Nearest Wall Flows – the Genuine Turbulence

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Abstract: *Traditionally, the wall-bounded flows are boundary-layer like flows where the velocity gradients normal to the wall ∂u/∂y are very large and the velocity monotonously increases from zero to about the velocity free flow, $U_e = 0.99\ U_\infty$. However, the instabilities of flow, resulting from the reaction of the solid boundary against onset of flow, originate in the nearest-wall region where the mean velocity varies around of the velocity U_b = 1m/s constituting the wavy microflow field dominated by the boundary vorticity-viscosity mutual changes. The paper is aiming at presenting the main features of the microflow field and a new vision of the turbulence phenomenon illustrated trough canonical boundary-layer flows.*

Key Words: *Boundary layer, Vorticity waves, Laminar-turbulent transition*

1. INTRODUCTION

The wall-bounded flow as boundary layer, pipe and channel flows present particular features relative to those in the free shear flows far-away from their boundaries, i.e. are slower and a little out of sync. Firstly, these flows are dominated by shear stresses everywhere in the flow field, where the physical presence of solid boundaries, inevitable for fluid media with a non-preferred shape, creates some restrictions and influences with important consequences on the development of shear flow including its turbulent behavior. These concerns the effect of initial and upstream conditions/starting conditions on the flow state when the motion starts from rest and the boundary vorticity and viscosity mutually adjust themselves continuously according to these conditions [1]. Secondly, the more-subtle issues relate to the effect of the wall on the inherent flow dynamics at both macroflow/ velocity and microflow/ vorticity scales. Such effects are non-perceptible at all scales in laminar flows, but they are easily observable in both macroflow and microflow fields of turbulent flows through the steep mean velocity gradients and the length and time scales of the local turbulence in the vicinity of the surface. The latter are associated with the high frequencies and small scales of the near-wall turbulence relative to free shear flows. Therefore, the scope of this paper is to study the question of how initial and upstream conditions affect the laminar-turbulent transition and what these might imply about understanding genuine turbulence. A holistic approach proposed for the whole evolution of turbulence phenomenon, i.e. from origin to a final state, is illustrated by the canonical Prandtl, Couette and Stokes flows [2]. They play a fundamental role in the development of turbulent shear flows in general, far beyond the considered particular examples. Moreover, it is shown that the Stokes's second problem is a fictional problem, i.e. it is physically ill-defined. In fact it comes back to the first problem as soon as the Reynolds effect is taken into account for supercritical values. The above model-problems also highlight physical phenomena behind the mathematical challenges.

2. PHYSICAL CONSIDERATIONS

The most common fluids, air and water in particular, behave easily different from a linear viscous Newton fluid when they are subjected to severe loads and their flows exceed a critical Reynolds number, i.e. for turbulent flows. They possess certain intrinsic elastic properties which at the onset of a motion/flow can absorb/store the kinetic energy during flow-solid boundary collisions as a "latent heat" of compressing, then this being recovered in the flow field by an intricate vorticity wave system involving an entropic spring process. Thus the knowledge of the microflow field near physical boundaries is vital for any real flow, where the flow-boundary interaction makes the essential difference between the instability features of turbulent shear flows of different kinds (wall-bounded flows: pipes/channels, boundary layers, and free: jets, wakes, mixing layers).

The deviant behavior of flows near boundaries arises only in motion as the reaction of a solid boundary on the flow inducing some anisotropy of the components of stress through the variations of the components of vorticity (angular acceleration) and shear viscosity (a thixotropic behavior).

The thixotropic-like behavior of the near-wall fluid means that a thin non-Newtonian fluid has a memory which affects its response to applied stresses in a manner different from that of the Newtonian viscous fluid. Instead of the Rivlin-Erickson's constitutive equation [3], based on a time expansion of the relative deformation tensor, the history of the deformation of shear-thinning fluid is taken into account by variations of the boundary vorticity and of the viscosity associated with a compliance/accommodation relationship [1], Fig. 1. This approach involves new entities that will be introduced in the sequel.

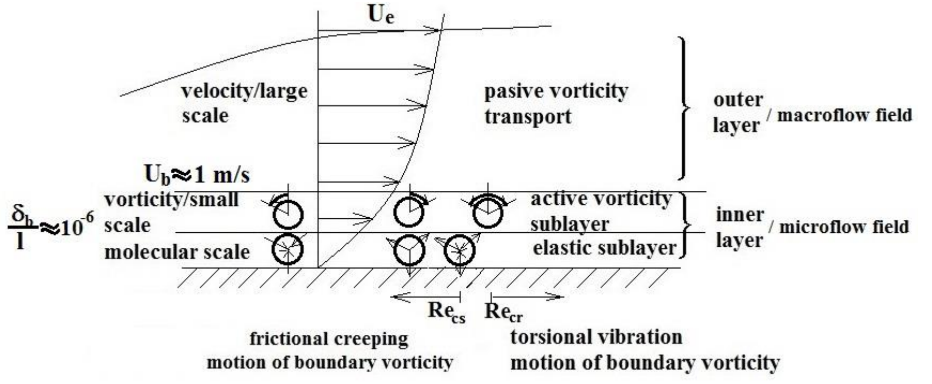

Figure 1. – Physical discrete-continuum model of wall-bounded flows; Re_{cs} – no slip condition, Re_{cr} – no rolling condition

The question of the fluid in a wall-bounded flow perturbed by its inherent limitations and influences needs to be placed in a deeper context, where the disturbance development at a fluid-solid interface is a complex process involving four near concomitant events: (1) the starting boundary vorticity creation (anisotropy of strains), (II) the change of fluid-microstructure (thixotropic-like behavior), (III) the localized mutual induction between vorticity and viscosity (compliance/ accommodation relationship), (IV) the action on the main flow field (self-sustaining mechanism of turbulence).

