Issues of Paradigmatic Nature of the Turbulence Origin

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Abstract: *The fluid such as deformable continuum, without own shape its motion, must be guided by some physical surfaces/boundaries, which are subjected to an action from fluid under the form of the wall pressure, equal to the reaction of the boundaries on the fluid. The fundamental law of equal action and reaction was easily misleading by assuming that the wall pressure is equal to the static pressure on the main motion. This result, known as Prandtl's hypothesis associated to the concept of a Newtonian fluid, i.e. viscous fluid obeying to a linear law of frictional shear stress and constant viscosity, could resolved the no-drag crisis (d'Alembert's paradox) and has dominated the paradigmatic development in research of Fluid Dynamics during the last century. However, the motions fluctuating in a disordered manner observed experimentally by Reynolds and known as turbulent/noisy motions or simply turbulence remain a physical and mathematical problem unsolved/nebulous up to the present. Indeed, the heaviest and most ambitious armory from theoretical physics and mathematics was tried for the last fifty years, but without much success: genuine turbulence, i.e. the big T (turbulence) – problem remains further as a new crisis/paradox. The physical/mathematical problems of turbulence being ones of paradigmatic nature concerning the behavior of flows close to boundaries, in this paper we propose a different approach of the fluid-boundary contact problem, i.e. a new law for the wall-bounded flows, complying better with the action-reaction law, responsible for most controversial aspects of turbulence.*

Key Words: *Boundary layer, Vorticity waves, Laminar-turbulent transition*

1. INTRODUCTION

At the beginning the research in turbulence was conducted almost exclusively by the engineering community to resolve practical problems in different fields, such as in hydraulics, aerodynamics and astrophysics and in atmosphere and ocean sciences. The last three decades have been marked by an increasing involvement of physicists and applied mathematicians who have tried without much success to elucidate the phenomenon of turbulence.

From the mechanical point of view, the idealized Prandtl's hypothesis [1], i.e. constant static pressure across the wall-bounded flow, eludes the law of equal action and reaction through using a misleading fluid-boundary interaction with consequences from the standpoint of continuum mechanics.

That is, the lack of a suited boundary reaction is reflected by the existence of a gap between the scales of molecular/laminar motions and the scales of the smallest relevant scales in fluid flows including turbulence. From the mathematical point of view, the fallacy of the action-reaction law finds expression in the ill-posed problem of the Navier-Stokes equations that fails to reliably describe the turbulence by a system of equations subject to initial and boundary conditions and forcing. At present, it is possible to obtain fully resolved solutions at modest Reynolds numbers via direct numerical simulations of Navier-Stokes equations, but hopeless to contribute to the understanding of the basic physics of turbulent flows [2]. The target for mathematicians to obtain solutions of the Navier-Stokes equations for $\mathrm{Re} \to \infty$ using the present paradigms is a futile one.

As the d'Alembert paradox [3] was solved by a paradigm change from the concept of ideal fluid to the concept of Newtonian (linear) viscous fluid, the same reason follows naturally that the T paradox/crisis is a paradigmatic nature one, resulting from the use of Prandtl's hypothesis and the ill-defined concept of Newtonian fluid, a concept too rigid (linear and constant viscosity) to support turbulent motions. There are two main drawbacks of the Newtonian fluid: on the one side the impossibility to describe the concentrated boundary vorticity, involving large accelerations i.e. its creation as the reaction of the boundary, and on the other side, impossibility to reflect the change from any one state of microstructure of fluid to another for different states of flow, i.e. the thixotropic/nonlinear effects [4]. Mechanically, the Newtonian fluid can describe only some simple/laminar flows (sparse vorticity/small angular velocity and constant viscosity/mass) analogous to the rigid body motion with both constant acceleration and mass, in contrast to the turbulent flow involving variations of both vorticity concentrations and viscosity.

In the sequel we will show how the dual concept of the torsional concentrated vorticity – thixotropic/viscoelastic fluid is a suitable paradigm to describe the nebulous part of turbulence: the origin and its self-sustaining at boundaries in a shear flow.

