Random Turbulence Versus Structured Turbulence

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Abstract: *The turbulence flows involve more or less the intermittent behavior of motions in two aspects. The first one is the so-called external intermittency associated with what is called partially turbulence flows, especially with the strongly irregular and convoluted structure (little understood) and random movement of the boundary between the turbulent and non-turbulent fluid. The second aspect is the so-called scale (small wave numbers) inertial or intrinsic intermittency and associated with spotty temporal ("bursting phenomenon") and spatial patterns of the small scale structures at the fluid – solid interface. This structured turbulence of molecular thermal nature is called Lagrangian turbulence, in which a volume-preserving flow has flowlines which fill up regions of space ergodically. The origin and the self – sustaining mechanism of the structured turbulence are described in the sequel by means of the starting impact inducing three self-sustaining wave configurations which further are propagated in the flow field as vorticity wave packets of soliton type. The result of the whole process is easily observed in the form of hairpin vortex packets so-called Ω – shaped coherent vortical structures, but is extremely difficult to interpret understand and explain. In this paper it is shown that hairpin vortex generation mechanism is governed by principle of "Ω" momentum – energy invariance for elastic collisions constrained to slide frictionless on the contact surface.*

Key Words: *Laminar-Turbulent Transition, Shear Turbulence, Coherent Hydrodynamic Structures, Vortex Dynamics.*

1. INTRODUCTION

When the Reynolds is increased, the flows of real fluids differ from the quiet smooth flows known as laminar flows, and their opposite, either internal flows or boundary layers adjacent to solid surfaces undergo a spectacular transition process from the laminar to the turbulent regime. In the turbulent flows the vorticity, pressure, temperature and other fluid mechanical quantities fluctuate in an observable disordered-manner with extremely sharp and irregular space and time variations [1]. The observation that the orderly pattern of flow ceases to exist at higher Reynolds numbers, and that the flow through a pipe becomes turbulent was firstly shown by O. Reynolds [2]. Unlike other complicated phenomena, turbulence is easily observed, but is extremely difficult to interpret, understand, explain and non in the last place to simulate [3]. Thus, the unknown origin of turbulence gave rise to a whole phenomenology of turbulence including most of semi-empirical approaches and turbulence modeling based on experimentation and painful efforts of its interpretation. During the last decades, research in turbulence has been marked by an increasing involvement of physicists and mathematicians, but without much success concerning the nature of turbulence: the origin and details of the mechanism of turbulence production and sustainment. Since the turbulence dynamics is a rapid process originated at the fluid-solid boundary interface ($y \to 0$) laying in packets of vorticity/shear waves caused by the onset of motion (i.e. the wave packet is the consequence of the starting impact [4]), the classical approaches based on the Navier-Stokes equations (N-S equations for short) with Stokes ($\lambda = -\frac{2}{3}\mu$) and Prandtl (constant viscosity μ) hypotheses could statistically describe (URANS, LES computations) only small-scale slow phenomena "en mass" (on mixed modes with large wave numbers) in outer-layer, produced by shear waves (on distinct modes with small wave numbers), expanding from the zero-thickness inner layer ($y \to 0$) at the wall. The true problem of turbulence dynamics generated by high frequency wall-vorticity waves, requests a new formulation of N-S equations near the wall, for taking into account the effects of wall-strained flow during the starting impact: the concentrated boundary vorticity (CVB) and the non-linear behavior of fluid (the thixotropic fluid hypothesis, $\nu(Re_l)$) [4]. However, since such a formulation is not available at this time, the whole flow field is described by the soliton solutions derived from similar solutions (localizing boundary singularities) associated with a mutual induction function, CBV – ν (Re_l), acting as a substitute for the Stokes's hypothesis which annihilates the turbulent solutions; *i.e. valid Stokes's hypothesis, there is no solution to turbulence*. The soliton solutions exhibit both average flow field and wavy flow pattern frozen at a given instant and transported by the main motion as a whole [5]. Compared with the previous approaches, the approximation of the rotor-translation motion/flow in the nearest wall region together with the dual concept of CBV – thixotropic fluid/non-linear viscous fluid is a more realistic prototype of various compressing-caused vortical waves in the wall-bounded turbulent shear flows, known as the soliton-like coherent structures (SCS) [6]. Moreover, this approximation explains the self-sustaining mechanism of turbulence by a local coupled longitudinal-transverse wave system, as well as the gross dynamic balance between the rotatory energy (wall torsion pressure) and the translational energy (dynamic pressure of free motion); *i.e. no valid law of equal action and reaction, no turbulence exists*.

2. THE PHYSICAL NATURE OF VORTICITY AT A SOLID BOUNDARY

The vorticity is a kinematical quantity and the equation governing its evolution, known as Helmholtz's vorticity transport equation, is derived from the N-S equation by a purely mathematical operation, so that this contains the same restriction as the original N-S equation, based indirectly on the concept of the point material (see §4) excluding any inertial rotatory effect. The most primary derived fields that describe the local spatial variation of a velocity field **u** are its divergence, a scalar field called dilatation and its curl, an axial vector called vorticity,

$$\theta \equiv \nabla \boldsymbol{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}, \tag{1a}$$

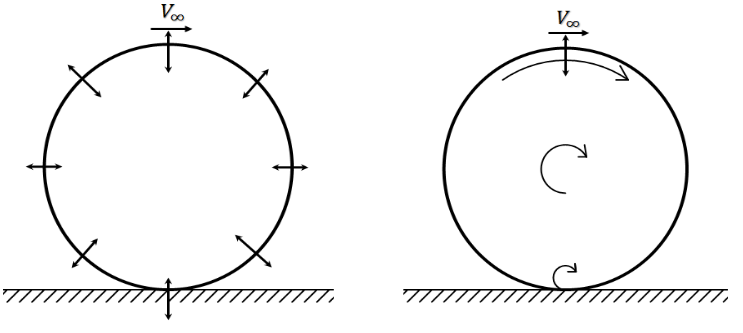
$$\boldsymbol{\omega} \equiv \nabla \times \boldsymbol{u} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right). \tag{1b}$$

The dilatation, θ, measures the isotropic expansion or compression of the fluid, while the vorticity $\boldsymbol{\omega}$ measures the rotation of fluid particles. From the generalized Gauss theorem, relations

$$\theta = \lim_{V \to 0} \frac{1}{V} \int_{\partial V} \boldsymbol{n} \cdot \boldsymbol{u} dS, \tag{2a}$$

$$\boldsymbol{\omega} = \lim_{V \to 0} \frac{1}{V} \int_{\partial V} \boldsymbol{n} \times \boldsymbol{u} dS, \tag{2b}$$

show that their net contributions are solely from the normal and tangential components of $\boldsymbol{u}$ on the boundary, as sketched in Fig. 1 with V being a small sphere.



a b

Figure 1: The velocity field associated with: a) dilatation - compressing process and b) vorticity - shearing process at the wall

Thus, the dilatation represents an isotropic compressing process, while the vorticity is a non-isotropic shearing process. The curling-up property of velocity can be associated with the vorticity by means of the circulation along a closed loop C (Stokes theorem)

$$\Gamma_C = \int_C \boldsymbol{u} \cdot d\boldsymbol{x} = \int_S \boldsymbol{\omega} \cdot \boldsymbol{n} dS, \tag{3}$$

where S is in any directional surface bounded by C.