I) The perturbation of a flowing fluid caused by the presence of a physical boundary is in fact a kind of impact process between the flow and its defining boundaries, where the momentum exchange between colliding fluid-solid surfaces takes place within a short time of the contact with a zero mass flux. As a result, the initial velocity distribution is rapidly skewed/ squeezed; the vorticity is created and organizes itself into more and more concentrated structures, so that at a boundary/wall there is a set of point-vortices/vorticity balls, Fig. 2. Stuart's solutions like Gaussian pulse (e^{τ} solutions, $\tau \in [0,2]$ [5]) can describe such strong vorticity concentrations developing in skewed shear layers, i.e. large angular accelerations of fluid. The concentration level of vorticity is estimated on an ln-scale e^{τ} from e^0 - sparse/weak vorticity, up to e^2 - highly concentrated vorticity, staying hydrodynamically stable; the index τ is a measure of the concentration of the vorticity local to the near-wall region.

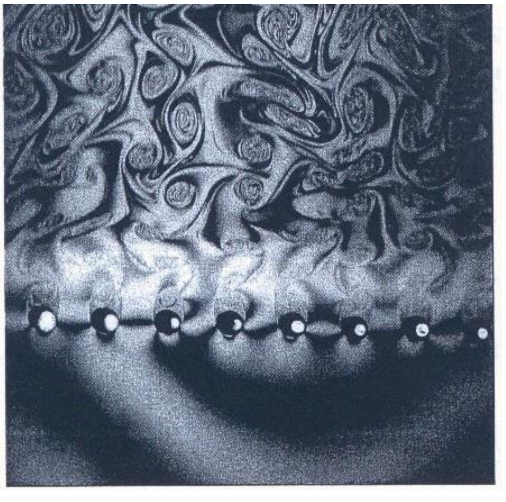

Figure 2. – Two-dimensional turbulent flow visualized in a soap film [4]

Generally, any type of initial perturbation evolves into Kelvin wave packets [6]. However, the weak fluid-boundary perturbations initiated at the onset of motion evolve differently depending on the nature of the fluid medium considered: in a Newtonian viscous fluid the sparse vorticity lines coincide with the streamlines, and the concentrated vorticity move as streak lines of particles building different patterns in terms of the concentration, and then when the Reynolds number exceeds its critical value suddenly generates an apparent disordered Brownian-like motion-turbulent flow. In a non-linear thixotropic fluid, the concentrated boundary vorticity and shear viscosity mutually adjust and mitigate continuously their stresses through a self-sustaining controlled process implying (longitudinal-transverse) shear wave packets-like solitons [1]. From the physical point of view, the concentration of vorticity as the reaction of boundaries is a local compression (suction) of flow induced by torsion of vorticity wires. The dual concept proposed here, the concentrated boundary vorticity (CBV)-thixotropic fluid, allows a better understanding of the boundary vorticity creation and its dynamics as vorticity waves developed in a viscoelastic medium/fluid. The true problem of the boundary vorticity dynamics is the problem of its origin as an initially localized perturbation and the successive development of vorticity waves (small amplitude-torsion vibrations) in the form of oscillating boundary/wall torsion pressures in a microflow field.

II) Concomitant with the boundary vorticity creation the impact process at the start up of flow induces microstructure changes and at large strains; the viscous behavior of the fluid itself becomes nonlinear resulting in a time dependent shear property $\nu = \nu(t)$, known as the thixotropic behavior of the flowing fluid [7].

That is, the fluid starting from rest behaves as the reactive/absorbent medium after it has been allowed to stand for a time, and if it is subjected to more severe loads, it behaves as a dispersive/elastic medium occurring fast longitudinal compressing/ expanding waves (L) and an intrinsic/molecular slip. This thixotropic behavior of fluid can be described by a Klein-Gordon-like wave equation [8]

$$\frac{d^2\nu(x)}{dx^2} = \frac{1}{U_e^2}\left(\omega_0^2 - \omega^2\right)\nu(x), \tag{1}$$

where $\omega_0 = \dfrac{c^2}{\nu_c(=1)} \approx \nu_0^{-1}$ (the inverse of steady/equilibrium kinematic viscosity or fluidity) is the natural angular frequency of a viscous fluid suddenly accelerated at sonic velocity *c*; ω_0 is the first zero of the Fourier coefficient $B(\omega)$ of a square pulse/impact.

