_2] \longrightarrow \vec{\kappa} = \left[\varepsilon \frac{\gamma - 1}{2}, \beta_1, \beta_2 \right], \quad \varepsilon = \pm 1,$$

where for $\vec{\beta}$ running through (7′) $\vec{\kappa}$ runs through (7).

- given $\vec{\kappa}$ [cf. (7)] we get from (2) [$\vec{\beta}$ in terms of $\vec{\kappa}$]

$$\vec{\kappa} = (0, -\kappa_2, \kappa_3) \longrightarrow \vec{\beta} = (v_x \kappa_3 - v_y \kappa_2, -\kappa_3, -\kappa_2),$$

$$\vec{\kappa} = \left[\varepsilon \frac{\gamma - 1}{2}, \kappa_2, \kappa_3 \right] \longrightarrow \vec{\beta} = [-(v_x \kappa_2 + v_y \kappa_3) - \varepsilon c, \kappa_2, \kappa_3], \quad \varepsilon = \pm 1,$$

where for $\vec{\kappa}$ running through (7) $\vec{\beta}$ runs through (7′).

- ■ It is easy to show that the hodograph characteristic curves along which, in a gas dynamic construction, $\kappa_1 \neq 0$ have a *genuinely nonlinear* character.

- Any smooth curve $\mathcal{C}$ placed in a plane c = constant $\neq 0$ appears to be a hodograph characteristic curve corresponding to $\kappa_1 = 0$ in (7).
- It is easy to show that the hodograph characteristic curves corresponding to $\kappa_1 = 0$ ● are *linearly degenerate* only if they are straightlined, and ● have a *genuinely nonlinear* character if they do not include straightlined arcs.

6. TWO SIGNIFICANT TWO-DIMENSIONAL SOLUTIONS

For the selfsimilar form of the system (6)

$$\begin{cases} (v_x-\xi)\dfrac{\partial c^2}{\partial \xi}+(v_y-\eta)\dfrac{\partial c^2}{\partial \eta}+(\gamma-1)c^2\left(\dfrac{\partial v_x}{\partial \xi}+\dfrac{\partial v_y}{\partial \eta}\right)=0 \\ \dfrac{\partial c^2}{\partial \xi}+(\gamma-1)(v_x-\xi)\dfrac{\partial v_x}{\partial \xi}+(\gamma-1)(v_y-\eta)\dfrac{\partial v_x}{\partial \eta}=0 \\ \dfrac{\partial c^2}{\partial \eta}+(\gamma-1)(v_x-\xi)\dfrac{\partial v_y}{\partial \xi}+(\gamma-1)(v_y-\eta)\dfrac{\partial v_y}{\partial \eta}=0 \end{cases} \qquad \xi=\frac{x-x_0}{t-t_0},\quad \eta=\frac{y-y_0}{t-t_0},$$

we consider two significant examples of local solutions for which

$$v_x=\mathfrak{a}\,\xi+\mathfrak{b}\eta+\mathfrak{c},\quad v_y=\overline{\mathfrak{a}}\,\xi+\overline{\mathfrak{b}}\,\eta+\overline{\mathfrak{c}};\qquad \text{real constant } \mathfrak{a},\mathfrak{b},\mathfrak{c},\overline{\mathfrak{a}},\overline{\mathfrak{b}},\overline{\mathfrak{c}}.$$

To the selfsimilar form of the system (6) we associate the *selfsimilar Mach number*

$$\widetilde{M}=\frac{1}{c}\sqrt{(v_x-\xi)^2+(v_y-\eta)^2}.$$

A first significant solution:

$$v_x=\frac{1}{\gamma}\xi+\mathfrak{c},\quad v_y=\frac{1}{\gamma}\xi+\overline{\mathfrak{c}};\qquad \text{arbitrary } \mathfrak{c},\overline{\mathfrak{c}},$$

$$c^2=\frac{1}{2}\left[\left(\frac{\gamma-1}{\gamma}\xi-\mathfrak{c}\right)^2+\left(\frac{\gamma-1}{\gamma}\xi-\overline{\mathfrak{c}}\right)^2\right].$$

This solution has a *conical* hodograph (Figure 1).