The circulation as a mathematical entity, without any dynamic implication has no experimental relevance for strong perturbations, i.e. for turbulence. The effort to explain vortical flows as diversely as possible, by means of coupling of compressing and shearing processes into a unitary approach [7], was not very successful: vorticity-creation from the wall. The shear turbulence processes remain further unknown as long as the fluid is assumed ideal or Newtonian; *i.e. no-elasticity shear in fluid, no turbulence exists*.

Historically, the viscosity property of fluid was used by Stokes [8] in the famous Navier-Stokes equation (NSE) as an intrinsic relation between the first and second viscosities ($3\lambda + \mu = 0$), for reducing the number of properties which characterizes the field of stresses in a flowing compressible fluid. The ignorance of the physical interpretation of the Stokes's hypothesis has led to the much disputed problem concerning the NSE solutions [9]. After half a century, the practical importance of NSE was proved for solving d'Alembert's paradox (D – drag crisis) by means of another famous mathematical development, Prandtl's boundary layer theory [10]. But, in spite of these heaviest and the most ambitious armory from theoretical physics and mathematics, the solution of the full time-dependent 3D NSE for turbulent flows is far (or impossible) from being found. The big mathematical problem of turbulence remains unsolved as a new challenge in Fluid Dynamics, T – turbulence paradox. Indeed, the approach of turbulence via the NSE with the Stokes approximation disregards the detailed wall-bounded flow structure at the starting moment ($y = 0, t \to 0$), where the initial and boundary conditions, $\boldsymbol{u}(0,t) = 0$ (no slip condition) and $\nu \equiv \mu/\rho = \nu_0$ (equilibrium value) corresponds exactly to one flow state called Blasius flow. The Stokes relation associated with the incompressible flow assumption (ρ = const.) obscure the easiest compressibility effects occurred during the short starting time ($t \to 0$) at the solid boundary, *acting like a fluid-solid collision called the starting impact* [4].

After impact, any flowing incompressible flow has a more or less shear non-constant/elastic viscosity at solid boundaries, and whereby the shearing is also a universal process causing transverse waves that expands in the boundary-layer flow; when the Reynolds exceeds a critical value, the shearing process becomes a self-sustaining one, generating vortices. The essence of turbulence is the self-sustaining shear waves created at solid boundary on three eigen modes with small wave numbers; the dispersion mode produces permanently contra-rotating vortex pairs/dipoles transported by the boundary-layer flow (see §4). The clear conclusion is that the scenarios usually based on linear theory (NSE) are too simple to describe the whole dynamic shearing process, and they must be reformulated based on a more realistic physical prototype that is able to embed weakly compressing microstructure elements.

3. THE DYNAMIC MODEL OF STARTING IMPACT-ROTOR-TRANSLATION MOTION AT WALL

Among various wall-bounded flows at large Reynolds numbers, the primary observable structure is thin boundary layers generated by and adjacent to solid surfaces. In these vortical flows generated by a moving body, the formation and evolution of boundary layer are closely related to the vorticity-creation process at a solid surface during the onset of motion.

The motion at impact ($t \to 0$). Even if a flow is incompressible, its starting produces weak compressibility effects with important consequences concerning the motion following start up. The flow of a gas can be considered incompressible when the relative change in density remains very small, $\Delta\rho/\rho_0 << 1/2\left(\frac{V_\infty}{c}\right)^2 = 1/2M_\infty^2$, and in the case of air usually a value of $M_\infty = \sqrt{0.1} \approx 1/3$ or 100 m/s can be considered as an incompressibility limit.

However, for $V_\infty = 100$ m/s and $l = 1\ m\ \ Re_l = 2/3 \cdot 10^7$, that is the flow is fully turbulent where there is the possibility that in starting condition (t = 0) the easy/early compressibility effects cause the oscillating behavior of viscosity and associated with an inherent fluctuating velocity field, near solid surfaces, produce a self-sustaining non-stationary flow termed generic shear turbulence [4].

For the description of the self-sustaining mechanism of turbulence, the concept of concentrated boundary vorticity (CBV) and the hypothesis of thixotropic/visco-elastic fluid will be introduced related to the starting event. Firstly, we define the elastic force of the thixotropic fluid by the maximum oscillating frequency of a virtual impulsively pure shear straining, with the acoustic velocity c and constant density ρ_0, as

$$c^2 = 3/2r_s^2\Omega_s^2 = 3/2(r_s^2\Omega_s)\Omega_s. \tag{4}$$

where $r_s^2\Omega_s$ is the angular momentum of a homogeneous small sphere.

For $r_s^2 \Omega_s = 1\mathrm{m}^2/s,\ \Omega_s = 2/3 \cdot 10^5 s^{-1}$ plays the role of a natural frequency of the thixotropic fluid, while for $V_\infty l = 1$its inverse $\Omega_s^{-1} = \nu_0 = 1.5 \cdot 10^{-5} m^2/s$is the equilibrium value of the kinematic viscosity, an outcome consistent with measured data. Thus, the critical Reynolds number $Re_c = \frac{1}{\nu_0} = \frac{2}{3} \cdot 10^5$ is well-defined as the maximum fluidity of thixotropic fluid; i.e. $R\,e_c$ is the boundary between the viscous behavior with viscosity adjusting itself continuously with the outer flow ($V_\infty l$) (reactive medium at low frequencies for $Re_l \leq R\,e_c$) and the elastic behavior with elastic response free of outer flow ($V_\infty l$) (dispersive medium at high frequencies for $Re_l > R\,e_c$).

Figure 2 shows the dependence of shear elastic viscosity due to the early compressibility effects in wall-bounded flows, where the crests of the longitudinal compressing/expanding process propagate with the group velocity $M_g = 2/3$.