2. MATHEMATICAL PROBLEM – MAIN CORNERSTONES

The first researcher worthy to be mentioned in the history of the fluid mechanics is Archimedes (287-212 B.C.). His result concerning the equilibrium of immersed bodies is well-known as the principle of Archimedes. The deeper concerns on the fluid mechanics are found in the Middle Ages due to Daniel Bernoulli (1700-1782). He introduced the term of hydrodynamics as the science of hydrostatics (equilibrium fluids) and the science of hydraulics (motion of fluids) and published the first hydrodynamics treatise (1738). But, the fluid dynamics was developed as the science itself only after Leonhard Euler first wrote in 1736 the motion equation of a material point – Newton's equation. After this date, d'Alembert wrote the treatise of the equilibrium and movement of fluids (1744) and discovered that the theory of perfect fluids fails to account for the drag of bodies (d'Alembert's paradox) [3]. However, the one who first wrote the equations of ideal incompressible fluid flows, in the definitive form, was Euler in 1757 [5],

$$\frac{\partial \mathbf{u}}{\partial t}+(\mathbf{u}\cdot\nabla)\mathbf{u}=-\frac{1}{\rho}\nabla p,\quad \nabla\cdot\mathbf{u}=0\,, \tag{1}$$

where $\mathbf{u}(\mathbf{x},t)$ is the velocity field of the fluid which is a time-dependent vector field of the fluid and $p(\mathbf{x},t)$ is the pressure function, which is defined by these equations up to a constant.

The solutions of Eqs. (1), $\mathbf{u}(\mathbf{x}(t),t)$, are called stream lines and the stream line pattern changes with time if the flow is unsteady. After a century, Helmholtz (1858) [6] rewrote the Euler's equations by means another time-dependent vector called vorticity and defined by

$$\boldsymbol{\omega}\equiv\nabla\times\mathbf{u}\,. \tag{2}$$

This quantity represents the rotational speed of the fluid either clockwise or counter clockwise which plays a crucial role in the vicinity of boundaries. The significance and importance of vorticity for the description and understanding of fluid flow stems from the facts, first, that Eq. (2) may be inverted to give the velocity field as an integral over the vorticity field, and, second, that when the viscous diffusion of vorticity is negligible, the vorticity can be transported by ideal flows,

$$\frac{\partial \boldsymbol{\omega}}{\partial t}=[\boldsymbol{\omega},\mathbf{u}]\,, \tag{3}$$

where $[\cdot,\cdot]$ is a commutator of vector field.

Using the transport equation of vorticity (3), it is easy to construct time-dependent solutions of the Euler equations with vortex lines of complex topology. Thus, it has been recently shown that the vortex lines $\boldsymbol{\omega}(\mathbf{x},t)$ at a time t are diffeomorphic to those at an initial time t_0 and from the initial vorticity $\boldsymbol{\omega}_0$ and the corresponding initial velocity $\mathbf{u}_0(\mathbf{x})$ one can construct smooth global solutions $\mathbf{u}(\mathbf{x},t)$ for large times [7].

However, the concept of the perfect fluid, i.e. frictionless and incompressible, fails completely to account for the drag of the body. This unacceptable result of the Euler equations of a perfect fluid known as the D (drag) – problem or d'Alembert's paradox, was removed in 1845, when Stokes published the definitive form of the motion equations for viscous fluids, referred as the Navier-Stokes model. Fluid dynamics studies the motion of continuous media with fluidity/viscosity generally considered as a material constant non-depending on the flow state. The fluid then has a dual feature and can be described in two ways. On the one hand, the fluid consists of continuously distributed material elements or particles, each of which retains its identity all the time so that can trace the fluid motion and evaluation by tracking each element. This way of description, known as particle or Lagrangian description, is a direct extension of Newton's particle kinematics.

On the other hand, fluid motion can be treated by a field theory where, like in electromagnetic field, the spatial positions $\mathbf{x}$ any time are independent variables. The fields of velocity $\mathbf{u}$, pressure p and other derived physical quantities are all functions of $f(\mathbf{x},t)$ and will by assumed sufficiently smooth except on certain surfaces of discontinuity. If the fluid is unbounded, except otherwise stated it is assumed to be at rest at infinity, or by Galilean transportation, to have uniform motion. However, the all flows are bounded by solid surfaces where large accelerations and vorticity concentrations occur so that a particle description of near-wall microflow relative to free shear flows is more suitable. The Navier-Stokes model is restricted from two points of view: the first drawback of paradigmatic nature is the concept of the Newtonian viscous fluid, i.e. the linear law $\tau = \mu \frac{du}{dy}$ and constant viscosity, which is too rigid to describe turbulent flows, and the second drawback of computational nature is the primitive variable formulation of Navier-Stokes equations which recast into diffusion-dominating problems with improper initial and boundary conditions.