A second significant solution [for $\frac{3-\gamma}{\gamma+1}<\mathfrak{a}<1$]

$$v_x=\mathfrak{a}\,\xi\pm\eta\sqrt{(1-\mathfrak{a})\left(\mathfrak{a}-\frac{3-\gamma}{\gamma+1}\right)}+K\sqrt{1-\mathfrak{a}},\ K=\frac{\mathfrak{c}}{\sqrt{1-\mathfrak{a}}}=\mp\frac{\overline{\mathfrak{c}}}{\sqrt{\mathfrak{a}-\frac{3-\gamma}{\gamma+1}}},$$

$$v_y=\pm\xi\sqrt{(1-\mathfrak{a})\left(\mathfrak{a}-\frac{3-\gamma}{\gamma+1}\right)}+\eta\left(\frac{4}{\gamma+1}-\mathfrak{a}\right)\mp K\sqrt{\mathfrak{a}-\frac{3-\gamma}{\gamma+1}},$$

$$c=\varepsilon\sqrt{\frac{(3-\gamma)(\gamma-1)}{2(\gamma+1)}}\left(\xi\sqrt{1-\mathfrak{a}}\mp\eta\sqrt{\mathfrak{a}-\frac{3-\gamma}{\gamma+1}}-K\right),\quad \varepsilon=\pm 1.$$

The hodograph of this solution consists in a pair of planes − a [double] *planar* hodograph (Figure 3).

Hodograph characteristic fields: a simple gas dynamic construction

For each of the two mentioned solutions the characteristic directions at each hodograph point result directly by intersecting the solution hodograph with the characteristic cone (7).

6.1. THE CASE OF THE FIRST SIGNIFICANT SOLUTION

For the first significant solution we find, around each point c^*, v_x^*, v_y^* of its conical hodograph, three families of hodograph characteristic fields: two families of *conical helices*

$$c=\frac{\gamma-1}{\sqrt{2}}\exp[-(R_+ + R_-)],\ V_x=\exp[-(R_+ + R_-)]\cos(R_+ - R_-),\ V_y=\exp[-(R_+ + R_-)]\sin(R_+ - R_-)$$

$$V_x = v_x - v_x^*,\quad V_y = v_y - v_y^*;\quad v_x^* = \frac{\gamma}{\gamma-1}\mathfrak{c},\quad v_y^* = \frac{\gamma}{\gamma-1}\bar{\mathfrak{c}}.$$

and a family of *horizontal circles* (Figure 1).

All these families appear to be *genuinely nonlinear* (cf. §4). Figure 1 has to be associated with Figure 2.

- We dispose in this case of *three* genuinely nonlinear systems of coordinates to present the first significant solution − in *three distinct manners!* − as a regular interaction of *multidimensional* simple waves solutions.
- To each manner a pair of Riemann invariants [(4)] contribute:

  $R_+(\xi,\eta)$, $R_-(\xi,\eta)$; $R_+(\xi,\eta)$, $R_0(\xi,\eta)$; $R_-(\xi,\eta)$, $R_0(\xi,\eta)$,

  which result when the solution is compared with its Riemann invariance structure mentioned above ([4]).
- The possibility of *several* Riemann structures appears to be a new fact of the multidimensional approach.
- For the first significant solution we compute $\tilde{M} \equiv \text{constant} = \sqrt{2} > 1$.

6.2. THE CASE OF THE SECOND SIGNIFICANT SOLUTION

For the second significant solution we find again, around each point c^*, v_x^*, v_y^* of its [double] planar hodograph, three families of *straightlined* hodograph characteristic fields (Figure 3): of which two are *non-horizontal* and appear to be *genuinely nonlinear* and the third is *horizontal* and shows a *linearly degenerate* character (cf. §5).