At the same time, The Reynolds number has a more general interpretation of the classical parameter

$$Re_l \equiv \frac{V_\infty l}{\nu_0} = \frac{V_\infty^2 \nu_0^{-1}}{V_\infty / l} = \frac{\text{frequency of wall-bounded flow}}{\text{frequency of outer flow}}. \tag{5}$$

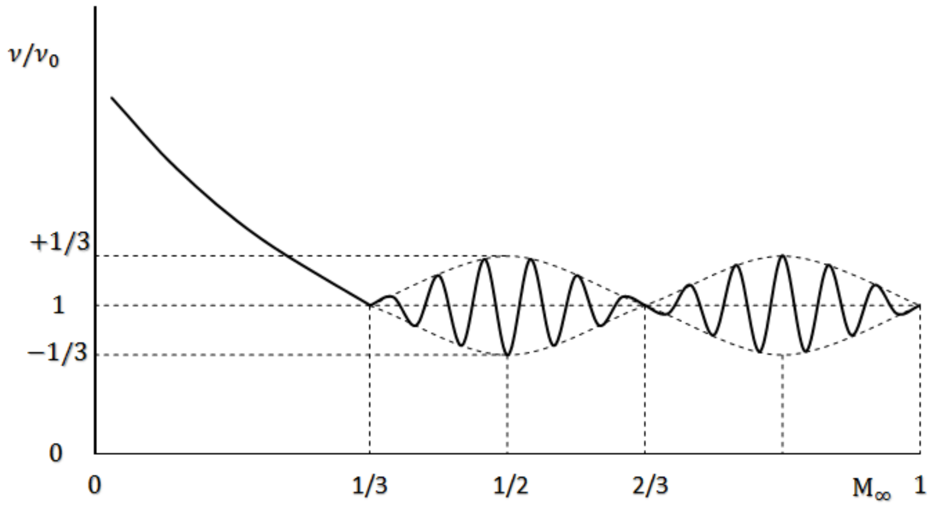


Figure 2: Elasticity effect in thixotropic fluid (longitudinal compressing/expanding process)

as a stability standard/control parameter of the flow state, the flow is stable for $Re_l \leq R\,e_c$ and unstable for $Re_l > R\,e_c$, and the Reynolds number is also a current reduced frequency for all real fluid motions. The universal finding concerning the viscoelastic nature of fluid in shear layer adjacent to solid surfaces is that the existence of shear elasticity in fluid is closely related to the enhanced fluidity of turbulent flows.

In contrast with the thixotropic fluid model, the previous concepts of ideal and viscous Newtonian fluid are less malleable and fall for describing turbulent flows; *no-elastic shear viscosity, no-turbulence exists*. Any fluid has more or less an elastic shear viscosity as a universal internal/intrinsic property, i.e. an externalizing form of internal/molecular thermal energy. Then, we define the start-up by the impulsive phase change from the natural translational motion to a pure shear straining rotational motion (i.e. in the limit of vanishing contacting time) with no-loss of mass, energy and momentum, as sketched in Fig. 3.

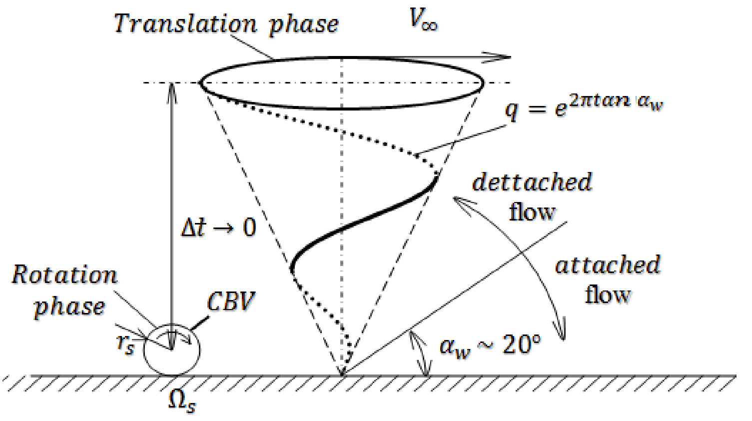


Figure 3: The impulsive phase change during the start-up with equal-energy partition condition (transverse shearing process)

Assuming that in such a fast shearing-compressing process the flow follows a logarithmic spiral $q_w = e^{2k\pi}$ $(k = tan\alpha_w)$ and the flow also remains attached to the solid surface only if $\alpha_w \leq 20^0$, or $q = e^2$, then the parameter of spiral q_w is a measure of the maximum torsion degree of a wall-skewed flow. This outcome physically defines the concept of concentrated boundary vorticity CBV. The CBV concept is consistent with the exact Stuart's solutions e^τ, $\tau \in [2-(-2)]$, found for a stable inviscid mixing layer in the form of a succession of point vortices [11]. Therefore, the onset of any motion of a continuous medium (solid and/or fluid) is a particular process of momentum exchange between colliding bodies within a short time of contact, with zero mass flux and energy preserving called starting collision/impact. During the starting impact, the initial velocity distribution is rapidly changed and the result of the coupling of some compressing and sheering processes is the creation of concentrated vorticity balls at solid surfaces as

$$\lim_{V(t)\to 0} \frac{1}{V}\int_{\partial V} \theta \boldsymbol{\omega} \cdot \boldsymbol{n}_s\, dS \equiv \frac{p_{w,torsion}}{\rho V_\infty^2} = \frac{e^\tau \nu}{V_\infty^2}, \tag{6}$$

where e is the CBV, ν is the kinematic shear viscosity of the thixotropic fluid adjusting itself mutually according to the stress state of flow, $\tau \in [2-(-2)]$ is the concentration/torsion index showing the stress degree of flow, and $p_{w,torsion}$ is the wall-perturbed pressure, generally greater than the pressure of free flow, termed torsion pressure.

The essence of starting impact is: (1) the creation of concentrated structures, like the CBV, through a fast compressing-shearing rotation process, accumulating internal/intrinsic energy in the form of inertial rotation potential; (2) the partition of post-impact motion in the proportion of 1/3 pure straining rotation and 2/3 natural translation is universal feature for any plane motion of restricted continuous media; (3) the wall-bounded motion after impact is a rotor-translational motion with two invariants: a kinematic one $V_\infty/r\Omega = 2$, and a dynamic other $2V_\infty^2/r^2\Omega^2 = 3 \approx \pi$ (azimuthal wavelength), which accelerates itself continuously at the wall. The two properties of the thixotropic fluid, CBV (e^τ) and shear viscosity (ν), have opposite tendencies "return force"/shearing process and "rotational inertia"/compressing process, where which are mutually dependent. The future evolution of any fluid more or less elastic, depends only on the present state (and not on its history), so that the wall torsion pressure $(p_{w,torsion}/\rho)$ plays the role of an order pressure pushing forward the non-autonomous velocity field of boundary-layer flow. More importantly, the wall torsion/twisted pressure must obey the law of equal action and reaction in the form of an equal-energy condition

$$e^{\tau}\nu/V_{\infty}^{2} \equiv 1. \tag{7}$$

The outcome of Eq. (7) is a consequence of the equivalence principle, heavy mass/inert mass ≡ 1; i.e. the effects due to the accelerated motion at the wall and gravitational force cancel each other out.