3. PHYSICAL PROBLEM

The problem of vorticity is a more complex question than the simple definition of boundary vorticity flux (BVF) (Lighthill, 1963) [8] that concern both the vorticity creation and boundary vorticity dynamics, inducing the non-autonomous velocity field of the laminar-turbulent flow. In the motion of a Newtonian fluid, two contacting layers though experience tangential forces (shearing stresses) they are not exact, especially at large Reynolds number in turbulent flows. This is equivalent to stating that a Newtonian fluid in turbulent flow offers approximate tangential forces inferred from the experience in relation to test flows (Couette-Poiseuille flows).

The huge effort to describe theoretically the details of the complex fluctuating motion superimposed on the main motion seems futile. However, at high Reynolds numbers in the vicinity of boundaries, there is an autonomous micro-field of vorticity that continuously generates concentrated vorticity in the form of torsion vibrating wires and wherefrom waves are emitted. The vorticity waves further push-forward the non-autonomous flow and its velocity field.

This autonomous wave mechanism is an intrinsic generation-dispersion process of the vorticity that assumes the existence of a thixotropic nonlinear medium/fluid, which can support both exponential waves (laminar flow) and sinusoidal waves (turbulent flow). The thixotropic fluid is a light damping one which in the post-impact flow, at Reynolds numbers above $\mathrm{Re}_x = \nu_0^{-1}$ (the natural frequency of fluid), is relaxed by longitudinal (compressing/expanding) and transverse (torsion shearing) vorticity waves [9].

The wave mechanism of the concentrated vorticity vibrating at a boundary is in contrast to Klebanoff's description where at high Reynolds numbers, energy constantly flows from the large eddies of the basic flow to the small eddies, in a narrow strip in the boundary layer in the neighborhood of the wall via a strongly dissipative process [10]. Without any consideration on the influence of concentration and large angular accelerations, any dynamic process could not exist.

The vortex stretching-rate of strain tensor kinematic mechanism where the large eddies of large-scale flow and their energy flux experience a dissipative process toward solid boundaries according to Kolmogorov's eddy cascade theory, is pretty limited one falling to describe the turbulence phenomenon.

Herein, the dispersive process of the vorticity concentrations follows an inverse path from the wall to the free flow. Therefore, the physical problem of vorticity should be approached as a whole process containing three linked events: the flow-solid surface colliding and starting vorticity creation, the non-dispersive creeping motion of vorticity (laminar flow) and the dispersive vibrating motion of vorticity (turbulent flow). Each perturbation small or large is in fact a kind of impact between flow and its boundaries exchanging momentum between bodies within a short time of contact. As a result, the initial velocity distribution is rapidly skewed/squeezed, the vorticity is created and organizes itself into more and more concentrated structures, clearly visible in Fig. 1 [11].

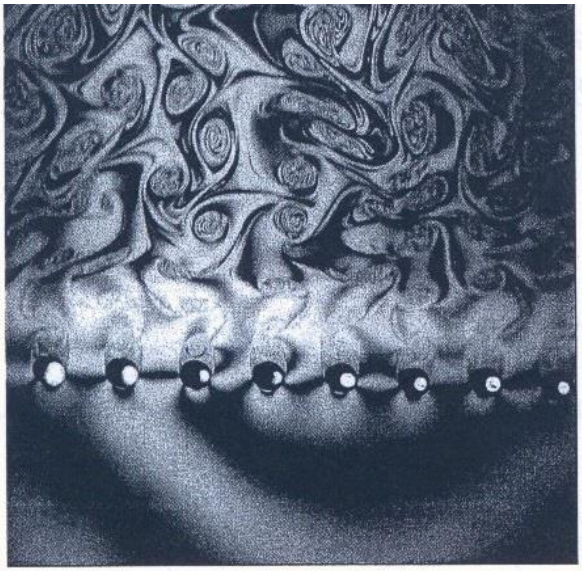

Figure 1 – Two-dimensional turbulent flow visualized in a soap film [11].

