Development of a laminar boundary layer model for curved wall jets

Valeriu DRĂGAN*

*Corresponding author
*COMOTI – Romanian Gas Turbine Research and Development Institute
220 D Iuliu Maniu Avenue, 061126 Bucharest 6, Romania, P.O. 76, P.O.B. 174
drvaleriu@gmail.com

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Abstract: *The paper addresses the issue of thin jets subjected to the Coandă effect and in particular the boundary layer modeling. An existing semi-empirical Coandă effect mathematical model is modified, with a more complex boundary layer model, in order to allow the estimative calculation of the detachment point and of other parameters such as friction coefficients, wall shear stress and the momentum and displacement integral thicknesses. The method used is analytical, based on the Rodman-Wood-Roberts model and the Pohlhausen boundary layer method. The work is significant for the pre-design calculations as well as for a quick checking of RANS CFD simulation results.*

Key Words:*Coandă effect, boundary layer, Pohlhausen method, SST RC, SA RC*

NOMENCLATURE

C – curvilinear distance from the blowing slot along the ramp
C_f – aerodynamic friction coefficient
h – initial jet thickness
k – turbulent kinetic energy
n – exponent for Rodman-Wood-Roberts model
R – ramp radius
u_j – initial jet velocity
u_m – maximal local jet velocity
$y_{1/2}$ – wall distance at which the local velocity equals half of local maximum velocity
y_m – local boundary layer thickness
δ^* – displacement thickness of the boundary layer
θ^* – momentum thickness of the boundary layer
τ_w – wall shear stress
ω – turbulent dissipation
SST – Shear Stress Transport
RC – Rotation and Curvature compensation

1. INTRODUCTION

As discussed in Ref [1-3], the current computational fluid dynamics methods have often proven to be inexact. Even the purpose tailored RANS models such as the Spalart-Almaras with Rotation and Curvature correction Ref [4] or the Menter SST RC Ref [5,6] were shown to have only limited success in describing the radial velocity distribution for thin Coandă jets [7,8]. Figure 1 shows a typical thin Coandă jet while Figures 2 and 3 reproduce data provided by Refs 7 and 8 for the comparison between current state of the art RANS methods and experimental research of Wygnanski Ref [9].

Existing semi-empirical models Saeed Ref [10], Rodman-Wood-Roberts Ref [11], CEVA Ref [12], Lewinsky-Yeh Ref [13] typically provide only basic boundary layer mathematical description. However, due to the way the majority of the semi-empirical curved wall jet models are expressed, more elaborate boundary layer models may be employed. It is therefore useful to implement a more complete boundary layer description which allows the computation of more elaborate parameters such as friction coefficient distribution and boundary layer separation.

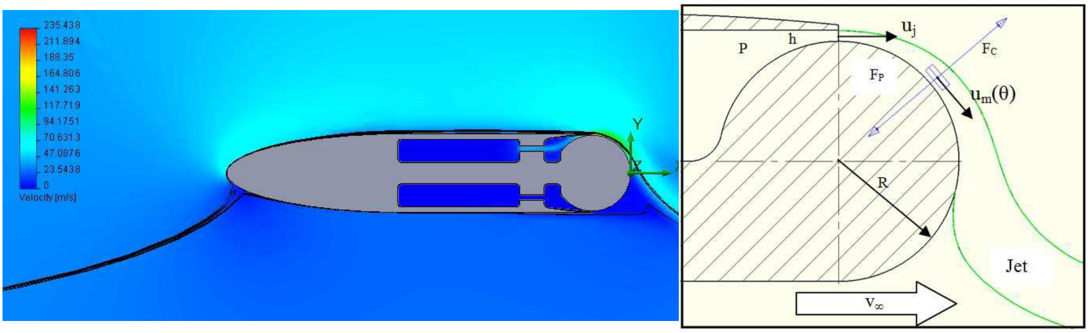

Fig. 1 – Trailing edge detail of a classical symmetrical entrainment wing cross section