Equation (1) has a time-dependent exponentially decaying amplitude solution for ω (low frequency) below the natural frequency ω_0, i.e. $\mathrm{Re}_l < \mathrm{Re}_c \approx \nu_0^{-1}$,

$$\nu = \nu_0^{\left(1-\frac{1}{n}\right)} \cos\left(\mathrm{Re}_l^{\frac{1}{2}} \frac{U_e}{l} t\right), n = \log \mathrm{Re}_l, \tag{2'}$$

and an oscillating lightly decaying amplitude solution for ω (high frequency) above ω_0 or $\mathrm{Re}_l > \mathrm{Re}_c$

$$\nu = \nu_0^{\left(1-\frac{1}{n}\right)} \cos(\omega_D t), \omega_D = \omega_0\sqrt{1-\varsigma^2}, \varsigma \le \frac{1}{4}. \tag{2''}$$

The damping coefficient ζ ($\varsigma = 1 - \dfrac{\log \mathrm{Re}_l^{\frac{1}{2}}}{\log \mathrm{Re}_c}$) separates the exponential decay in Equation (2’) (power law, ζ = 1) from the oscillating lightly damped decay of viscosity in Equation (2’’) ($\varsigma \le 0,25$ is the model of an oscillating lightly damped thixotropic fluid).

Figure 3 illustrates the response of the thixotropic fluid impacted at the onset of motion: at small Reynolds numbers $\mathrm{Re}_l < \mathrm{Re}_c$, the fluid microstructures cannot respond quickly, and we see an exponentially decaying amplitude response, while at high Reynolds number, the microstructure can adjust itself continuously, i.e. it has an oscillating lightly damped response showing viscoelastic effects.

The nonlinear viscoelastic effects of fluid shows a more general interpretation of the Reynolds number as the ratio of maximum frequency at boundaries, $U_e^2\nu_0^{-1}$ to the mean angular frequency in free shear flow, U_e / l.

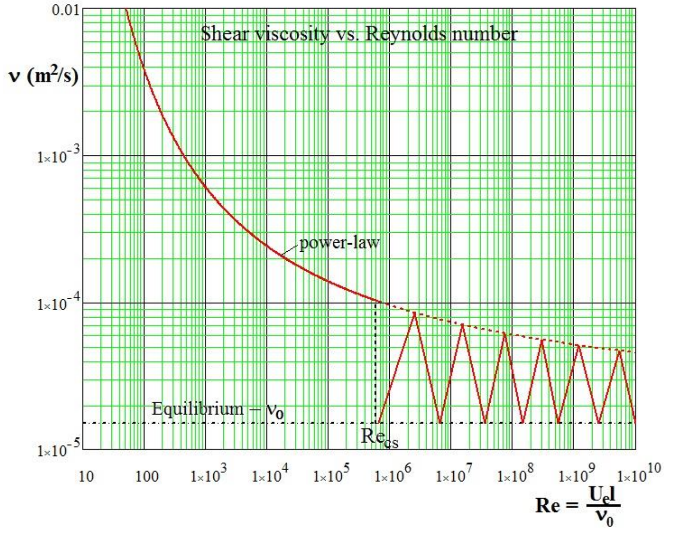

Figure 3. – The thixotropic-like behavior of fluid after the “start-up” of flow

$$\frac{U_e^2 \nu_0^{-1}}{U_e / l} \equiv \frac{U_e}{\nu_0 / l} = \mathrm{Re}_{cs}\,, \tag{3}$$

which for $U_e = 1$ m/s, $l = 1$ m is a critical value related to the boundary $\left(BC : \nu_0^{-1}\right)$ and initial $\left(IC : U_e, l\right)$ conditions. When the Reynolds number is below this value the frictional shearing stresses (with low frequency) are large enough to smooth instabilities in the flow, while above Re_{cs} the strong shear waves (with high frequency) in the microflow largely overcomes the inertia of the macroflow, exponential growth of instabilities occurs, and the flow en masse becomes turbulent. This critical value theoretically found from the thixotropic fluid hypothesis will be next subjected to a vorticity correction. An important conclusion drawn from Eq. 2’’ is that the zero-shear viscosity, the target of mathematicians, is a false one physically impossible to attain, being a misconcept. The conclusion is supported by transient measurements of the viscosity for different thixotropic materials.

III) A viscoelastic model can have thixotropy introduced if the particles that give the viscous and elastic responses, in this case the concentrated/twisted vorticity, are made to change after a time to a purely viscous behavior. An expression for the rate of change of fluidity (the inverse of viscosity) can be obtained by the coupling of intrinsic properties of the thixotropic fluid (concentrated vorticity and time depending viscosity) in the form of the boundary/wall torsion pressure,

$$\frac{1}{\rho} p_{torsion,w} \equiv \omega_w \nu = U_e^2 \text{ on } \partial \mathrm{B}, \tag{4}$$

where $p_{torsion,w}$ is the torsion pressure at the wall, $\omega_w = e^{\tau}$ is the vorticity concentrated into a point on a solid surface (∂B), τ is a torsion/concentration index $\tau \in [0, 2]$, and $\nu(t)$ denotes the change of viscosity during the post-impact flow which is able to adjust itself continuously. The model of the thixotropic fluid based on the shear compliance/accommodation relationship (4) entails some comments. The left hand equality is a product of the torsion deformation of vorticity and the response of viscosity (or decreasing fluidity) defining a torsion pressure (suction) at the walls. The right hand equality shows firstly, that the law of equal action (U_e^2 - dynamic pressure) and reaction ($p_{torsion,w}$ - torsion pressure) is satisfied for laminar flow as Newton's law of friction, but in fluctuating near-wall viscous flows this has been wrongly interpreted and applied up to now. Secondly, the mutual accommodation of vorticity and viscosity can easily be rewritten in terms of the acceleration normal to the direction of the main flow