The exact solutions of the equations of inviscid motion found by Stuart [12] (e^{τ} - solutions: $\tau \in [0,2]$) can describe strong vorticity concentrations developing in skewed shear layers, where at the boundary there is a set of point-vortices. The concentration level of vorticity is estimated on a natural logarithmic scale e^{τ} from e^{0} - sparse/weak vorticity, up to e^{2} – concentrated vorticity, where the index τ is a measure of the concentration of vorticity. In contrast to the sparse vorticity transported by the ideal fluid flow according to the laws given by Helmholtz [6], the concentrated vorticities are transported by waves, their shape depending mostly on the concentration of vorticity, Fig. 2.

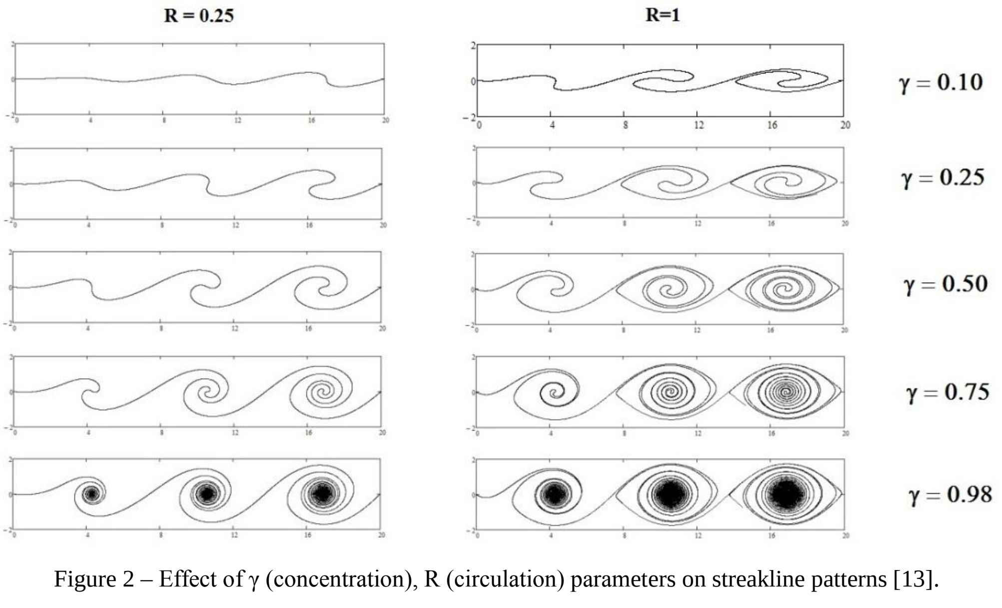

Figure 2 – Effect of γ (concentration), R (circulation) parameters on streakline patterns [13].

From the physical point of view the concentration of vorticity at boundaries is a local compression of flow induced by the torsion of vorticity wires. The concept of torsion of the concentrated vorticity allows a better understanding of the boundary vorticity creation and its dynamics, which becomes an active one just after impact.

Concomitantly with the vorticity creation the impact process induces microstructure changes of the flow properties resulting in a time dependent shear stress, $\nu = \nu(t)$, known as the thixotropic behavior of the flowing fluid.

It is experimentally shown that the transient viscosities follow the line of the complex viscosity versus angular frequency [4]. This behavior can be described by a Klein-Gordon like wave equation [14]

$$\frac{d^2\nu(x)}{dx^2} = \frac{1}{U_e^2}\left(\omega_0^2 - \omega^2\right)\nu(x), \tag{4}$$

where ν_0 is steady shear viscosity and $\omega_0 = \nu_0^{-1}$ is the natural angular frequency of fluid, defined as the first zero of the Fourier coefficient B(ω) of a square pulse/impact. Thus, the solutions of Equation 4, describe well enough the dual behavior of the thixotropic fluid, as a reactive medium, $\omega < \omega_0$, at impact inducing exponential waves (without energy dissipation), and as a dispersive medium which can support sinusoidal waves for ω above the natural frequency ν_0^{-1}. Equation 4 also shows that the microstructure takes time to respond to the flow/stress and its elastic response at low frequency is faster as the flow velocity U_e increases.

The disturbed post-impact flow is a boundary-layer type flow which is relaxed through a complicated wave system, which transports concentrated vorticity from boundaries to the flow field and rebuilds the flow microstructure.

