Vorticity Waves in Shear Flows

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Abstract: *The main purpose of this paper is to bring about the vorticity-creation process at the boundary surfaces on one hand and to understand the interaction of concentrated vorticity structures on the shear flow on the other hand. The paper address the simulation of flow visualization of the vortical structures accounting for both the circulation and the degree of concentration of vorticity via the exact nonlinear solution of Stuart for an inviscid unsteady mixing layer. At a fixed volume of circulation an increased concentration of vorticity is provoked by decreasing the area containing most of the vorticity, and then it spreads over an increased area of the flow. A concentration parameter characterizing vorticity is defined and flow patterns, representing waves travelling in the flow direction are found for different combinations of vorticity concentration and circulation.*

Key Words: *Vorticity waves, Mixing layer, Inviscid flow*

1. INTRODUCTION

Flow visualization is a tool in experimental fluid mechanics that renders certain properties of flow field directly accessible to visual perception. It has always been believed that observation of a process pattern facilitates the development of an understanding and the subsequent analysis of such a phenomenon.

In general and under normal circumstances, most fluids, gaseous or liquid, are transparent media, and their motion must remain invisible to the human eye during direct observation, unless a technical allowing visualization of the flow is applied.

A great variety of methods has been reported that allow the fluid flow to be visible, in the fluid mechanical laboratory, in industrial environments and for field experiments.

Local marking techniques, such as injection of dye or smoke at a particular location in the flow have been used to generate streaklines or timelines, i.e. lines made up of particles generated at the same instant [1]. Tracer material realized into the flow only for a short instant of time might be collected into a flow regime where the residence time of the fluid is high, while the rest of the material is carried away with the main flow.

In this way, flow structures such as vortices and regimes of separated flow became visible. However, interpreting the flow visualization in unsteady flow has remained confusing. A cornerstone of all interpretations of flow visualization is the degree of concentration of vorticity in the unsteady shear flow.

In this paper, the flow visualization of mixing layer based on an exact solution of the Euler equations found by Stuart [2], is simulated numerically by tracking fluid markers. The effects of circulation and concentration of vorticity are discussed.

2. PERTURBATION AS AN IMPACT PROBLEM

The perturbation caused by the motion of the solid bodies into a flow is firstly an impact problem related to its start up which generally is drastically simplified by idealization. The most important idealizing assumption is that of a sudden jump in the velocity distribution and, thus of the momentum in the limit of vanishing contacting time.

However, the impact is a process of momentum exchange between two colliding bodies that one is the moving fluid, within a short time of contact. With respect to a moving fluid impacted by a solid surface or a mixing layer, loading such a process acts with high intensity during this short period of time.

As a result, the initial uniform velocity distribution is rapidly changed and the vorticity and a torsion pressure waves generate due to the intrinsic torsion property of the motion of fluids. The torsion pressure for a unit perpendicularly to shear layer vorticity, $\omega_n = \frac{1}{s}$, is equal to the dynamic viscosity μ, $p_{torsion} = \mu\omega_n$, $G \equiv \mu\left[\frac{Ns}{m^2}\right]$, where G denotes the elasticity modulus of torsion of a fluid. The torsion pressure is a suction pressure proportional with the vorticity.

The perturbations of the fluid impacted by boundaries (physical surfaces or mixing layers) in laminar shear layers propagate in the form of traveling vorticity waves at different speeds depending on the concentration of vorticity contained in the shear layer. Within the short time of the impact the torsion (suction) pressure reaches its peak value, when for any circulation the vorticity is concentrated into a point vortex set on the boundary.

$$p_{torsion}\left(\omega_n\right) = \mu\omega_n,\ \omega_n \in \left[1, e^2\right] \text{ on } \partial B, \tag{1}$$

The maximum torsion pressure found by Stuart [1] through hydrodynamic stability reasons is the source of the vorticity waves that are emitted and propagated with finite speed through fluid.

Therefore, the boundary vorticity dynamics concerns a two-step process: the vorticity creation from an impacted boundary followed by vorticity waves that propagate at different speeds depending on their concentration.

Due to the dual property of fluids circulation, $\nu = \frac{\mu}{\rho}$, and, the torsion, $p_{torsion} = \mu\omega_n$, besides the circulation parameter (strength of vorticity source) a second parameter is required, i.e. the concentration of vorticity, describing its spread, directly related to the wave length.

The degree of concentration of the vorticity will be defined in the sequel based on the Stuart's vorticity model.

3. STUARD'S VORTICITY SOLUTIONS

Any origin of perturbation is the change of pressure or number Reynolds; these induce the changes of vorticity in the flow. Based on the hydrodynamic stability of a laminar shear layer $u = \tanh y$, Stuart found the flow patterns and vorticity distributions for the two-dimensional inviscid shear layers containing strong vorticity concentrations without the loss of flow stability.