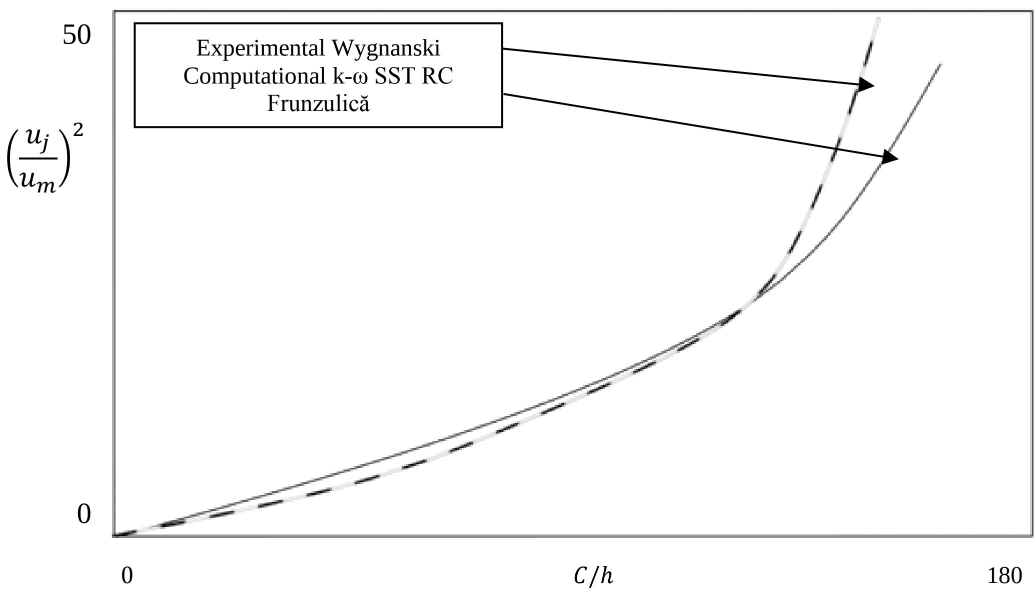

Fig 2 – Comparison between the computational results obtained by Ref [8] and the experimental results of Ref [9], boundary layer thickness y_m vs ramp circumference

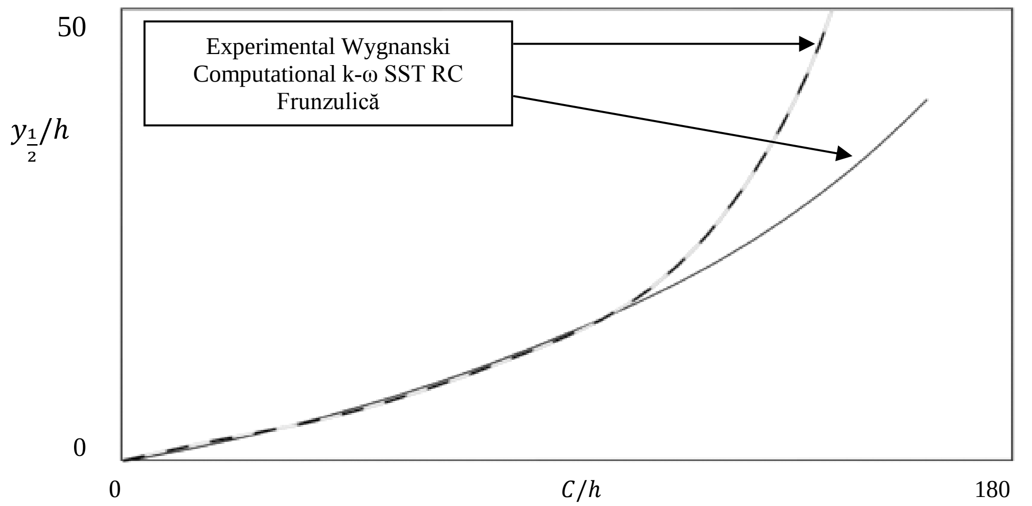

Fig 3 – Comparison between the computational results obtained by Ref [8] and the experimental results of Ref [9], maximum local velocity u_m vs ramp circumference

2. THE BOUNDARY LAYER MODELLING

One of the main goals of modeling Coandă effect jets is determining a point for boundary layer separation. This has proven to be problematic due to the surface curvature that the flow must adhere to. For the purposes of this paper, the Rodman-Wood-Roberts (RWR) model will be used since it is both accurate within a reasonable jet thickness range of up to 3-4% and simple enough in its mathematical expression.

A similar semi-empirical model, which extends the validity of the model to higher jet to radius thickness ratios - up to 10% is described in Ref [12].

The formulation is, however, considered too complicated for the purposes of this paper.

Due to the fact that, in its original form, the Rodman-Wood-Roberts model uses a boundary layer with the law

$$\frac{u}{u_m} = 2\left(\frac{y}{y_m}\right)^{\frac{1}{n}} - \left(\frac{y}{y_m}\right)^{\frac{2}{n}} \quad , \ y < y_m \tag{1}$$

the derivative parameters such as the wall shear stress

$$\tau_w = \mu \left(\frac{\partial u}{\partial y}\right) |_{y=0} \ , \tag{2}$$

$$C_f = \frac{2 \cdot \tau_w}{\rho \cdot u_m^2} \ , \tag{3}$$

cannot be accurately computed, not even through numerical derivation.