There is a non-dispersive transport of vorticity performed by exponential waves in the form of the laminar flows dominated by the frictional shear stress and a dispersive one which involves lightly damped sinusoidal waves by dry friction in turbulent flows. Hence, it is evident that the analysis of the impact-relaxation process requests another constitutive relation to describe the intricate behavior of viscous fluid.

4. VISCOELASTIC MODEL

An expression for the rate of change of fluidity (the inverse of viscosity) can be obtained by the coupling of intrinsic properties of the thixotropic fluid (concentrated vorticity and time depending viscosity) in the form of the boundary/wall torsion pressure,

$$\frac{1}{\rho} p_{torsion,w} \equiv \omega_w \nu = U_e^2 \text{ on } \partial\mathrm{B}, \tag{5}$$

where $p_{torsion,w}$ is the torsion pressure at wall, $\omega_w = e^{\tau}$ is the vorticity concentrated into a point on a solid surface (∂B), τ is a torsion/concentration index $\tau \in [0, 2]$, and ν(*t*) denotes the change of viscosity during the post-impact flow which is able to adjust itself continuously.

The model of the thixotropic fluid based on the shear compliance/accommodation relationship (5) entails some comments: the left hand equality is a product of the torsion deformation of vorticity, and the response of viscosity (or decreasing fluidity) defining a torsion pressure (suction) at walls, the right hand equality shows firstly, that the law of equal action (U_e^2 - dynamic pressure) and reaction ($p_{torsion,w}$ - torsion pressure) is satisfied for laminar flow as Newton's law of friction, but in fluctuating near-wall viscous flows this has wrongly interpreted and applied up to now, and secondly, that the mutual accommodation of vorticity and viscosity can easily be rewritten in terms of the acceleration normal to the direction of flow

$$\omega_w \nu / l = \frac{U_e^2}{l} > or < g \approx e^2, \tag{5'}$$

where $\frac{U_e^2}{l}$ is the starting acceleration of the fluid compared with the acceleration of gravity g[m/s^2].

Using Stuart's solutions e^τ, Eq. (5') can define a boundary Reynolds number as

$$R_b \equiv l^2 e^\tau \left(\nu_0^{-1}\right)^{1/\tau} = \frac{U_e^2}{e^2 \nu(0)} \approx \frac{U_e^2}{l\,\mathrm{g}}, \tag{6}$$

where $U_e, e^2, \nu(0) = 1$ are starting conditions at t = 0.

For $\frac{U_e^2}{l} \le g$, the start-up flow is a slow/smooth one where the microflow field induces a low frequency – creeping motion of CBV governed by the frictional shearing stress, $\left(\tau_{ss} / \rho U_e^2\right)^{1/2} = \frac{V_f}{U_e}$, with the friction velocity, V_f, i opposite phase with U_e. The most important result of the hypothesis of the thixotropic fluid is related to its capability to describe the setting in a circulation at fluid-solid boundaries. The near-wall viscous flow can never be circulation-creating.

For $\frac{U_e^2}{l} > g$, the start-up flow is a fast/impulsive one inducing the high frequency oscillating motion that obeys the law of angular momentum and circulation-preserving with the invariant potential (molecular thermal) energy towards a rotating reference system.

At $t = 0$, the CBV is broken in two-contra-rotating halves with size preserving (e, e^{-1}) and the possibility of direction change of the common rotation axis, Fig. 1.

This halving feature of CBV is the result of a torsion fatigue process. The oscillating motions of the ordered vorticity elementary pairs/dipoles generate longitudinal-transverse vorticity waves penetrating the macroflow field where induce the divers observable instabilities of flow, known as a flow in bulk/en mass or turbulent.

The high frequency oscillating motion of microflow field, in phase with the macroflow field, is described by the sign change of the index τ as

$$R_b \equiv U_e^2 e^{\frac{1}{|\tau|}} \left(\nu_0^{-1}\right)^\tau \text{ for } \mathrm{Re} > R_{bcr} = l^2 e^2 \nu_0^{-1},$$

where $U_b e^{-\frac{1}{|\tau|}}$ and $U_b^2\left(\nu_0^{-1}\right)^\tau$ can be identified as a wave length and frequency of the vorticity dipole, and the critical boundary Reynolds number, $R_{bcr}\left(l=1\right)$, represents the starting conditions for the impulsive start-up as a non-rolling condition for concentrated vorticity which separates the non-periodic creeping motion-laminar flow, from the torsion vibration motion-turbulent flow.

