Notes regarding the definition and applicability of supercirculation

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Abstract: *The term supercirculation refers to an active, fluidic circulation control method used for a variety of aeronautical applications. In particular, this paper refers to supercirculation of entrainment wings. Using experimental data as well as some basic analytical equations, a mathematical expression is used to define supercirculation; subsequently an optimization calculation is provided for the minimal blowing pressure ratio needed for achieving supercirculation. Also, based on the proposed definition, a screening equation is determined in order to discern whether or not a certain airfoil has a geometry suited for supercirculation.*

Key Words: *Coanda effect, supercirculation, entrainment airfoil, experimental data*

NOMENCLATURE

h – initial jet thickness
k = adiabatic constant
$\dot{m}$ = mass flow
u_j = blowing jet velocity *m/s*
v_∞ = freestream velocity
t= fitting coefficient
C_L = airfoil lift coefficient
C_μ = momentum coefficient
$C_{\mu\,max}$ = maximal momentum coefficient for the given geometry
L = lift force exherted on the airfoil section
M = molar mass of the gas used *kg/kmol*
R – ramp radius
R_{gas} = 8.314 *kJ/kmolK*
T_{5c} = outlet temperature
ρ_∞ = freestream density
Γ = circulation around the airfoil

1. INTRODUCTION

As a high lift method, supercirculation has been in use since 1927, when professor Ion Stroescu first described a synthetic jet blowing over the top of a conventional airfoil Ref [1-3]. Numerous CFD Ref [4-7] as well as experimental research Ref [8-10] studies have been performed on similar configurations, featuring airfoils with leading or trailing edge blowing in order to increase aerodynamic performance.

Figure 1 presents one of the most common embodiments of the modern day supercirculation airfoil Ref [11].

This supercirculation airfoil is comprised of a symmetrical parabolic leading edge region, a circular trailing edge region of radius R and two thin slots of height h, on the pressure and suction sides of the airfoil, which accelerate the airflow from a pressurized plenum chamber inside the wing.

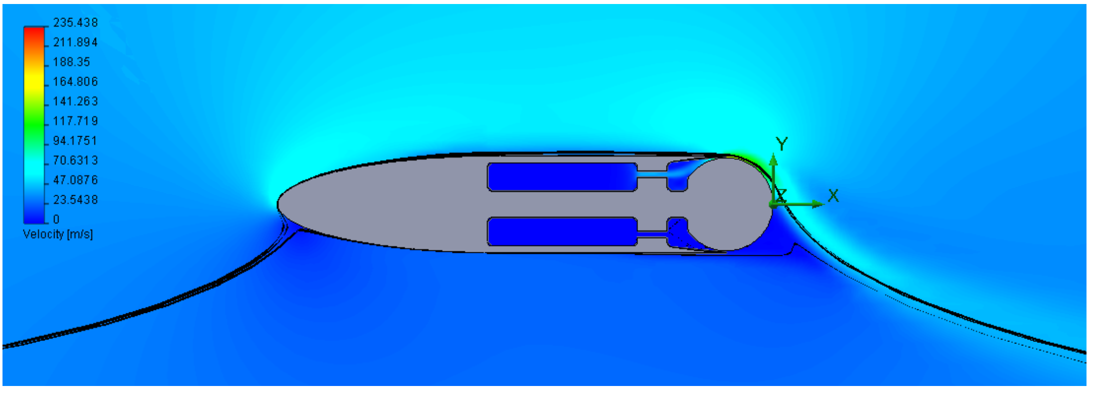

Fig. 1 - Velocity flow field of a CFD simulation based on Economou's geometry.
A similar CFD test has been performed in Ref [12]

The term supercirculation is reminiscent of Kutta-Joukovski theorem which links the value of circulation to lift.

$$L = \rho_\infty v_\infty \Gamma \tag{1}$$

Although counterintuitive, the dominant effect of the thin jet blown from a slit near the trailing edge of the airfoil is to entrain the air on the top of the airfoil – due to the Coanda effect – and thus to increase the value of the circulation Γ. In Figure 1 this effect is easily seen since the otherwise symmetrical airfoil has the frontal stagnation point on its underside rather than in the exact leading edge. It must be said that this is due – to a lesser extent – to the „fluidic trailing edge flap" effect.