In fact, the Eq. 7 expresses a gross dynamic balance governing the whole dynamic process of motion. Physically, the wall torsion pressure is a rotatory energy, where the fluid strained by the wall has rather a solid body-like behavior with angular velocity e^{τ} and angular momentum ν. Since their product must be equal to the kinetic energy $\rho/2V_{\infty}^{2}$(Eq. 7), the wall torsion pressure excepting of a scale factor l^2 defines a boundary Reynolds number (per m^2) which is a kind of normal angular acceleration,

$$Rb \equiv e^{\tau}\nu/l^{2}\ [s^{-2}]. \tag{8}$$

In contrast to the well-known Reynolds number *Re* which is a control parameter of flow state, this new boundary Reynolds number *Rb* is an order parameter switching the flow state. Its critical value,

$$Rb_{cr} = e^{2}\nu_{0}^{-1},\ \nu_{0}^{-1} - \text{ the natural frequency of thixotropic fluid,} \tag{9}$$

is a non-rolling condition for the CBV, which separates the non-periodic creeping motion/laminar flow from the non-linear torsional vibration motion/turbulent flow.

The parameter *Rb* is an autonomous parameter depending only on the intrinsic state of fluid at the wall, where a first approximation of the starting condition of equal-partition of energies, (Eq. 7), is $Rb = Re_l$.

Using the end properties of the thixotropic fluid $(e^2, 1\,;1\,,\nu_0)$ and the approximation $Rb = Re_l$, the parameter *Rb* describing the local state of fluid can be predicted by means of three power law-like relationships, depending on the intensity of starting impact [5] as

$$e^{\tau}(\nu_0^{-1})^{\frac{1}{1+\tau}},\ \tau \in \{2,1,0\} \text{ for the inelastic impact,} \tag{10a}$$

$$e^{\tau}\nu_0^{-1},\ \tau \in \{0,1,2\} \text{ for the linear/elastic impact,} \tag{10b}$$

$$e^{\tau}(\nu_0^{-1})^{\frac{3}{2}},\ \tau \in \{2,1,1/2\} \text{ for the nonlinear/ballistic impact.} \tag{10c}$$

Equations (10) act as a substitute for the Stokes's hypothesis which is a relation to restricted (valid only for an elastic impact). In fact, *the CBV* e^{τ} *and viscosity ν are the first and second viscosities for a thixotropic fluid*.

The post-impact motion (*t*> 0). In contrast to the no-acceleration parallel flow approximation for a wall-bounded flow, the wall flow model of the rotor-translational motion generating itself accelerations, is a more realistic prototype of various compressing-caused vortical waves. The motion following impact is a boundary-layer structure with 2/3 translational motion and 1/3 rotational motion embodying shear concentrated vorticity at the wall, in a compact texture of laminar flow, and dispersed vorticity "en mass", i.e. with a large number of vorticity dipoles/micro vorticity pairs, across a turbulent boundary layer with a less pressed texture. The rapid loading in the contacting area of the solid boundary, during the starting, is a source where instabilities are produced and then propagated as linear/elastic and nonlinear/dispersion body waves. The local instabilities at the fluid-solid boundary interface, propagate as three wave packets/groups containing fast compressing/expanding longitudinal (L) waves and slower shearing transverse (T) waves that are mutually dependent (see § 4).

The kinematics and the dynamics of shear turbulence can be conceptually synthesized by means of the rotor-translational motion model as follows:

- The cycloidal trajectory of a fluid particle at the wall generates itself a circular instability $(\boldsymbol{u}(0) = 0,\ \boldsymbol{a}(0) \neq 0)$ stronger than the static instability $(\boldsymbol{u}(0) = 0,\ \boldsymbol{a}(0) = 0)$ in laminar flows;
- The post-starting motion develops a boundary-layer structure set up into a non-autonomous outer layer/inertial phase with the average group velocity of $\frac{V_g}{V_\infty} = \frac{2}{\pi}$, and an active autonomous inner layer/non-inertial phase at the fluid-solid boundary ($y = 0$) with the phase velocity of $\frac{V_{p\square}}{V_\infty} = \frac{4}{\pi}$ (the jump of average normal velocity) in the circular/azimuthal plane $(0, 2\pi)$;
- The wall wave packets have a phase velocity twice the group/convection velocity;
- The wave pattern is differentiated by the intensity of impact as shown in Fig. 4.

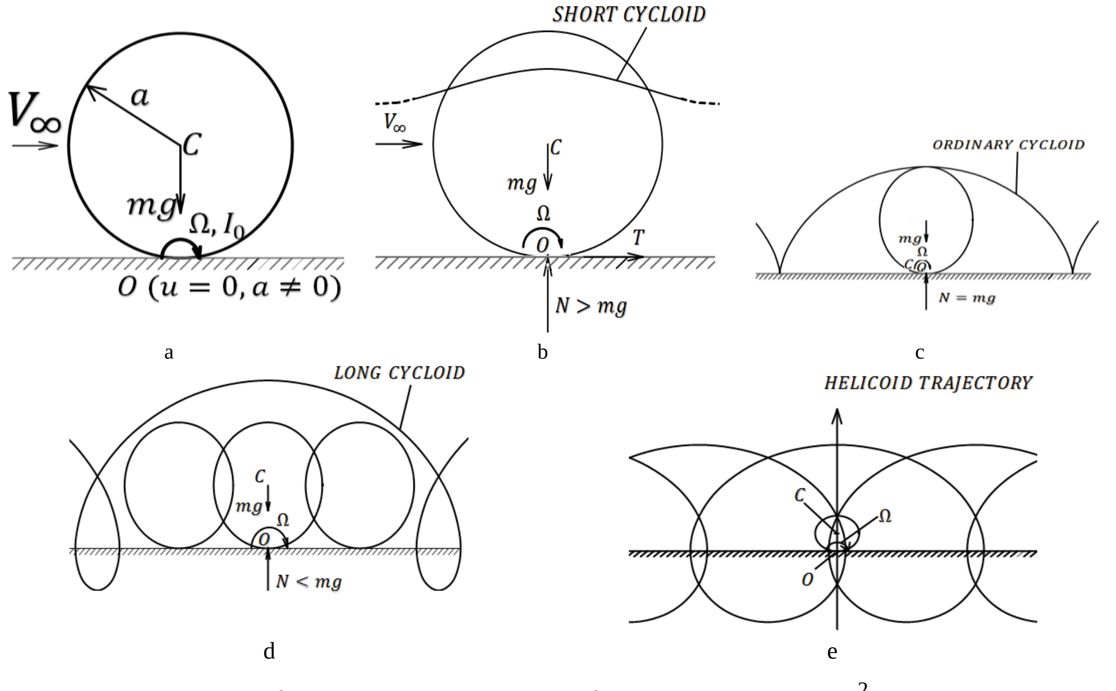


Figure 4: The causality of a plane motion (the evolution of an energy perturbation (V_∞^2) carrying a change ($I_0\Omega^2, e^{\tau}\nu$) concomitantly with its damping): a) starting impact (equal-energy partition condition; b) inelastic impact (λ<1) with shearing friction; c) inelastic impact ($1 \leq \lambda < \lambda_1 = 3\pi/2$), steady rolling without slip; d) linear/elastic impact ($\lambda_1 \leq \lambda < \lambda_2 = 5\pi/2$), unsteady rolling/torsional elastic waves without slip; e) nonlinear/ballistic impact ($\lambda \geq \lambda_2$), unsteady rolling/nonlinear torsional waves with slip