The oscillating motions at high frequency undergo a mixing process and forget their initial/starting conditions, i.e. the intrinsic properties of the thixotropic fluid (concentrated vorticity and viscosity) and the local perturbation stimulus *l*, so that the elastic fluid layer oscillates as a whole with a wavelength $\lambda_l = U_b e^{-\frac{1}{|\tau|}}$ and angular frequency $f_l = U_b^2\left(\nu_0^{-1}\right)^\tau$ along the near-wall flow.

This wall high frequency longitudinal wave is governed by the relation $\lambda_l f_l = U_b = 1$ (phase velocity) and for large wave numbers, $k_l = 1/\lambda_l$, the wave number along the mean flow direction can be replaced with the frequency and vice versa, $k_l = \frac{2\pi f_l}{U_l}$ (Taylor's hypothesis).

The longitudinal compressing/expanding wave of thixotropic fluid is slightly attenuated by a factor $e^{1/\tau}$.

At wall, the high frequency oscillations of fluid are accompanied by transverse standing shear waves whose frequency decreases with the distance from wall.

The transverse shear waves are attenuated at the micro-macro-flow interface by coming back to the initial meaning of vorticity $e^{1/\tau}$ as the angular velocity and the transverse wave number $k_t \approx y$ as a phase lag to the microflow field. The attenuation of transverse shear waves with the distance from wall is by a factor $2\pi^{-1}$.

A more subtle observation is related to the circulation preserving that shows a smooth retarded setting in of the boundary circulation as in Fig 3.

$$\frac{\Gamma_b}{\Gamma_e} \equiv 1 = \left(\frac{\delta-\delta_1}{\delta} + \frac{V_f}{U_e}\right) \text{for laminar flow,}$$

$$\frac{\Gamma_b}{\Gamma_e} \equiv 1 = \left(\frac{U_s}{U_e} + \frac{\bar{V}_f}{U_e}\right) \text{for turbulent flow,}$$

where $\frac{\delta-\delta_1}{\delta} \approx \frac{2}{3}$ is the dilatation/free circulation, $\frac{V_f}{U_e} \approx \frac{1}{e}$ is the friction velocity/CBV, $\frac{U_s}{U_e}$ is the molecular slip velocity/rotation defect and $\frac{\bar{V}_f}{U_e} \approx \frac{1}{e}$ is the average shear stress/molecular thermal effect.

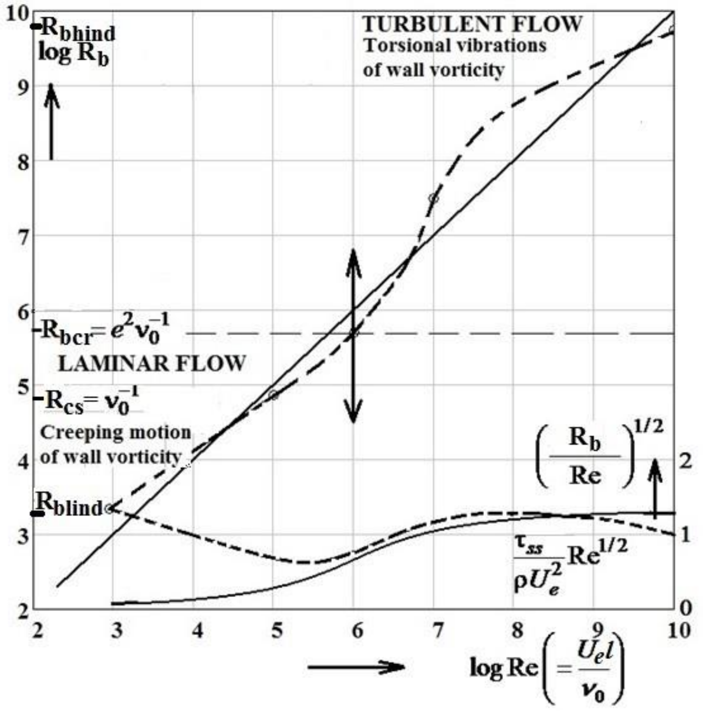

Figure 3 – Non-linearity correction for Re – thixotropic effect.