Although in the technical literature a distinction is made between circulation control and supercirculation, the latter being a sub-class of the former, an exact physical definition for supercirculation has not yet been given. In their paper, Jones et al [13] make an exhaustive (at the time) presentation of the state of the art of supercirculation, stipulating their preferred threshold for supercirculation. Similar assessments have been presented in Refs [14] and [15], providing cvasi-arbitrary definitions for supercirculation.

2. PROPOSAL FOR MATHEMATICAL DEFFINITION FOR SUPERCIRCULATION

Figure 2 reproduces the original chart plotted by Jones et al in Ref [13]. In his paper, Jones distinguishes between the effects the jet blowing has on the airfoil lift coefficient (of a similar airfoil as the one presented in Fig. 1). Thus, he notes – based on experimental results – that for smaller values of the momentum coefficient C_μ, the effect of the blowing jet is rather to limit the boundary layer separation on the top of the airfoil.

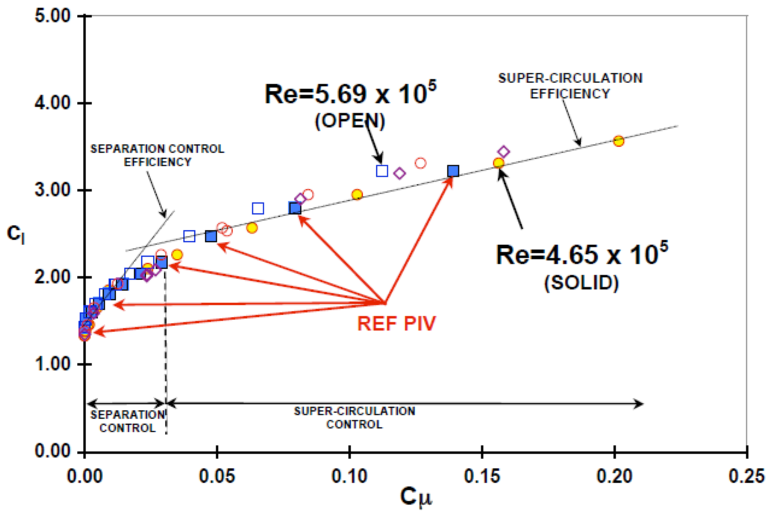

Fig. 2 The limits for separation control and supercirculation as described by Jones et al Ref. [13]

The momentum coefficient is defined as the ratio between the thrust of the fluidic jet (which is considered to be fully expanded) and the product between the lift surface aria and the dynamic pressure of the free stream.

$$C_{\mu} = \frac{u_j \cdot \dot{m}}{S\,\frac{\rho_{\infty} v_{\infty}^2}{2}} \tag{2}$$

In the spirit of the common generic definitions from the cited literature, which are based on the interpretation of the $C_L(C_\mu)$ chart, we begin with a mathematical description of the chart itself.

Thus far, as seen from Fig. 2, the evolution of the lift coefficient versus the momentum coefficient has been described in two regions (boundary layer control and supercirculation), using linear interpolations. Although this is a straight forward approach, it lacks in accuracy and cannot be used for extrapolations, since the second slope will imply that the higher the momentum coefficient the higher the lift – which is only valid up to a point.

A more accurate approach is to use a single mathematical law to describe the entire evolution of the lift coefficient, bearing in mind that the function must have a horizontal asymptote – to keep the lift coefficient from growing to infinity.

By analyzing the chart traced by Jones using the available experimental data, we note that it can be described by an exponential equation of the form:

$$C_{L\,cc} = C_L + (C_{\mu\,\max.cc} - C_L)\left[1 - \exp(-t \cdot C_\mu)\right] \tag{3}$$

where C_L is the lift coefficient without the circulation control, $C_{\mathrm{Lmax.cc}}$ is the maximal value of the lift coefficient attainable by circulation control and t is a positive coefficient.