The intensity of a starting impact is measured by a non-dimensional normal rotational acceleration $\frac{a_n}{\Omega^2 r} \equiv \lambda$ and units of $\sqrt{g} \approx \pi$ (g is the acceleration of gravity and π is wavelength of motion), on a scale of 1 to 10. The parameter λ plays the role of a stability parameter for moving bodies in a plane motion, showing the instability level (π) closely related to the intensity of its cause $\sqrt{g}$ (the starting impact)

$$\lambda_i = \frac{(2i+1)\pi}{2}, \text{ with } i = 1,2,3. \tag{11}$$

as follows:

- $\lambda_1 = 4.71$ is the Feigenbaum's criterion (4.669…) indicating the onset of the linear instability state at the molecular scale; for $\lambda \leq \lambda_1$ the impact is an inelastic impact without microstructure change.
- $\lambda_2 = 7.79$ indicates the onset of the nonlinear instability state; for $\lambda_1 < \lambda \leq \lambda_2$ the impact is a linear/elastic/random impact preserving the Gaussian behavior of the molecular microstructure without inertia changes/deformations at the macroscale;
- for $\lambda > \lambda_2$ the impact is a nonlinear/ballistic impact involving irreversible microstructure changes with nonlinear material behavior and structural damping/hysteretic (remanent deformations), [12].

Therefore, any energy perturbation propagates at high frequencies in the localized form of a three-wave packet of inertial nature (a "lifting" effect, Fig. 4).

In the case of following fluids, the local stability parameter is $log\,R\,b$ where

- $log\,R\,b_{in} \equiv logRe_{cr} = log(\nu_0^{-1}) = 4.82$ is the onset of instability state;
- $log\,R\,b_{cr} = log(e^2\nu_0^{-1}) = 5.7$ is the onset of transition process;
- $log\,R\,b_{st} = log\left(e^{-1/2}\nu_0^{-\left(1+\frac{1}{2}\right)}\right) = 7.0$ is the full/statistic turbulent state.

The analogy between the stability state of solid and fluid media shows some difference in the elastic range (shorter for fluids) because of the less elasticity of incompressible flows by comparison with the solid bodies. However, the surprising similitude between the stability parameters λ and *Rb*, in fact both wave numbers, shows *the universal character of the starting process of a moving continuous medium*. The concept of CBV as a wall angular velocity is crucial for the dynamics of turbulence closely related to the nonlinear non-isotropic effects, induced at starting, where these are offset not by viscous diffusion, but by weak dispersion of the CBV by means a shear wave system, called the self-sustaining mechanism of turbulence described in the sequel.

4. THE SHEAR VORTICITY WAVES AND SELF-SUSTAINING MECHANISM OF TURBULENCE

Commonly it is assumed that the transition process from laminar to turbulent flow occurs because of an incipient instability of the basic flow field. This non-defined instability intimately depends on subtle and obscure details of the flow [13]. Thus, the some small disturbances in the freestream enter the boundary layer from where a variety of different instabilities can occur and grow up to the breakdown of laminar flow. The scenario following various linear stability approaches within the framework of the classical Navier-Stokes theory [14], however, failed to explain the origin and the mechanism of the transition process. *The "original sin" lies just in the ignorance of the initial impulse triggering off a primary instability state*. This drawback is removed in the sense that any flow is started at some moment in time from rest, where the initial impulse/starting impact is occurred, and as long as the Reynolds number or a similar stability parameter λ doesn't exceed a critical value (Rb_{cr}, λ_2), the flow/motion remains laminar/in elastic regime. As the Reynolds number/λ increases, some instability sets firstly in the linear/elastic range, being is followed by the transition to nonlinear instabilities, at the critical Reynolds number $Rb_{cr} = 5.69$(or $\lambda_2 = 7.79$), and a fully developed turbulent/hysteretic damping state, Figs. 4, 10. With the above considerations the transition process becomes a well-defined process quantified as the jump from linear to nonlinear behavior in the case of moving continuous media strained by surface forces.

The critical values (Rb_{cr}, λ_2) are closely related to the natural frequency of material continuum, and represent the approaching to the resonance state, (60%, 75%). The essential difference between solid and fluid continuum is the range of elastic behavior. In the case of sharp loading of a material solid sensitive to tension, the elastic behavior is relatively long $(\lambda_1 - \lambda_2)$ followed by a sudden transition where tensile fracture may occur; an effect known as "spalling". The transition process of fluid occurs earlier $(R\,b_{cr})$ and its effect is called "turbulence". The fluid transition seems a quiet sudden process in far field/outer layer, but locally in near field/inner layer, at the fluid-solid interface, the fluctuating flow of fluid particles is steep, an effect known as "bursting phenomenon" [15]. From the mathematical point of view, transitions from linear behavior to nonlinear behavior occur when a threshold regime is attained and is closely related to the boundary singularity at y = 0, which in laminar regime is a static singularity $(\boldsymbol{u}(0) = 0,\ \boldsymbol{a}(0) = 0)$, while in wall-bounded turbulent flow is a weak circular singularity with high frequency $(\boldsymbol{u}(0) = 0,\ \boldsymbol{a}(0) \approx cos(\Omega t/2 - \pi))$. The circular singularity at the contact point (y = 0) is analyzed in circular/azimuthal plane for an elastic/deformable structure, as sketched in Fig. 5. Any dense structure, like the CBV, has more or less inertia depending on the circular frequency according to angular momentum-preserving, where at the limit, near the resonance regime (full/statistic turbulent flow) a remanent inertia $(\pi/2 \approx e/2)$ is embedded in the fluid microstructure, changing the physical state of fluid (fluid compressible). Figure 5 shows the main differences between the models of zero-thickness inner layer (y = 0): inertial material points for elastic dense structures (laminar flow) and non-inertial material point for hysteretic less pressed structures (turbulent flow).