The change of the flow pattern with the level of concentration is discussed by reference to the vorticity which is given by

$$\omega_n = -e^{-2\psi} = -\left[\cosh y + A\cos x\right]^{-2}, \quad (2)$$

where $\psi = \ln\left(\cosh y + A\cos x\right)$ the stream function, x is the coordinate in the direction of mean flow and *y* is coordinate normal to that direction; *x* and *y* velocity components are $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$.

The Gauss function like solution named e^{ω_n} - solution, represents a spatially periodic mixing layer, consisting of a single row of vortical structures, with constant circulation, Γ=-4π around the contour: $y = \pm\infty,\ x = 0, 2\pi$, Fig. 1.

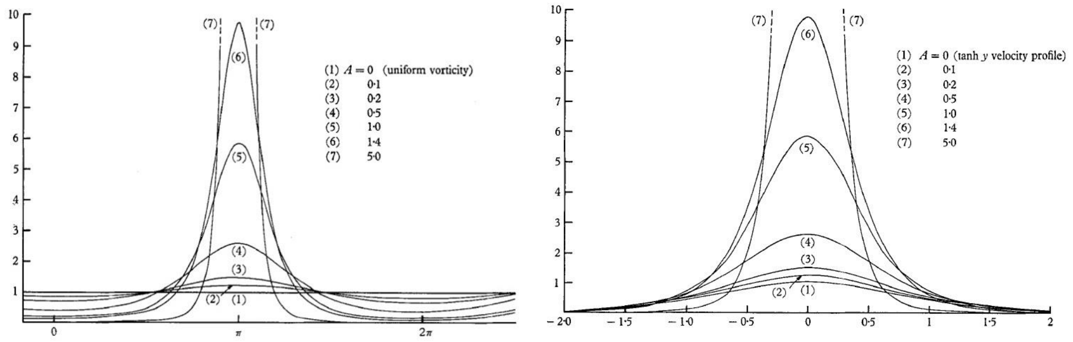

Figure 1 – Gauss like vorticity distributions

e^{ω_n} - solution shows some features:

1) for the constant vorticity contained into the shear layer they can describe different distributions of ω_n with *x* and *y* changes and values from e^0 (uniform vorticity) to e^2;

2) at initial impact conditions the each distribution can be transformed into a set of point vortices on the boundary, $\int_{e^0}^{e^2} d\left(\ln\omega_n\right) = 2$;

3) they simulate a class of flows involving small or large periodic perturbations from a skewed shear layer. These are time-dependent solutions, representing waves travelling in the *x* direction obtained by translation of axes.

Two important parameters are defined to describe the mixing layer: the extreme velocity ratio $R = \left(U_1 - U_2\right)/\left(U_1 + U_2\right)$, and the vorticity concentration parameter γ, which can take values from 0 to 1.

If the velocity is nondimensionalized by the average velocity $\bar{U} = \left(U_1 + U_2\right)/2$, the streamwise and cross-stream velocity components can be expressed as

$$\bar{u} = 1 + R\frac{\sinh y}{\cosh y + \gamma\cos(x-t)};\ \bar{v} = R\frac{\alpha\sin(x-t)}{\cosh y + \gamma\cos(x-t)}, \quad (3)$$

As $y \to \infty, u \to 1+R$, whereas when $y \to -\infty, u \to 1-R$. When R = 1, only one stream is present. As the vorticity concentration parameter $\gamma \to 1$, a single row of point vortices is obtained, while for $\gamma << 1$, the unsteady shear flow corresponding to the linearized stability analysis of the tanh *y* profile is recovered. The dimensionless vorticity is given as:

$$\omega_n = -R \frac{\left(1-\gamma^2\right)}{\left(\cosh y + \gamma \cos(x-t)\right)^2}, \tag{4}$$