A first observation can be made, that the above equation admits a horisontal asymptote - corresponding to the maximal value of the lift coefficient with the fluidic jet, $C_{\mathrm{Lmax.cc}}$.

$$\lim_{C_\mu \to \infty} C_{\mathrm{L}} = C_{\mathrm{L\,max.cc}} \tag{4}$$

As seen in papers [13-15], a good interval for considering the lower boundary of supercirculation is ranging between 58% -63% of $C_{Lmax.cc}$ value.

Therefore, we can define supercirculation as *"a fluidic circulation control method which achieves or exceeds 60% of the asymptotical maximal value of the lift coefficient obtainable through fluidic means"*.

A mathematical expression of the above statement can be expressed as

$$0.6 \cdot C_{L\max.cc} = C_L + (C_{\mu\max.cc} - C_L)\left[1 - \exp(-t \cdot C_\mu)\right] \tag{5}$$

Re-writing the expression we obtain,

$$1 - \frac{0.6 \cdot C_{L\max.cc} - C_L}{C_{L\max.cc} - C_L} = e^{-t \cdot C_\mu} \tag{6}$$

and after further processing, we reach the minimal momentum coefficient required for the supercirculation definition:

$$C_{\mu_{S.C.}} = \frac{\ln\left[1 - \dfrac{0.6 \cdot C_{L\max.cc} - C_L}{C_{L\max.cc} - C_L}\right]}{-t} \tag{7}$$

Note that this equation permits the imposing of any arbitrary limit to supercirculation, other than the 60% limit proposed herein (which is based on the recommendations found in the cited literature).Reference [13] can be used for a non-linear regression that fits the above analytical equation to the experimental data synthesized by Jones et al.

Thus, the actual coefficients of the general Eq. (3) are:

$$C_{L\,cc} = 1.483 + 2.2 \cdot \left[1 - \exp(-13.1 \cdot C_\mu)\right] \tag{8}$$

Figures 3 depicts the overlapping scatter plots of the experimental data and Eq. (8), while Fig. 4 presents the extrapolation of Eq. (8), proving that the lift coefficient tend to reach a plateau as the momentum coefficient increases beyound a certain limit.

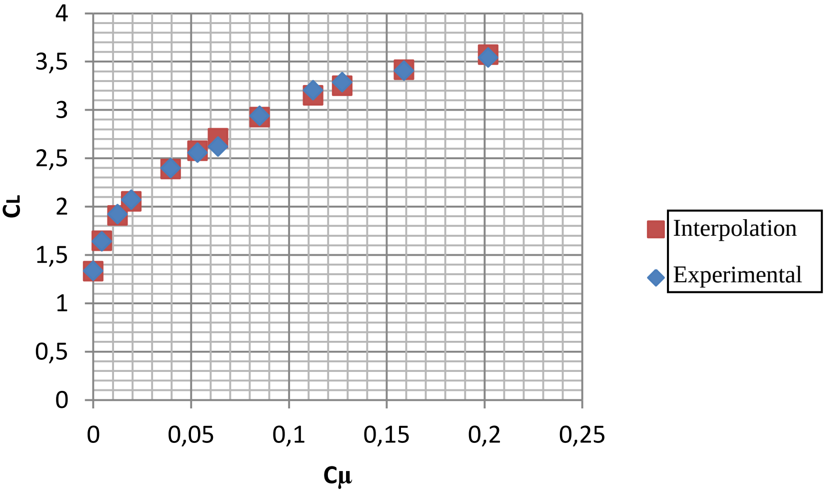

Fig. 3. - The overlapped scatter plots of the experimental results and Eq.(8) which links the lift coefficient C_L to the momentum coefficient C_μ.