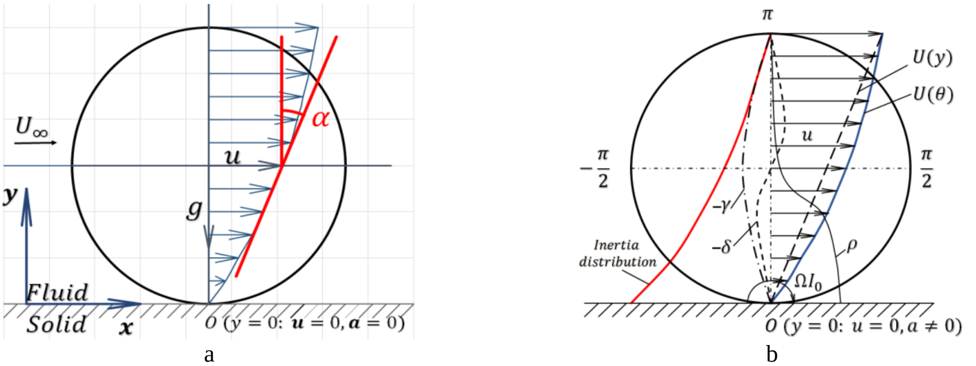


Figure 5: Local models of zero-thickness inner layer ($y = 0$): a) inertial material point in the 2D plane (zero-acceleration parallel flow model); b) non-inertial material point in the azimuthal plane (rotor-translation motion model with (ρ, γ, δ) shear waves)

As we move on from inertial rectilinear (x, y) to consider curvilinear shear flow we have defined shearing/torsion (γ), compressing/inertia (ρ) and their rates as

$$\gamma \equiv \frac{dV(\theta,t)}{dy} = \frac{V_\infty}{4a\sin\theta/2} = \Omega\frac{1}{2\sin\theta/2}, \tag{12a}$$

$$\dot{\gamma} \equiv \frac{d\gamma}{dt} = \Omega^2\frac{\cos\theta/2}{\sin^2\theta/2}, \tag{12b}$$

$$\rho \equiv I(\theta,t)\Omega = a^2\Omega\frac{3}{2}\cos^3\frac{\theta}{2}, \tag{12c}$$

$$\dot{\rho} \equiv \frac{d\rho}{dt} = -\frac{9}{8} a\Omega^2 \cos\theta \,/2 \sin\theta. \quad (12d)$$

From Eqs. (12a) and (12b) we can obtain the torsion degree τ of the wall-bounded flow as

$$\tau \equiv \frac{1}{\Omega}\frac{d}{dt}(\ln\gamma) = 2\cot\frac{\theta}{2}. \quad (13)$$

where for $\theta \geq \pi/2$ and $\tau \leq 2$ the torsion is linear while for $\theta \geq \pi/2$ and $\tau > 2$ the torsion is nonlinear. The maximum torsion is$\tau_{max} = \pi$ for θ=65^0 after which the CBV evolves into a point vortex structure called vortex sheet or Helmholtz discontinuity surface. Apart from some dimensional constants the shearing (γ), compressing (ρ) and the mutual dispersion function $\delta\left(\dot{\gamma} \rightleftarrows \dot{\rho}\right)$ represent dynamic processes with high frequency on different azimuthal mode shapes in the form of three coupled shear waves: the shear wave (γ), inertial wave (ρ) and dispersion wave (δ) induced by the circular/azimuthal boundary singularity. In fact, the shear waves (γ, ρ, δ) are respectively, relative, transport and Coriolis components of acceleration **a**(0) of the non-inertial point y = 0 ($\boldsymbol{u}(0) = 0,\ \boldsymbol{a}(0) \neq 0$).The mode shapes ($\varphi$) of shear waves (γ, ρ, δ) for the linear behavior/laminar flow and nonlinear behavior/turbulent flow are shown in Fig. 6; *the shear waves lie in the origin of coherent structures in shear flows and their dynamics is associated with the phenomenon of bursting (see Fig. 7).*

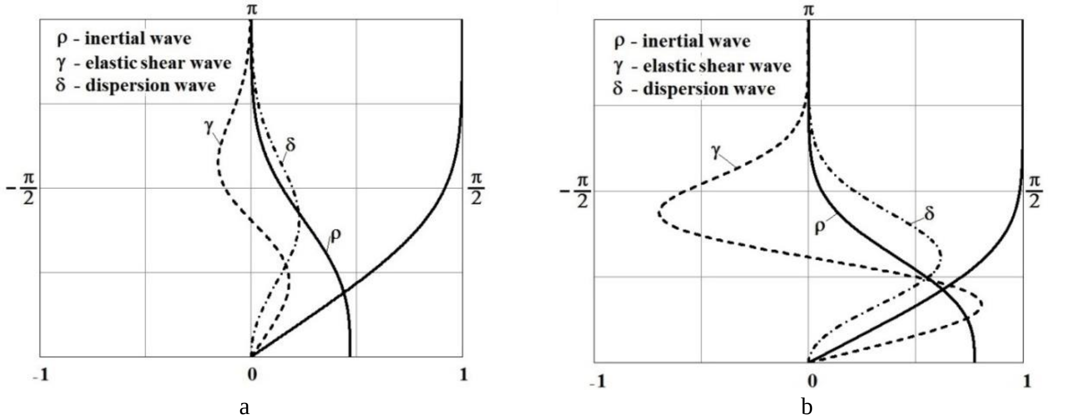


Figure 6: Shapes of azimuthal modes: a) linear/elastic impact; b) nonlinear/ballistic impact (the envelope of (γ, ρ, δ) curves is the transverse variation of torsion pressure $\frac{p_{torsion}}{\rho V_\infty^2}$)

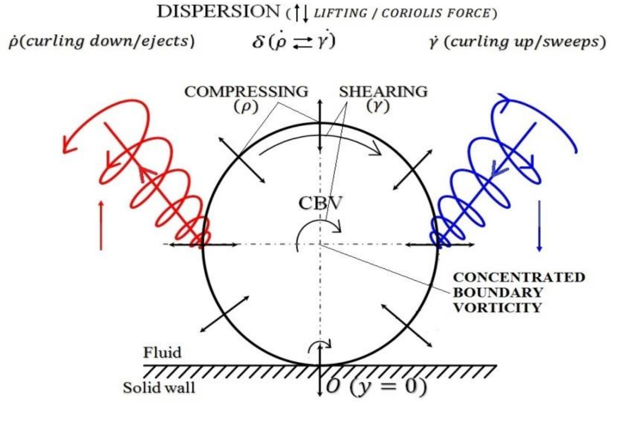


Figure 7: Local coupling of shearing (γ), compressing (ρ) and dispersion (δ) (self-sustaining mechanism of wall turbulence)