- We dispose in this case of a *single* genuinely nonlinear hodograph system of coordinates to present the solution as a regular interaction of simple waves solutions. For this unique *gnl* hodograph system Figure 3 has to be associated with Figure 2. The corresponding pair of Riemann invariants [$R_+(\xi,\eta)$, $R_-(\xi,\eta)$] results again when the solution is compared with its Riemann invariance structure.

Precisely:

- Let us consider in the hodograph space corresponding to the system (6) (n = 3) the plane

$$c-c^{*}=A(v_x-v^{*}_x)+B(v_y-v^{*}_y) \tag{8}$$

through the point $u^{*}=(c^{*},v_x^{*},v_y^{*})$.

Two families of characteristic straightlines could be drawn in this plane if the intersection of (8) with the circular branch of the cone (7),

$$(c-c^{*})^2=\frac{(\gamma-1)^2}{4}[(v_x-v_x^{*})^2+(v_y-v_y^{*})^2], \tag{9}$$

is real.

The straightlines of these families appear to be the coordinate lines of a characteristic system of coordinates around u^{*}. The reality of the intersection of (8) and (9) is guaranteed by requiring

$$A^2+B^2-\frac{(\gamma-1)^2}{4}>0. \tag{10}$$

- A third family of hodograph characteristics on a plane (8) will result from the intersection of this plane with the *planar* branch of (7). As the hodograph characteristic curves of the third family are horizontal straightlined arcs, these curves have a *linearly degenerate* character.
- A coordinate system R_+, R_- on (8) around the point u^{*} is described by

$$c-c^{*}=\kappa_c^{+}R_+ +\kappa_c^{-}R_-,\quad v_x-v_x^{*}=\kappa_{v_x}^{+}R_+ +\kappa_{v_x}^{-}R_-,\quad v_y-v_y^{*}=\kappa_{v_y}^{+}R_+ +\kappa_{v_y}^{-}R_-$$

where the vectors $\vec{\kappa}^{\pm}$ correspond to intersection of (8) and (9).

We compute

$$(\kappa_c^{\pm},\kappa_{v_x}^{\pm},\kappa_{v_y}^{\pm})=$$

$$\left\{\frac{\gamma-1}{2}\left[A\frac{\gamma-1}{2}\pm B\sqrt{A^2+B^2-\frac{(\gamma-1)^2}{4}}\right],-\left[B^2-\frac{(\gamma-1)^2}{4}\right],\left[AB\pm\frac{\gamma-1}{2}\sqrt{A^2+B^2-\frac{(\gamma-1)^2}{4}}\right]\right\}$$

where

$$A=\varepsilon\sqrt{\frac{\gamma^2-1}{2(3-\gamma)}}\sqrt{1-\mathfrak{a}},\ B=\mp\varepsilon\sqrt{\frac{\gamma^2-1}{2(3-\gamma)}}\sqrt{\mathfrak{a}-\frac{3-\gamma}{\gamma+1}};\quad \varepsilon=\pm1$$

$$v_x^{*}=K\sqrt{1-\mathfrak{a}},\ v_y^{*}=\mp K\sqrt{\mathfrak{a}-\frac{3-\gamma}{\gamma+1}},\ c^{*}=-\varepsilon K\sqrt{\frac{(\gamma-1)(3-\gamma)}{2(\gamma+1)}}.$$

We calculate in (10)

$$A^2+B^2-\frac{(\gamma-1)^2}{4}=\frac{\gamma+1}{3-\gamma}\cdot\frac{(\gamma-1)^2}{4}>0.$$

- For the second significant solution we compute $\tilde{M}\equiv \text{constant}=\frac{2}{\sqrt{3-\gamma}}>1$.