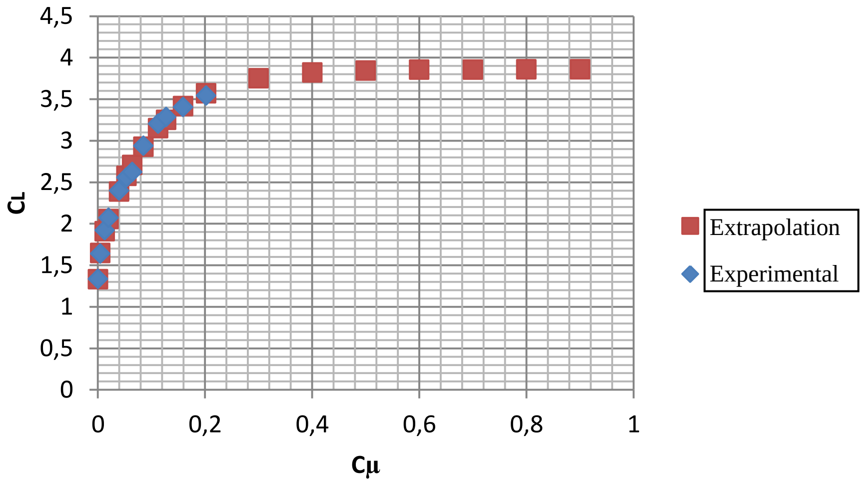

Fig. 4. - The overlapped plots of the experimental results and an extrapolation Eq.(8) which shows the asymtotical increase in the lift coefficient C_L with the momentum coefficient C_μ.

A tempting optimization is to determine at which point the circulation control is most effective. However, expressing this as a ratio,

$$\frac{C_L + (C_{L\max.cc} - C_L)\left[1 - \exp(-t \cdot C_\mu)\right]}{C_\mu} \tag{9}$$

the first derivative yields

$$\frac{\mathrm{d}}{\mathrm{d}C_\mu}\frac{C_L + (C_{L\max.cc} - C_L)\left[1 - \exp(-t \cdot C_\mu)\right]}{C_\mu} = \frac{e^{-t \cdot C_\mu}}{C_{\mu^2}}\left[t \cdot C_L \cdot (1 + C_\mu) + C_{L\max.cc}\left(-k \cdot C_\mu + e^{t \cdot C_\mu - 1}\right)\right] \tag{10}$$

It can be seen that the equation above does not only have no zeros given by positive values of the momentum coefficient but is, in fact, asymptotical.

Therefore an optimal balance between the momentum and the lift coefficient cannot be determined using this formalism.

3. THE MAXIMUM MOMENTUM COEFFICIENT AND MINIMAL PLENUM PRESSURE

Since an efficiency optimization is not permitted by the proposed formalism, another tradeoff can be computed i.e. the minimal blowing pressure for supercirculation - as it was defined in the previous section.

Experimental data provided by Kizilos and Rose [16] and Lowry [17], show that there is a limitation to the blowing pressure - hence outlet velocity- depending on the h/R ratio. This is a sensible conclusion since the jet needs to be attached to the cylindrical ramp in order to entrain the ambient air; this is done mainly through the Coanda effect which is characterized by adverse pressure gradients - this in turn often leads to boundary layer separation.

The semi-empirical equation deduced by Lowry et al. is:

$$\frac{P}{P_{atm}} = \frac{k}{\sqrt[3]{h/R}} \tag{11}$$

considering that the blowing slot acts as a convergent-divergent nozzle, the maximal blowing velocity can easily be calculated

$$u_j = \sqrt{\frac{T_{5c} R_{gas}}{M} \cdot \frac{2k}{k-1} \cdot \left[1 - \left(\frac{P}{P_{atm}}\right)^{\frac{k-1}{k}}\right]} \tag{12}$$

therefore

$$u_{j\,\max} = \sqrt{\frac{T_{5c} R_{gas}}{M} \cdot \frac{2k}{k-1} \cdot \left[1 - \left(\frac{k}{\sqrt[3]{h/R}}\right)^{\frac{k-1}{k}}\right]} \tag{13}$$

Since the definition of the momentum coefficient is Eq. (2), we write the expression for the maximal momentum coefficient allowed by the airfoil geometry and the nature of the gas used (generally air).