A priori transition, wave number $k < k_{tr}$, the dispersion process can be ignored and the local dynamic balance between “compressing inertial force (ρ)” and “shearing elastic force (γ)”is described kinematically by linear torsional vibrations

$$\ddot{\tau} + k^2\tau = 0, \tag{14}$$

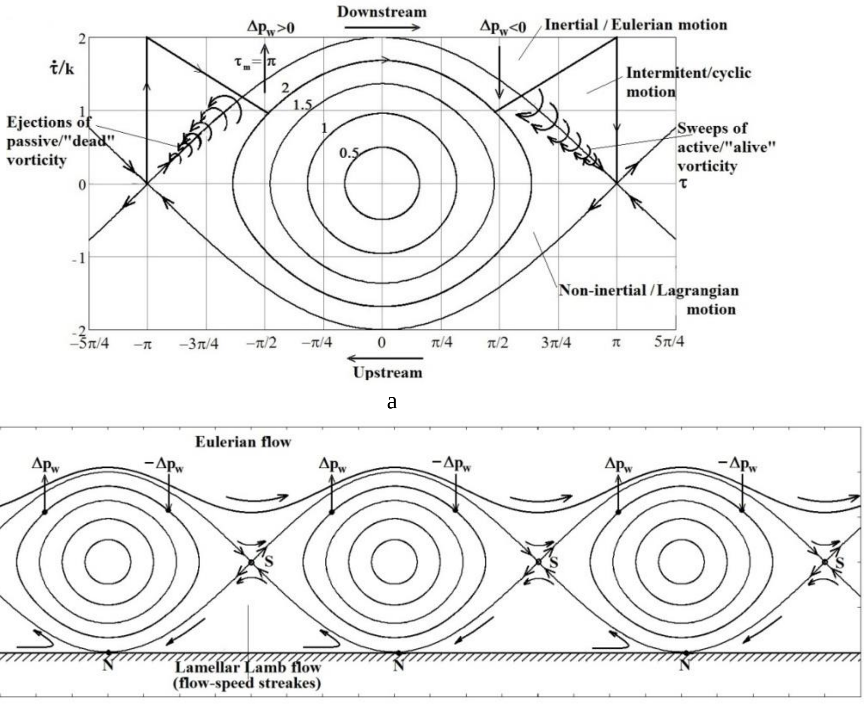
with the wave number $k < k_{tr} = \pi/4 + 2\pi = 7$ (6 for fluids) and the torsion index $\tau \leq 2$.

After starting (equal-energy partition), the gross/energy dynamic balance between the kinetic energy (E_k) and the inertial/elastic potential (E_p) is expressed by the Rayleigh’s quotient/ratio [16]

$$k^2 = \frac{E_{p\,max}}{(1/k^2)E_{k\,max}} = R(\phi). \tag{15}$$

depending on the mode shape φ.

At the transition $k \geq k_{tr}$, the starting impact is a ballistic one where the maximum shearing exceeds the accepted physical value of e^2, followed by its halving, fluid microstructure changes with a nonlinear/hysteretic damping behavior (i.e. non Gaussian) of internal/molecular thermal energy and an abrupt intensifying of dispersion process (δ) on account of the intrinsic energy (compressing “latent” heat) of fluid. All coupled processes (shearing, compressing and dispersion) are running as a whole (the self-sustaining mechanism of turbulence) up to the point where the starting energy perturbation (e^2) is offset/damped by its dispersion and embedding in a new microstructure of fluid (compressible fluid at $Re_{indiff} \approx 10^{10}$). The dispersion process is a kind of inertial Coriolis force producing intermittent lifting effects as sketched in Fig. 7



a

b

Figure 8: The fundamental “cat-eyes” coherent structures (CES) at the wall: a) torsion pendulum mechanism: phase curves and separatrix ($\tau_m = \pi$) of motions in the phase plane; b) topology of wall-bounded motion in the azimuthal plane (y = 0) and the bursting phenomenon (Δp_w- jump of wall pressure)

The topology of the lamellar Lamb flow is of the stable node (N)/saddle (S)/saddle type.The local dynamic balance between the shearing, hysteretic compressing and dispersion processes can be kinematically described by the torsion pendulum with hysteretic damping, Fig. 8.

$$\ddot{\tau} + k^2 \sin\tau = 0, \tag{16}$$

where the limit of small amplitudes $\tau_m \leq 2$ renders the linearized solution and constant period $T_l = 2\pi/\Omega = 10^{-k}$.

In the phase plane $(\dot{\tau}/k(\tau))$ a limit cycle exists for $\tau_m < \pi$, indicating a rotational motion, while the solution for $\tau_m > \pi$ is no longer closed and is extended to infinity as a translational motion. A separatrix for $\tau_m = \pi$ in the phase plane, separates the two kinds of motion (Fig. 8a), [5].

Note that the most primary concentrated vorticity (e^{τ}) structure nearest the wall (y = 0), moving independently from the streamlines (translation motion), is a “cat-eyes”-like coherent structure (CES), educed fromEq. 16, Fig. 8b.

The CES is the fast inertial/compressing wave (ρ) of Lagrangian nature, extracting the molecular thermal energy of fluid for the sustenance of the slower shear (γ) and dispersion (δ) waves.

The whole shear wave packet, i.e. the soliton coherent structure (SCS) penetrates then the outer inertial layer activating the wall-bounded flow.

The self-sustaining mechanisms of turbulence. It is important to clarity the distinction between the standing coherent structures and their major relevant physical mechanism close to the wall, which have universal characters, so that up to some yet to be specified conditions, the results can be carried over the regions, of general turbulent flows, close to the wall. Thus, herein exists two fundamental coherent structures triggered off by a ballistic impact ($Re_l > Rb_{cr}$): the CES generated by a primary torsion pendulum-like mechanism (TP) running at the natural frequency of fluid ($Re_c = \nu_0^{-1}$) and the SCS (2D wave packets or “hairpin vortex” structures) responsible for a secondary non-inertial mechanism so-called the self-sustaining mechanism of wall turbulence (SST) at easily lowered frequencies($R\, e_l^{1/2}$).

The nonlinear TP with hysteretic light damping produces the periodic forcing of SST accompanied by a “bursting”/splashing phenomenon. This name was given by Kline et al. [15] to the sequence of events happening to the near-wall structures: lift up, oscillation and break up, referred also to as an “ejection-sweep cycle” (Fig. 8a).

In fact, this intermittent cyclic motion accommodates the wall Lagrangian motion with the Eulerian flow field. *The coherent structures (CES, SCS) and bursting-like intermittent motion are the outcome of the non-inertial processes of Lagrangian nature, all these creating the complicated self-sustaining mechanism of wall shear turbulence*.

The gross/energy dynamic balance must take account of the internal energy (rolling friction) of fluid E_{in}, which sustains the turbulence process through the wall shear wave system, known as soliton-like coherent structure (SCS), [4], [6].