The molecular-slip velocity represents different expressions of the rotational relativity effect for plane incompressible motions

$$\frac{U_s}{U_e} \approx \frac{\log \mathrm{Re}_l}{2\log \mathrm{Re}_c} \approx \frac{2}{\pi} \approx e^{-1/2}$$

where $\frac{\log \mathrm{Re}_l}{2\log \mathrm{Re}_c}$ is the approaching to resonance state, $2/\pi$ is the rotation defect (period loss) and $e^{-1/2} = M_{inc}$ is the incompressibility condition.

The ratio $\Gamma_b / \Gamma_e \equiv 1$ is a circulation-preserving kind showing the law of translation-rotation partition for plane flows, with constant partition ($2\pi^{-1}$, e^{-1}) both in laminar and turbulent flow state.

The wavy microflow field evolves along a wide frequency spectrum from the low indifference Reynolds number $R_{blind} = e^2 \nu_0^{-1/2}$ - the onset of the weakest waves (TSM-Tolmien-Schlichting waves) up to the high indifference Reynolds number $R_{bhind} = e^{1/2} \nu_0^{-2}$ - the soliton-like wave packet, where the wave system is a slightly damped one with the resonance close to the natural frequency ($U_b^2 \nu_0^{-1}$) emitting vorticity dipoles with this frequency.

The transitional flow displays a strong beat phenomenon with varicose aspect of vorticity for above $R_{bcr} = e^2 \nu_0^{-1}$ where its frequency (≈ 3kHz) is far from the resonance frequency. For above $R_b = e^{2/3} \nu_0^{-3/2}$ a new crumbling of concentrated vorticity is produced and their frequency approaches the resonance frequency followed by a full/en masse turbulent flow. The boundary Reynolds number controls the wave packets playing a role of tuning button switching the frequency band of turbulence.

4. CONCLUSIONS

The splitting of the wall-bounded flow field in the macroflow field dominated by the Newtonian boundary-layer flow and the wavy microflow field $u \le U_b$, obeying the vorticity transport equation and the compliance relation associated to a thin thixotropic fluid hypothesis, has the advantage of using specific methods for the detailed investigation of the near-wall region.

The early local decompositions/representations of the flow field based on the Navier-Stokes equations failed to discover the origin of turbulence and its self-sustaining process which lies much nearer the solid boundary than the present computational methods (DNS, LES) can get the right starting details. Besides the technical difficulties the scarce theoretical understanding of how initial conditions can affect the phenomenon of turbulence led to an ill posing of the problem of turbulence [9]. The cornerstone of turbulence mechanism is the molecular/intrinsic energy of the thixotropic fluid able to store it during the short time-start up of flow via frictional shearing stresses and a molecular thermal process inducing high frequency vorticity wave packets/groups in the microflow field, that transfer its rotation kinetic energy on large wave-numbers to the translation motion of the macrofield with smaller wave-numbers. This complex rotation-translation change process on account of the molecular energy of fluid, with high frequencies and small scales in the near-wall region is a self-sustaining process termed generic turbulence. More exactly the self-sustaining process of turbulence is the mutual induction between the wave-length $\lambda_l = U_b e^{-1/\tau}$, and the frequency $f_l = U_b^2 \left(\nu_0^{-1}\right)^{\tau}$ of the vorticity dipoles as $\lambda_l \cdot f_l = 1$. The turbulence represents an organized microscopic light dissipative world, acting on modes with very large wave numbers, which transfer their molecular thermal energy to the macroflow field.

The perturbation of the translation motion U_e, of a fluid at the fluid-solid boundary breaks off the continuity of motion, that can be restored smoothly as laminar flows with low frequency U_e / l, acceleration $U_e^2 / l \le g$, large scales (small wave numbers) and the discontinuity of wall-bounded flow (u = 0), or impulsively as turbulent flow involving large accelerations $U_e^2 / l > g$, high frequency, small scales (large wave numbers), and the molecular slip at wall.

The high frequency large wave numbers proportionality ($2\pi f_l / U_b = k_l$) for turbulent wall-bounded flows known as Taylor's hypothesis is a more general one that shows that any incompressible flow disturbed by the boundaries guiding flow, into a constant potential energy region, is restored in time without momentum and energy loss, but accompanied by small amplitude - high-frequency fluctuations superimposed on the main flow.

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