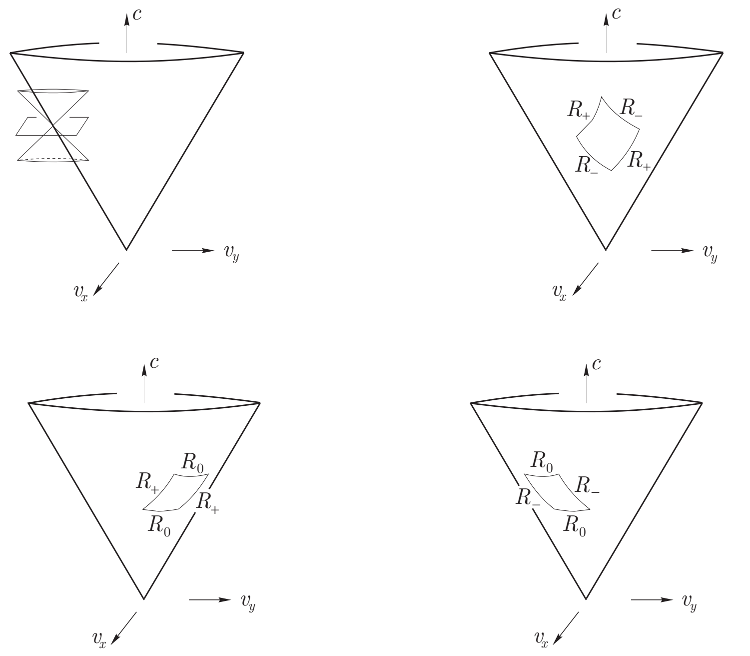

Figure 1 Hodograph details of the first significant solution

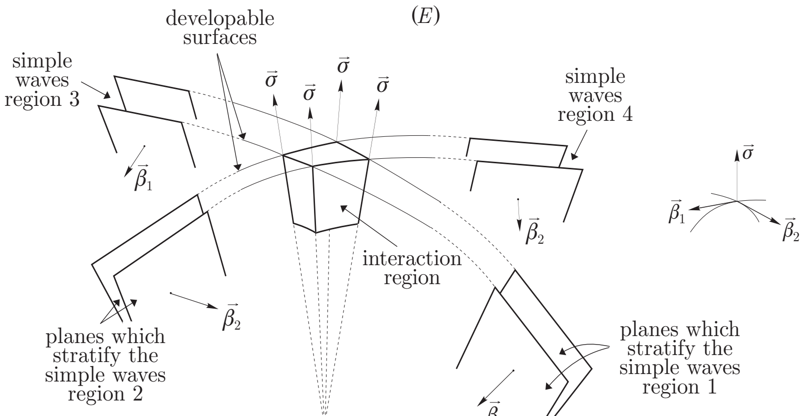

Figure 2 Physical details

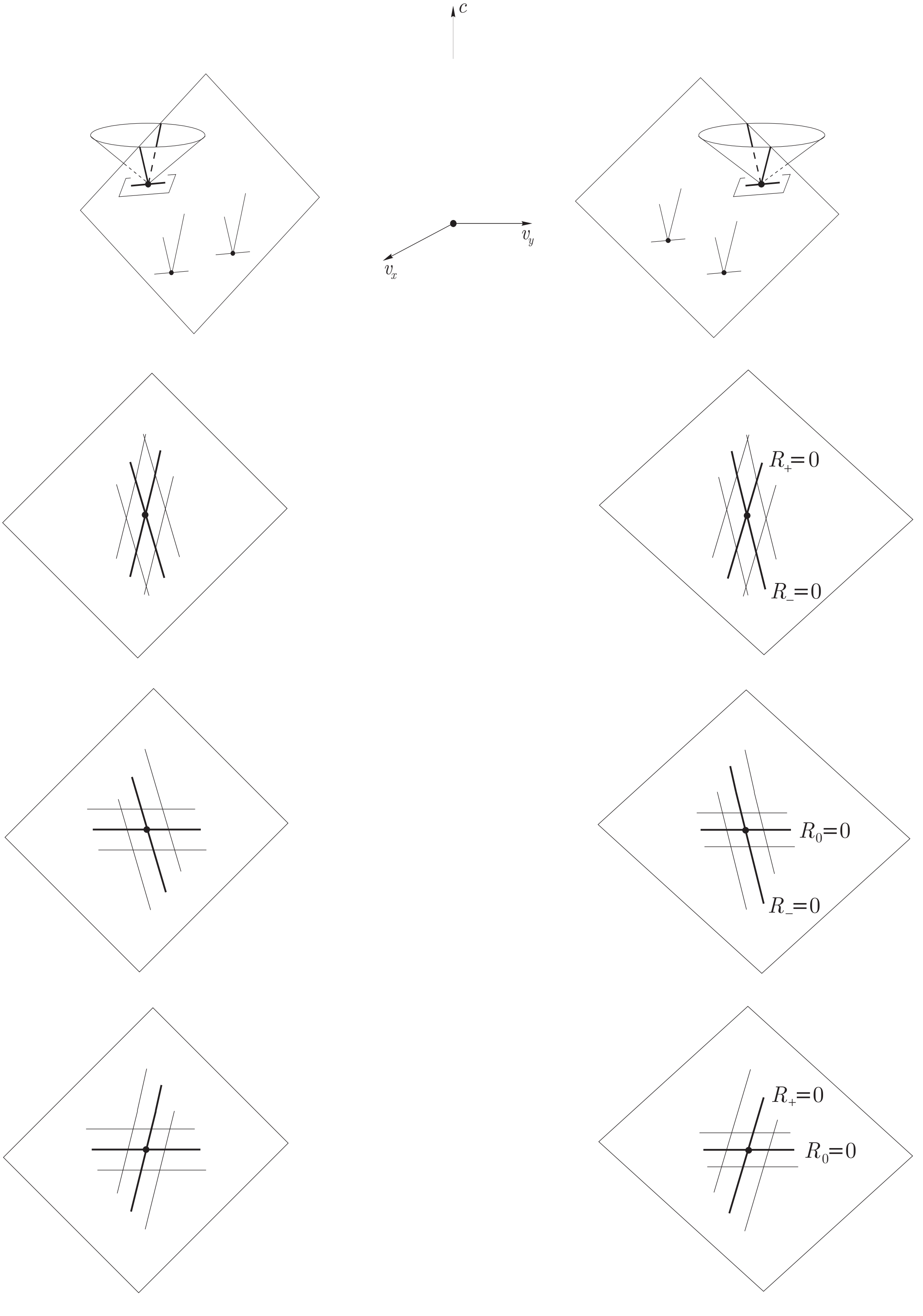

Figure 3 Hodograph details of the second significant solution

7. FINAL REMARKS

- A non-gas dynamic example of these authors is available for which all the hodograph characteristic fields are linearly degenerate [see the circumstance (*c*) in Remark 9, §4]. The *formal* character [as a regular structure] of a wave-wave interaction solution with a *ldg* characteristic hodograph [see the circumstances (*b*) and (*c*) in Remark 9, §4] is described in [3].
- The isentropic wave-wave interactions constructed parallel, from an *analytic, local* and *regular* prospect, some details [interactions of simple waves solutions] of the Zhang and Zheng two-dimensional *qualitative, global* and *irregular* construction.
  - The regular character reflects, in presence of a *gnl* character, a *multidimensional* and *skew* construction generally. In Zhang and Zheng's approach the contributing waves are *one-dimensional* and the interaction structure is *orthogonal* ([11]).
  - The *complicated* character of the Zhang and Zheng irregular interaction is particularly suggested if one describes the variation of the pseudo Mach number $\tilde{M}$ [= the selfsimilar Mach number, §6] by means of some level curves, we call them $\tilde{M}$- curves, in the plane ξ,η. The plane ξ,η appears to be divided into a pseudo subsonic region (for $\tilde{M}<1$) − associated to a convex and compact configuration − and a pseudo supersonic region (a *qualitative proof* of this partition is included in [11]; a *numerical study* of it is considered in [10]).
  - We notice that the two regular wave-wave interactions in the examples here above are both pseudo supersonic [with $\tilde{M}\equiv \text{constant}=\sqrt{2}>1$ and $\tilde{M}\equiv \text{constant}=\frac{2}{\sqrt{3-\gamma}}>1$, respectively].

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