$$\omega_w \nu / l = \frac{U_e^2}{l} > or < g \approx e^2, \tag{4'}$$

where $\frac{U_e^2}{l}$ is the starting acceleration of the fluid compared with the acceleration of gravity g[m/s^2]. Using Stuart's solutions e^τ and Eqs. 2 for the $\nu(t)$ evolution, Eq. (4') can define a boundary Reynolds number as

$$R_b \equiv l^2 e^\tau \left(\nu_0^{-1}\right)^{1/\tau} = \frac{U_e^2}{e^2 \nu(0)} \approx \frac{U_e^2}{l\mathrm{g}}, \tag{5}$$

where $U_e, e^2, \nu(0) = 1$ are starting conditions at t = 0. For $\frac{U_e^2}{l} \le g$, the start-up flow is a slow/smooth one, where the microflow field induces a low frequency – creeping motion of CBV governed by the frictional shearing stress, $\left(\tau_{ss} / \rho U_e^2\right)^{1/2} = \frac{V_f}{U_e}$, with the friction velocity, V_f, in opposite phase with U_e. The most important result of the hypothesis of the thixotropic fluid is related to its capability of describing the setting in a circulation at fluid-solid boundaries. The near-wall viscous flow can never be circulation-creating. For $\frac{U_e^2}{l} > g$, the start-up flow is a fast/impulsive one inducing the high frequency oscillating motion that obeys the law of angular momentum and circulation-preserving with the invariant potential (molecular thermal) energy towards a rotating reference system [9]. At $t = 0$, the CBV is broken into two-contra-rotating halves with size preserving (e, e^{-1}) and the possibility of direction change of the common rotation axis, see Fig. 2. This halving feature of the CBV is the result of a torsion fatigue process. The oscillating motions of the ordered vorticity elementary pairs/dipoles generate longitudinal-transverse vorticity waves penetrating the macroflow field where they induce the divers' observable instabilities of flow, known as a flow in bulk/en mass or turbulent flow. The high frequency oscillating motion of the microflow field, in phase with the macroflow field, is described by the sign change of the index τ as

$$R_b \equiv U_e^2 e^{\frac{1}{|\tau|}} \left(\nu_0^{-1}\right)^\tau \text{ for } \mathrm{Re} > R_{bcr} = l^2 e^2 \nu_0^{-1},$$

where $U_b e^{-\tau}$ and $U_b^2 \nu_0^{-(1+\tau)}$ can be identified as a wave length and frequency of the vorticity dipole, and the critical boundary Reynolds number, $R_{bcr}(l = 1)$, represents the starting conditions for the impulsive start-up as a non-rolling condition for concentrated vorticity which separates the non-periodic creeping motion-laminar flow, from the torsion vibration motion-turbulent flow. The oscillating motions at high frequency undergo a mixing process and forget their initial/starting conditions, i.e. the intrinsic properties of the thixotropic fluid (concentrated vorticity and viscosity) and the local perturbation stimulus *l*, so that the elastic fluid layer oscillates as a whole with a wavelength $\lambda_l = U_b e^{-\tau}$ and angular frequency $f_l = U_b^2 \nu_0^{-(1+\tau)}$ along the near-wall flow. This wall high frequency longitudinal wave is governed by the relation $\lambda_l f_l = U_b = 1$ (phase velocity) and for large wave numbers, $k_l = 1/\lambda_l$, the wave number along the mean flow direction can be replaced with the frequency and vice versa, $k_l = \frac{2\pi f_l}{U_l}$ (Taylor's hypothesis). The longitudinal compressing/ expanding wave of thixotropic fluid is slightly attenuated by a factor $e^{-\tau}$. At the wall, the high frequency oscillations of the fluid are accompanied by transverse standing shear waves whose frequencies decrease with the distance from the wall. The transverse shear waves are attenuated at the micro-macro-flow interface by coming back to the initial meaning of vorticity $e^{1/\tau}$ as the angular velocity and the transverse wave number $k_t \approx y$ as a phase lag to the microflow field. The attenuation of transverse shear waves with the distance from the wall is by a factor of $2\pi^{-1}$. A more subtle observation is related to the circulation preserving that show a smooth retarded setting in of the boundary circulation as in Fig 4.

$$\frac{\Gamma_b}{\Gamma_e} \equiv 1 = \left(\frac{\delta - \delta_1}{\delta} + \frac{V_f}{U_e} \right) \text{ for laminar flow,}$$

$$\frac{\Gamma_b}{\Gamma_e} \equiv 1 = \left(\frac{U_s}{U_e} + \frac{\bar{V}_f}{U_e} \right) \text{ for turbulent flow,}$$

where $\frac{\delta - \delta_1}{\delta} \approx \frac{2}{3}$ is the dilatation/translation inertia, $\frac{V_f}{U_e} \approx \frac{1}{e}$ is the friction velocity/ rotary inertia, $\frac{U_s}{U_e}$ is the molecular slip velocity/convection velocity and $\frac{\bar{V}_f}{U_e} \approx \frac{1}{e}$ is the turbulent skin friction shear stress/rotary inertia effect.