$$C_{\mu\,\max} = \frac{\dot{m}\sqrt{\frac{T_{5c} R_{gas}}{M} \cdot \frac{2k}{k-1} \cdot \left[1 - \left(\frac{k}{\sqrt[3]{h/R}}\right)^{\frac{k-1}{k}}\right]}}{S \cdot \frac{\rho_\infty v_\infty^2}{2}} \tag{14}$$

For the following demonstration it is implicit that the geometry satisfies the condition:

$$\frac{h}{R} = \left\{ k \left\{ 1 - \left[S \cdot \frac{\rho_\infty v_\infty^2}{2} \cdot \frac{\ln\left[1 - \frac{0.6 \cdot C_{L\max.cc} - C_L}{C_{L\max.cc} - C_L}\right]}{-k} \cdot \frac{1}{\dot{m}} \right]^2 \frac{M}{T_{5c} R_{gas}} \cdot \frac{k-1}{2k} \right\}^{\frac{k-1}{k}} \right\}^3 \tag{15}$$

Equation (15) represents the geometric requirement for the airfoil in order for the supercirculation to be available as an active control method.

Assuming that the maximal momentum coefficient, $C_{\mu\,\max}$, is high enough (i.e. that supercirculation is attainable for the geometric configuration tested), the following reasoning applies:

$$\frac{\ln\left[1-\dfrac{0.6\cdot C_{L\max.cc}-C_L}{C_{L\max.cc}-C_L}\right]}{-k}=\frac{\dot{m}\sqrt{\dfrac{T_{5c}R_{gas}}{M}\cdot\dfrac{2k}{k-1}\cdot\left[1-\left(\dfrac{P_{\min}}{P_{atm}}\right)^{\frac{k-1}{k}}\right]}}{S\cdot\dfrac{\rho_\infty v_\infty^2}{2}} \tag{16}$$

this leads to the minimal blowing pressure ratio

$$\frac{P_{\min}}{P_{atm}}=\left\{1-\left[S\cdot\frac{\rho_\infty v_\infty^2}{2}\cdot\frac{\ln\left[1-\dfrac{0.6\cdot C_{L\max.cc}-C_L}{C_{L\max.cc}-C_L}\right]}{-k}\cdot\frac{1}{\dot{m}}\right]^2\frac{M}{T_{5c}R_{gas}}\cdot\frac{k-1}{2k}\right\}^{\frac{k-1}{k}} \tag{17}$$

This corresponds to a minimal energy expense required to reach supercirculation for a given airfoil. Note that the above equation does not include the geometric parameters of the airfoil. Other methods can be employed to determine the maximum pressure ratio for blowing, including the use of a more accurate analytical description of turbulent boundary layers subject to the Coanda effect Ref [18], [19] or laminar boundary layer, if applicable, Ref [20, 21]. It must be stated that the calculations presented herein must be also validated by either experimental or computational fluid dynamics simulations for each individual case.

4. CONCLUSIONS

The paper addresses the problem of defining in a mathematical, standardized, way the concept of supercirculation as part of the active circulation control methods used by entrainment wings. Based on the existing literature, the inferred limit for which the trailing edge blowing circulation control method becomes supercirculation was used, in conjunction with the proposed mathematical description Eq.(3), for determining a minimal momentum coefficient for supercirculation Eq.(7).

Due to its form, Eq. (3) can accurately describe the entire evolution of the lift coefficient as a function of the momentum coefficient. Also, it is constructed in such a way that for high values of the momentum coefficient, the lift coefficient becomes asymptotical; therefore the extrapolation of Eq. (3) does not infringe the physical behavior of the airfoil by limiting the maximal value of the circulation control lift coefficient.

Section 2 presents two important aspects of entrainment supercirculation airfoils, the first being the minimal pressure ratio required in the plenum chamber. Since pressurization requires energy consumption, a minimization of the pressure can be considered an optimization of the configuration. The second aspect treated in section 2 is the geometrical requirements of a given entrainment airfoil (i.e. an airfoil with a trailing edge blowing slot over a cylindrical ramp) in order for it to reach supercirculation.

Further improvement of the proposed definition, optimization and criterion can be made by accumulating more experimental data and by employing more advanced methods, especially in determining the maximal pressure ratio in the plenum chamber.

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