Since the internal energy of molecular nature, the inertial rotatory potential and the kinetic energy are a mixture at high frequency, close to the resonance frequency their partition can be obtained only by statistical models. Such as model frequently used in physics is the mixture Lorenz curve [17], Fig. 9, where the Gini coefficient [18]

$$G \equiv \frac{E_p}{E_p + E_{in}} = \frac{1}{3}. \tag{17}$$

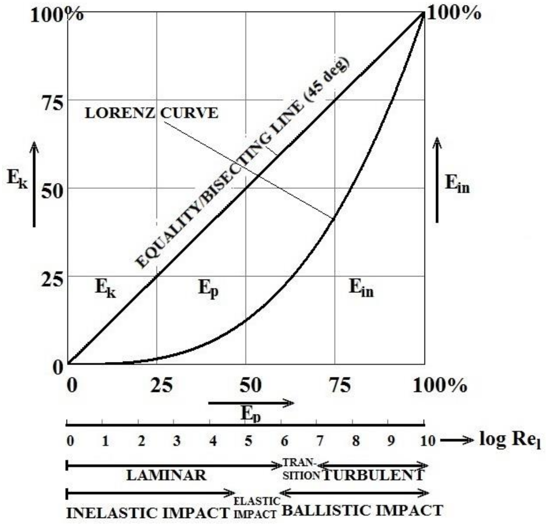


Figure 9: Lorenz curve of energy mixture in the wall-bounded flow

The parametric evolution of the transition process, from laminar to turbulent flow in the Prandtl boundary layer flow, can be visualized by means of the similar soliton solutions [5] and the generalized Stokes's hypothesis from Eqs. 10, Fig. 10.

Figure 10 shows the projection of the shear wave field with high frequency on small wave numbers ($k < \pi$), frozen at a given instant, onto the mean velocity field. It is worth marking that at $\mathrm{Re}_l = 10^7$, the separatrix of motions is achieved (a wall effect termed "intrinsic/molecular slip") and further the Reynolds number is practically indifferent towards flow and the local isotropy is restored.

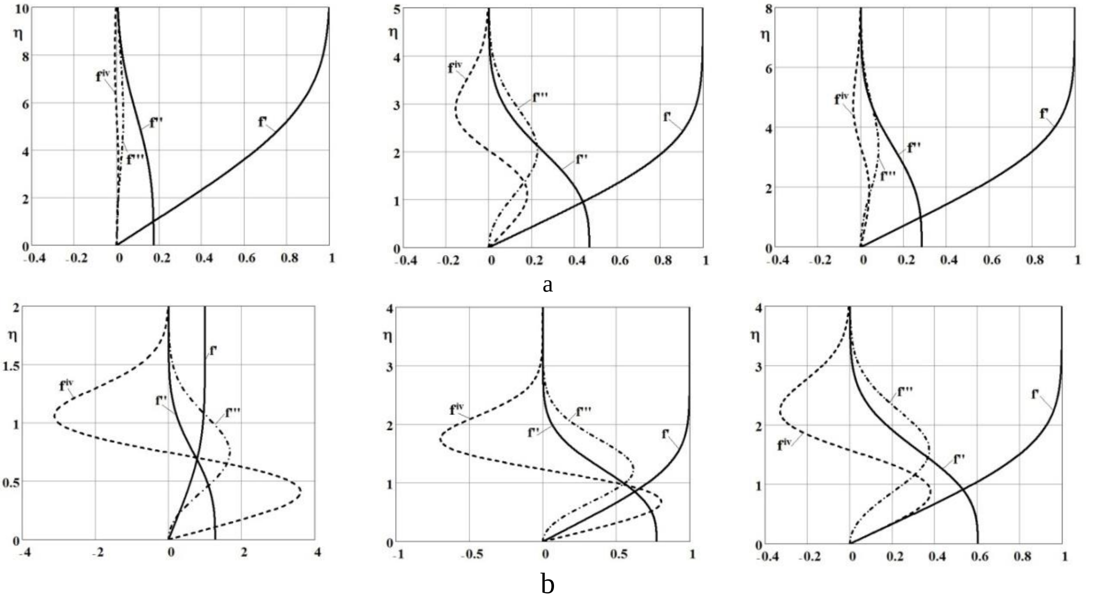


Figure 10: Similar soliton solutions of mean velocity and shear wave fields with a generalized Stokes's hypothesis (Eqs. 10) for the Prandtl boundary-layer flow: a) laminar flows (linear/elastic impact): $Rb = e^{0}\nu_0^{-1}(\approx 10^5) - e^{2}\nu_0^{-1}(\approx 5\cdot 10^5)$; b) transitional-turbulent flows (nonlinear/ballistic impact): $Rb = e^{-2}\nu_0^{-(1+1/2)}(\approx 2\cdot 10^6) - e^{-1/2}\nu_0^{-(1+1/2)}(\approx 10^7)$

5. CONCLUSIONS

The paper presents a unitary approach of the plane motions strained by boundaries for a continuous medium, fluid or solid. The key hypothesis for such motions is the fact that the onset of motion is a kind of impact (starting impact) which governs the subsequent evolution of motion. The starting impact is an energy perturbation localized in a wave packet at the fluid-solid boundary interface, which has the important advantage concerning the transport of the perturbation energy concomitantly with its damping. The ignorance of this initial impulse as the primary instability state triggering off further (secondary, tertiary, …) instabilities (bifurcations), followed by transition and a hysteretic damping, led to a branching in the phenomenology of turbulence which sometimes obscured physical and mathematical justification.

The present approach follows a direct reasoning from the origin/causes to diverse facts/effects where in the case of complicated turbulence phenomenon, the ignorance of its causes gave birth, in exchange to a number of beautiful images of turbulent flows easily observed, but extremely difficult to interpret, understand and explain.

By means of the model of starting impact-rotor translational motion at the wall, the paper proposes a universal stability criterion/parameter which governs the stability state along the successive development of motion, from the origin up to when an ultimate (statistical) state is restored. Thus, the transition process becomes a well-defined state as the jump from linear to nonlinear behavior for both solid and fluid plane motions.

One of the common approaches, both in theory and data analysis, is a reduction one, i.e. some decomposition of the flow field. The first known decomposition was given by Reynolds, in which the instantaneous flow field is represented as a sum of a mean and its fluctuations, associated with URANS computations and the traditional observation instruments like the hot-wire and laser-Doppler anemometer. These analyses of the flow field are one-point techniques not able, to reveal the instantaneous spatial structures of a flow. But the turbulence is not a completely random process, and as shown above consists of coherent flow structures, i.e. wall shear waves. In spite of the lack of a valid theory concerning the formation of coherent structures (hairpin-vortex packets [19]) and their major relevant physical mechanisms, new observation techniques like PIV/LSV (particle image/laser speckle velocimetry), were developed to study the formation and dynamics of coherent flow [20]. The present model of weak starting shocks (i.e. early compressibility effects) with energy-preserving and light damping, explains a lot of above intricate events from a shear flow. At the same time, the model also gives insight into some controversial mathematical issues concerning the problem of turbulence as nonlinearity, non-integrability, non-locality, large Re, zero-viscosity limit, Lagrangian versus Eulerian acceleration, etc. Comparative results on the transitional process are shown for two canonical wall-bounded motions: the rolling elastic wheel and the boundary layer on a flat plate.

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