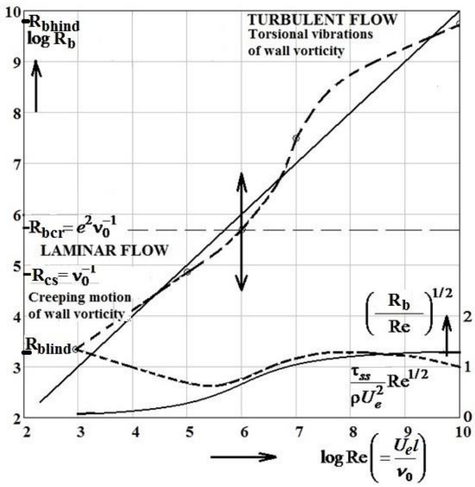

Figure 4. – Non-linearity correction for Re – thixotropic effect

The molecular-slip velocity represents different expressions of the rotational relativity effect for boundary rotor translational motions

$$\frac{U_s}{U_e} \approx \frac{\log \mathrm{Re}_l}{2\log \mathrm{Re}_c} \approx \frac{2}{\pi} \approx e^{-1/2}$$

where $\frac{\log \mathrm{Re}_l}{2\log \mathrm{Re}_c}$ is the approaching to resonance state, $2/\pi$ is the rotation inertia effect and $e^{-1/2} = M_{inc}$ is the incompressibility condition.

The ratio $\Gamma_b / \Gamma_e \equiv 1$ is a circulation-preserving kind showing the ratio of translation-rotation partition for plane flows, with constant partition ($2\pi^{-1}$, e^{-1}) both in laminar and turbulent flow state. The wavy microflow field evolves along a wide frequency spectrum from the low indifference Reynolds number $R_{blind} = e^2 \nu_0^{-1/2}$ - the onset of the weakest waves (TSM-Tolmien-Schlichting waves) up to the high indifference Reynolds number $R_{bhind} = e^{-1/2} \nu_0^{-2}$ - the soliton-like wave packet, where the wave system is a slightly damped one with the resonance close to the natural frequency ($U_b^2 \nu_0^{-1}$) emitting vorticity dipoles with this frequency. The transitional flow displays a strong beat phenomenon with varicose aspect of vorticity for above $R_{bcr} = e^2 \nu_0^{-1}$ where its frequency (≈ 3kHz) is far from the resonance frequency. For Reynolds number above $R_b = e^{2/3} \nu_0^{-3/2}$ a new crumbling of concentrated vorticity is produced and their frequency approaches the resonance frequency followed by a full/en masse turbulent flow. The boundary Reynolds number controls the wave packets playing a role of tuning button switching the frequency band of turbulence.

IV) The splitting of the wall-bounded flow field in the macroflow field dominated by the Newtonian boundary-layer flow and the wavy microflow field $u \le U_b$, obeying the vorticity transport equation and the compliance relation associated to a thin thixotropic fluid hypothesis, has advantage to use specific methods for the detailed investigation of the near-wall region.

The early local decompositions/representations of the flow field based on the Navier-Stokes equations failed to discover the origin of turbulence and its self-sustaining process which lie much nearer the solid boundary than the present computational methods (DNS, LES) can get the right starting details [10]. Beside the technical difficulties the scarce theoretical understanding of how initial conditions can affect the phenomenon of turbulence that led to an ill posing of the problem of turbulence [1]. The cornerstone of the turbulence mechanism is the molecular/intrinsic energy of the thixotropic fluid able to store it during the short time-start up of flow via frictional shearing stresses and a molecular thermal process inducing high frequency vorticity wave packets/groups in the microflow field; these transfer its rotation kinetic energy on small distinct wave-numbers to the translation motion of the macrofield with random large wave-numbers. This complex rotation-translation change process on account of the molecular energy of fluid, with high frequencies and small scales in the near-wall region is a self-sustaining process termed generic turbulence. More exactly the self-sustaining process of turbulence is the mutual induction between the wave-length $\lambda_l = U_b e^{-\tau}$, and the frequency $f_l = U_b^2 \nu_0^{-1}$ of the vorticity dipoles as $\lambda_l \cdot f_l = 1$. The turbulence represents an organized microscopic light dissipative world, acting on modes with small distinct wave numbers, which transfer their molecular thermal energy to the macroflow field. The perturbation of the translation motion U_e, of a fluid at the fluid-solid boundary breaks off the continuity of motion, that can be restored smoothly as laminar flows with low frequency U_e/l, acceleration $U_e^2/l \le g$, large scales (small wave numbers) and the discontinuity of wall-bounded flow (u = 0), or impulsively as turbulent flow involving large accelerations $U_e^2/l > g$, high frequency, small scales (large wave numbers), and the molecular slip at wall. The high frequency – large wave numbers proportionality ($2\pi f_l/U_b = k_l$) for turbulent wall-bounded flows known as Taylor's hypothesis is a more general one that shows that any incompressible flow disturbed by the boundaries guiding flow, into a constant potential energy region, is restored in time without momentum and energy loss, but accompanied by small amplitude – high-frequency fluctuations superimposed on the main flow.