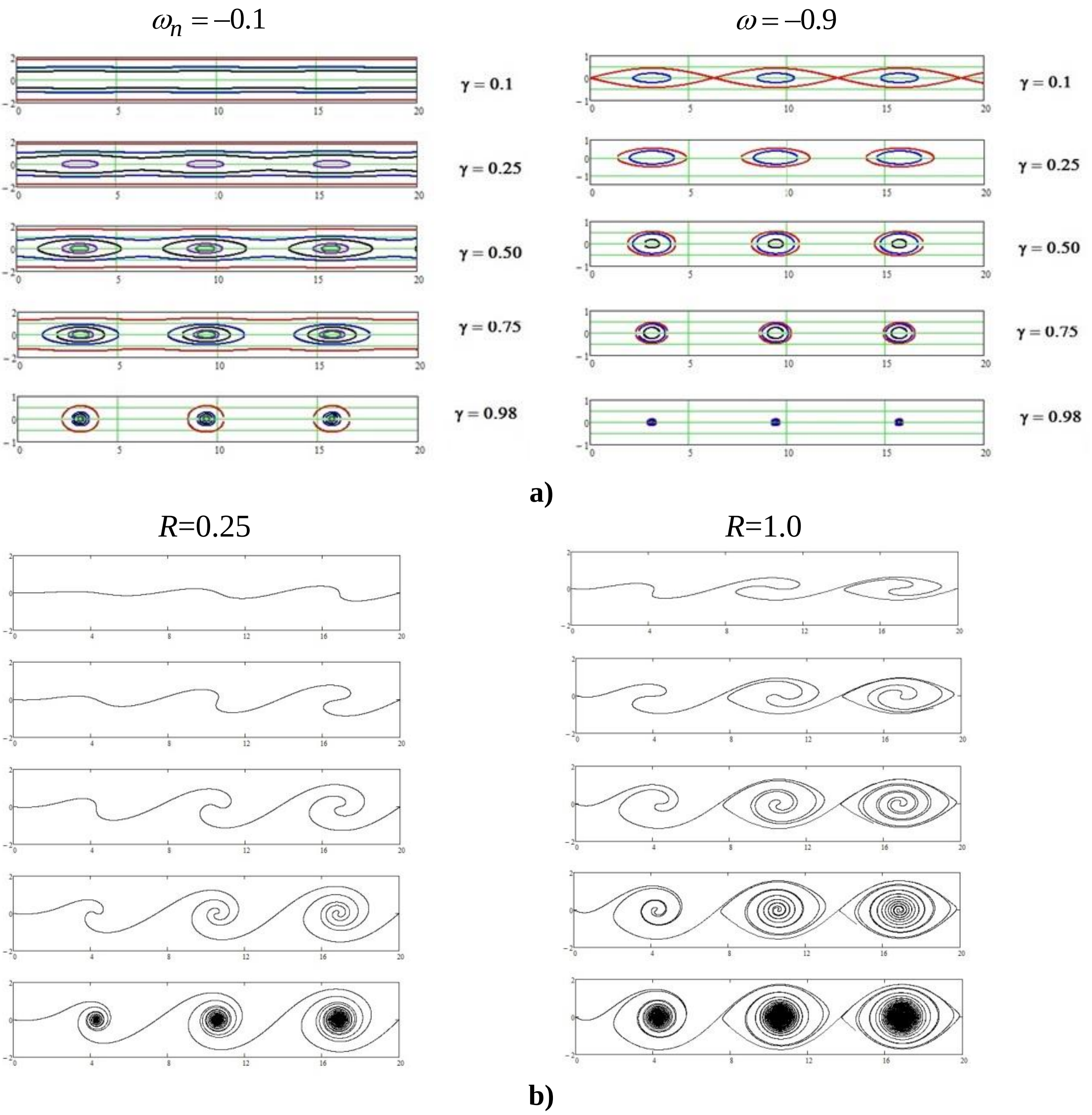

Figure 2 – a) Contours of constant vorticity for several $\omega_n = -0.1, \omega = -0.9$ b) Effect of γ on streakline patterns $R = 0{,}25$ and $R = 1$, any ω_n

The vorticity distribution for $R = 1$ at time $t = 0$ is shown in Fig. 2a for several values of vorticity concentration parameter γ.

For all vorticity contours, the outermost contour has the same value $\bar{\zeta} = -0.5$ (the increment between contour lines is $\delta\bar{\zeta} = -0.5$). The circulation of each vortical structure is $\Gamma = 4\pi R$ with $\Gamma = 4\pi$ for $R = 1$ (Fig. 2a).

From the vorticity distribution, time-averaged values at any point can be calculated. If we form the ratio of maximum total vorticity to maximum mean vorticity (at $y = 0$, $x - t = \pi$), we obtain

$$\frac{\omega_{max}}{\Omega_{max}} = (1+\gamma)\sqrt{(1+\gamma)/(1-\gamma)} \,. \tag{5}$$

Equation (5) is a one-to-one relationship between this ratio and the vorticity concentration parameter γ, which can be used to estimate the vorticity concentration from the value $\frac{\bar{\zeta}_{max}}{\bar{Z}_{max}}$ found experimentally.

There is a wide range of reported values of γ or $\frac{\bar{\zeta}_{max}}{\bar{Z}_{max}}$ in the literature, ranging from $\gamma = 0.25$ to $\gamma = 0.70$, depending upon the type of shear layer and the Reynolds number [3], [4], [5]. Furthermore, it is straightforward to visualize the flow using the tracking particles of the flow field via its material derivative to obtain the streakline patterns, Fig. 2b.

As expected, the rate of roll up of the particles increases with increasing concentration, and the cross-stream extent of the rolled-up particles approaches the same value for all cases, showing no dependency on circulation.

4. CONCLUSIONS

Based on Stuart's vorticity solutions for Gauss function like perturbations of the two dimensional vorticity equation a class of flows involving small and large periodic perturbations from a skewed shear layers has been simulated. These are time dependent solutions representing waves travelling in the longitudinal direction of the flow.

Two important parameters describing the mixing layer: the circulation parameter and the vorticiy concentration parameter are discussed via a tracking particles method used to visualize the flow field patterns.

The rate of roll up of the particles (shape of vorticity wave) increases with increasing concentration and the cross-stream extent of the rolled-up particles approaches the same values for all cases showing weak dependency on circulation parameter and strong dependence on concentration parameter.

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