Also, the integral boundary layer thicknesses cannot be used since, in this case, it yield inaccurate results

$$H = \frac{\delta^*}{\theta^*} \tag{4}$$

displacement thickness

$$\delta^* = \int_0^{y_m} \left(1 - \frac{u}{u_m}\right) dy \tag{5}$$

momentum thickness

$$\theta^* = \int_0^{y_m} \frac{u}{u_m}\left(1 - \frac{u}{u_m}\right) dy \tag{6}$$

In order to eliminate this problem, another boundary layer method must be employed.

Since the flow boundary layer is assumed to be laminar, the Pohlhausen method Ref [14] may be used since it is both complex enough and relatively easy to compute.

In its theoretical form, the Pohlhausen boundary layer type can be expressed through a 4 degree polynomial equation

$$\frac{u}{u_m} = \left(2 + \frac{\Lambda}{6}\right)\frac{y}{y_m} + \frac{\Lambda}{2}\left(\frac{y}{y_m}\right)^2 + \left(\frac{\Lambda}{6} - 2\right)\left(\frac{y}{y_m}\right)^3 + \left(1 - \frac{\Lambda}{6}\right)\left(\frac{y}{y_m}\right)^4 \tag{7}$$

where the Pohlhausen coefficient has the expression

$$\Lambda = \frac{y_m^2}{\nu}\frac{du_m}{dC}. \tag{8}$$

After the analytical computation for this case, we obtain

$$\Lambda = \frac{1}{\nu}\left\{6.26 \cdot 10^{-3} \cdot R^2 \left[\exp\left(0.49 \cdot \frac{C}{R}\right) - 1\right]^2 \cdot \frac{u_j}{h}\left[-2.695 \cdot \left(\frac{C}{h} + 9.6\right)^{-1.55}\right]\right\} \tag{9}$$

One of the immediate advantages of this method is that, by definition, the boundary layer detachment can be calculated with the condition

$$\frac{1}{\nu}\left\{6.26 \cdot 10^{-3} \cdot R^2 \left[\exp\left(0.49 \cdot \frac{C}{R}\right) - 1\right]^2 \cdot \frac{u_j}{h}\left[-2.695 \cdot \left(\frac{C}{h} + 9.6\right)^{-1.55}\right]\right\} = -12 \tag{10}$$

furthermore, the integral thicknesses and shape factor can then be derived as follows;
the displacement thickness

$$\delta^* = y_m\left(0{,}3 - \frac{\Lambda}{120}\right) \tag{11}$$

the impulse thickness

$$\theta^* = y_m \frac{1}{315}\left(37 - \frac{\Lambda}{3} - \frac{5\Lambda^2}{144}\right) \tag{12}$$

the shape factor

$$H = \frac{0.3 - \Lambda/120}{0.12 - \frac{\Lambda}{945} - \frac{\Lambda^2}{9072}} \tag{13}$$

Perhaps the most important difference between the proposed method and the original RWR model is the ability to calculate the wall shear stress

$$\tau_w = \mu \frac{u_m}{y_m}\left(2 + \frac{\Lambda}{6}\right) \tag{14}$$

and the aerodynamic friction coefficient

$$C_f = \frac{2\upsilon}{u_m y_m}\left(2 + \frac{\Lambda}{6}\right) \tag{15}$$

Because the equation which gives the local value of the friction coefficient in the Pohlhausen model cannot be applied for the detachment point Ec. (10) confirmation – as this is a mathematical implication of the model – we must resort to an alternative empirical equation Ref [15].

$$C_f = 0{,}058 Re_{\theta^*}^{-0{,}0268}(0{,}93 - 1{,}95\log_{10} H)^{1{,}705} \tag{16}$$

The detachment point is obtained by imposing the shape factor H=3.

3. CONCLUSIONS

The paper provides a development of the Rodman-Wood-Roberts model for the Coandă effect, through the mathematical re-modeling of the boundary layer. The initial assumptions of the RWR model are maintained (low jet thickness to curvature ratio, low Reynolds number, laminar flow), thus allowing the application of the Pohlhausen method which is more complete than the original 1/n boundary layer method.

The development presented herein allows the calculation of the flow detachment point, the friction coefficient and the other parameters of interest for flow analysis - the momentum and displacement integral thicknesses and the wall shear stress.

Further work may include the application of the same boundary layer model for the CEVA model which extends the maximum thickness ratio h/R to 10%.

Also, more elaborate models for laminar boundary layers, presented in Refs [16-17] could be employed in order to increase the accuracy of the method.

The work is significant for quick calculations involving the Coandã effect, such as super circulation airfoils with trailing edge entrainment Ref [18-19], thrust augmenting ejectors Ref [20] or other fluidic control devices [21-22]

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