3. FORMULATIONS FOR MICROFLOW FIELD

This flow kind belongs to the fluid domain in the vicinity of solid surfaces bounding a flow with small velocities, $U_b \le 1$ m/s, but large angular velocities $\omega_w > 1$, where the flow is effectively two-dimensional and is dominated by the rotation of fluid, shear stresses and concentrated vorticities, resulting from the reaction of solid boundaries on the flowing fluid. The physical presence of walls introduces the constraints and influences on the behavior of flow with important consequences on the boundary vorticity dynamics, but unfortunately poorly understood and essentially not known. The basic issue is the no-slip boundary condition at a solid-fluid interface, since this assumption violates the law of equal action and reaction and this is impossible to derive from first principles. The center of understanding of microflows is the creation of the concentrated vorticity associated with the thixotropic like behavior of fluid able to describe right starting details, obscured by early ill-defined concepts (linear Newtonian fluid, boundary vorticity flux (BVF) [11] and its creation, empirically defined Reynolds number [12], etc.). The microflow field is a complex behavior at a fluid/solid interface involving a mutual induction between the large concentrations of vorticity occurring in wall-bounded flows and the changes of physical properties of fluid, including shear rate, the effect of torsion pressure and molecular slip. All these processes are directly related to the action of initial and boundary or starting conditions on the development of turbulent shear flows, including the birth of genuine/ shear turbulence. For the understanding the mechanism of the turbulence phenomenon and its origin, two simple formulations of microflow field are shown in the sequel. The first one concerns the 1-D vorticity equations yielding the boundary vorticity state and the starting/impact conditions general valuable for any wall-bounded flow, and the second is the similar solutions solely describing the complete evolution of flows from wall-collision (y = 0) up to their post-impact full relaxation/ attenuation, i.e. the exterior boundary condition $U_\infty(y \to \infty)$. Any experimental and/or computational turbulence tests can get intermediate deactivating degrees of a flow, resulting in much diversity of the observed turbulence phenomenon, mostly influenced by the size of the investigation domain. The truncating approximations at boundaries ($u\left(y^+ \approx 1\right) = 0$ and $0.99U_\infty\left(y \to \infty\right)$) are plausible explanations for many controversial results in the research of turbulence. State of the art for both the experimental and computational methods shows that these boundary conditions are far from reaching a full attenuation state and probably the 2-D turbulent flows are the fictional ones. Since the light damping/ decay of turbulence is easily observed in the case of free shear flows, the jet like flows are mostly experimentally and computationally investigated. Sometimes, the poor knowledge of the starting conditions and the intrinsic mechanism of turbulence makes many results extremely difficult to interpret, understand and explain [13].

The CBV formulation for the microflow field yields the starting conditions ($SC \equiv IC + BC$) for any wall-bounded flow from

$$\frac{\mathrm{d}\omega}{dt} = \nu \frac{\partial^2 \omega}{\partial x^2} \text{ - vorticity equation}, \quad (6)$$

$$\omega\nu = U_e^2 \text{ - mutual-induction relation}, \quad (7)$$

and a new interpretation for the issue of Taylor's frozen-field hypothesis, where U_e is seen as a phase velocity and the microflow field is "out of phase", $\frac{\partial}{\partial t} = U_e \frac{\partial}{\partial x}$, in laminar flow, $\left(\mathrm{Re}_l \le R_{bcr}\right)$ and "in phase", $\frac{\partial}{\partial t} = -U_e \frac{\partial}{\partial x}$, in turbulent flow.

The true problem of turbulence dynamics is the problem of its origin and successive development from start-up at $R_{bcr} = e^2 \nu_0^{-1}$ up to an ultimate (statistical) state, $R_{bhind} = e^{-1/2}\left(\nu_0^{-1}\right)^2$, whence the turbulence dies away in time. Since the CBV is the backbone of shear turbulence, the microflow field is effected approximately with zero vorticity diffusion ($\frac{\partial u}{\partial y} \approx 1$ is the light diffusion condition) and this is seen as an oscillating thin concentrated vorticity layer adjacent to the solid wall (∂B), in which the flow is highly rotational and causes an oscillating skin friction stress (molecular slip effect). In an unidirectional shear flow on the semi-infinite plane, $x = l, y \ge 0$, the velocity and vorticity fields taken the form

$$\mathbf{u} = \left(u(y,t), 0, 0\right), \boldsymbol{\omega} = \left(0, 0, \omega(x,t)\right), \omega\left(0\right) = e^2, \omega\left(R_{bcr}\right) = e^0. \quad (8)$$

For $\mathrm{Re}_l \le R_{bcr}$ the CBV at y = 0 is given by Eqs. (6, 7) as

$$g''\left(\xi\right) - gg'\left(\xi\right), g\left(\xi\right) = \frac{e^\tau}{e^2}, \xi = \log \mathrm{Re}_l, \quad (9)$$

indicating in laminar flows clearly a creeping/retarded motion, out of phase, with a temporal light dispersion effect at solid boundaries [14].

For $\mathrm{Re}_l > R_{bcr}$ the near harmonic motion of CBV is described by

$$g''\left(\xi\right) + \omega_0^2\left(g\left(\xi\right) + s\,\mathrm{sgn}\, g'\left(\xi\right)\right) = 0, \quad (10)$$

indicating a temporal strong dispersion effect of the concentrated boundary vorticity, where *s* is a characteristic length related to the intrinsic/molecular friction of the thixotropic behavior of fluid $(s = e^{-2})$. The mutual induction between the CBV and the viscoelastic fluid produces a fast longitudinal compressing/expanding wave that propagates along the solid wall with high frequency close to the natural frequency $\omega_0 \approx \nu_0^{-1}$ and has a lightly damped oscillating decay, Fig. 5.

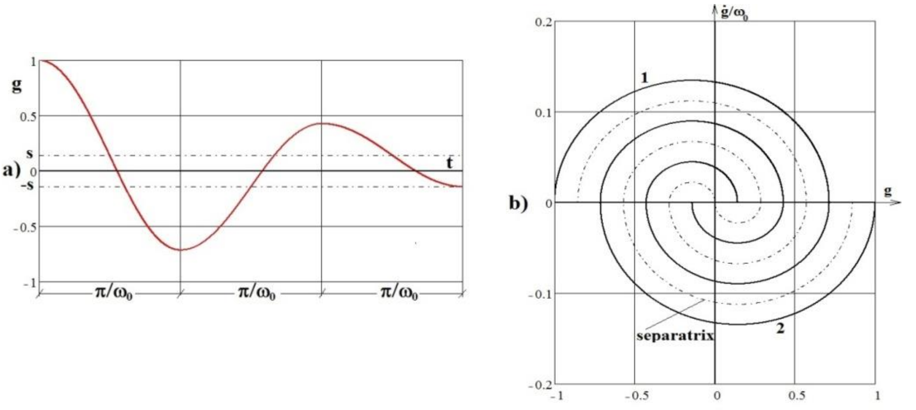

Figure 5. – The oscillating motion of CBV: a) longitudinal lightly damped wave in thixotropic fluid;
b) phase curves

The velocity distribution $u(y,t)$ in the microflow field is a sinusoidal travelling wave

$$u(y,t)=U_b e^{\frac{1-k_t y}{2}}\cos(nt-k_t y),\ n=\log \mathrm{Re}_l,\ k_t=\sqrt{\frac{n}{2\log \nu_0^{-1}}}\ , \tag{11}$$

where $U_b e^{\frac{1-k_t y}{2}}$ is the amplitude decreasing outwards and the distance from the wall $k_t y$ is a phase lag of microflow compared to the fast motion the CBV. The velocity distribution for different times is shown in Fig. 6 indicating that two layers at a distance $2\pi/k_t$ from each other oscillate in phase. This distance is just the wave length of oscillation called the penetration depth or attenuation length. Therefore, the CBV produces three vorticity wave packets that propagate along the normal/transverse and decays exponentially.

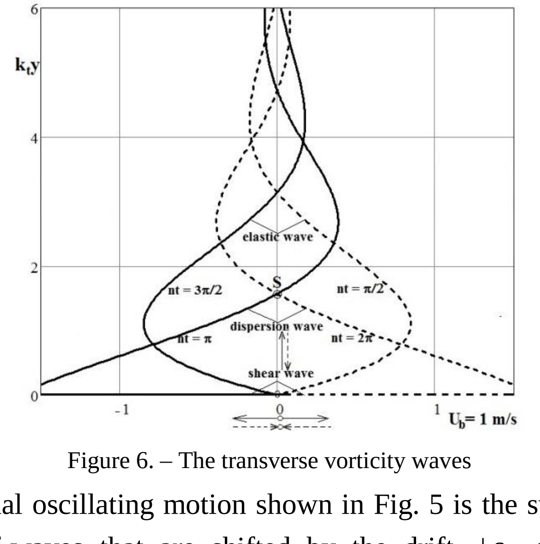

Figure 6. – The transverse vorticity waves

The fast longitudinal oscillating motion shown in Fig. 5 is the successive superposition of near harmonic half-waves that are shifted by the drift $\pm s = e^{-2}$, leaving the linear frequency unaltered, while the slower torsional vibrations/vorticity waves propagates along the normal by wave packets containing three waves: shear wave (T_s), dispersion wave (T_d) and elastic wave (T_e). The dynamic process of the evolution of the three vorticity waves and their interactions play a crucial role in the turbulence phenomenon: the shear wave is responsible from the molecular thermal energy transfer to microflow velocity field, the elastic wave damps down the high acceleration at wall, and the dispersion wave carries the attenuated vorticity dipoles to macroflow field. More subtle observations relate to the points 0 ($y = 0$) and S ($y = \pi/2$) with vanishing friction and velocity that are, the centre of rotational oscillation with angular frequency $f_l = U_b^2\left(\frac{2}{\pi}\nu_0^{-1}\right)\left[s^{-1}\right]$ and center of translational/ slipping oscillation with $\lambda_l = e^{-\tau}U_b\left[m\right]$ respectively. The transverse shear waves from Fig. 6 represent moving and fixed centrodes in rolling contact.

The holistic formulation concerns the approach of wall-bounded flows as a whole: the motion equations governed by the basic principles, and the accurate initial and boundary conditions, where the wall boundary (∂B_w) is given by R_b and the exterior boundary (∂B_∞) tends to infinity, so that the flow perturbed by the start-up/impact on ∂B_w can be exactly attenuated/ damped on the ∂B_∞, and its non-altered/genuine response to the starting condition it is found in the vicinity of the wall (∂B_w). Any fluidic system entailing three indispensable elements: a motion/flow (U_∞), a physical surface/wall (l) bounding the flow and a medium/fluid (ν), evolves differently in terms of what medium hypothesis is chosen for the paradigm of flow. In this context, the unsolved problem of turbulence (T – paradox/ crisis) is a paradigmatic nature one due to the ill-posed starting conditions and the misconception of the medium (linear-viscous/ Newtonian fluid) [1]. The approach of wall-bounded flows uses the similar solutions of the Navier-Stokes equations in the boundary-layer approximation, that are a kind of diffeomorphisms in flow field related to the existence of some smooth global solutions $\mathbf{u}(\mathbf{x},t)$ that evolve, for the large times, into an equilibrium state, characterized by a stationary solution, to Euler $\mathbf{u}_\infty(\mathbf{x})$ [15]. According to [13], the various methods of describing and studying turbulent flows, including the direct numerical simulations of Navier-Stokes equations, are more or less incomplete formulations failing to describe the "essence" of turbulence: the origin and its self-sustaining mechanism. Kolmogorov's idea [16] of disregarding the origin of turbulence and the details of its mechanism in favor of a noisy/ spectacular flow far from boundaries is a harmful, obscuring the micro-world of turbulence, i.e. the residence of the origin and the self-sustaining process. In contrast to Kolmogorov's point of view, we show that just by disregarding these detailed flow representations so hopelessly complex to describe, the self-similarity leads to localized singularities at solid boundaries, where the details of the turbulence production-dispersion mechanism are found. The mechanism involves the CBV-viscosity mutual induction generating longitudinal-transverse waves in a thixotropic fluid.

4. RESULTS FOR CANONICAL BOUNDARY-LAYER FLOWS

The coordinate transformation from the variables x, y to new dimensionless variable

$$\xi = \frac{x}{l},\ \eta = \frac{y}{l}\frac{\mathrm{Re}^{1/2}}{\delta(\xi)} = \frac{y}{\overline{\delta}(\xi)}$$

hides the physical importance of solid surfaces as the origin of the reaction forces on the flow directly related to the creation of boundary vorticity and the change of microstructure of fluid. Thus, the boundary-layer equations for plane steady incompressible flow reduce to the ordinary differential equation (ODE) for the stream function $\psi(\xi,\eta) = \frac{lU_N(\xi)}{\sqrt{\mathrm{Re}}}\overline{\delta}(\xi)\,f(\eta)$ [17]

$$f''' + \alpha_1 ff'' + \alpha_2 - \alpha_3 f'^2 = 0, \tag{12}$$

where the constants $\alpha_1, \alpha_2, \alpha_3$ defined as

$$\alpha_1 = \frac{\bar{\delta}}{V}\frac{d}{d\xi}\left(U_N \bar{\delta}\right), \alpha_2 = \frac{\bar{\delta}^2}{V}\frac{U}{U_N}\frac{dU}{d\xi}, \alpha_3 = \frac{\bar{\delta}^2}{V}\frac{dU_N}{d\xi}$$

are unknown functions $U(\xi), U_N(\xi)$ and $\bar{\delta}(\xi)$, determined once the basic/outer flow is specified. Here the Reynolds number $\mathrm{Re} = \frac{Vl}{\nu_0}$ is formed with the reference velocity *V*, the reference length *l* and the equilibrium value ν_0 of the kinematic viscosity or the inverse value of the natural frequency of fluid. The term f''' must be corrected with the starting acceleration effect according to Eq. (4), as $\alpha_4 f''', \alpha_4 = e^{\pm\tau}$.

A. Prandtl boundary-layer flow $\left(\frac{dU}{d\xi} = 0\right)$.

In this case we set $\frac{dU_N}{d\xi} = \frac{dU}{d\xi} = 0, \left(\alpha_2 = \alpha_3 = 0\right)$, and $\alpha_1 = \frac{\bar{\delta}}{V}\frac{d}{d\xi}\left(U\bar{\delta}\right) = 1$, since the thickness scale $\bar{\delta}$ is only fixed up to a numerical factor. With $\alpha_1 = 1, \alpha_2 = \alpha_3 = 0, \alpha_4 = e^{\pm\tau}$, it then follows that

$$\alpha_4 f''' + ff'' = 0, \text{ (Blasius soliton)} \tag{13}$$

with the boundary conditions

$$\begin{aligned} \eta = 0: &\quad f = 0,\ f' = 0, \\ \eta \to \infty: &\quad f' = 1 \end{aligned}. \tag{14}$$

Equation (13) with $\alpha_4 = 1$ early known as Blasius equation, describes transverse standing vorticity/shear waves, called solitons, which retain their identity upon the flow-solid boundary collision. The vorticity solitons depending on the function α_4 are directly related to Re_l via the mutual induction relation (5). The mutual induction function α_4 is responsible for the coupling between the autonomous fast motion of the CBV and the slower non-autonomous macroflow field flow, Fig. 7

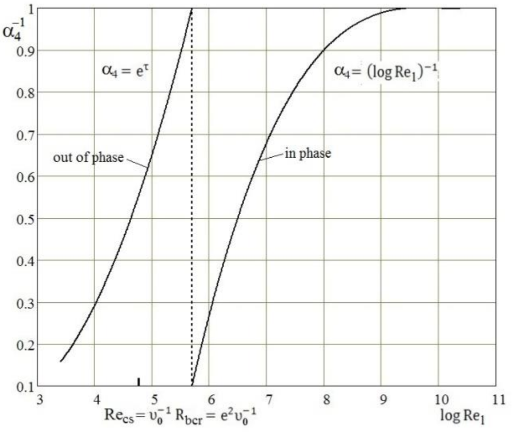

Figure 7. – The mutual induction function $\alpha_4\left(\mathrm{Re}_l\right)$

$$\begin{aligned}\alpha_4 &= e^{\tau} && \text{for } \mathrm{Re}_l < R_{bcr} \text{ (weak coupling)} \\ \alpha_4 &= e^{-\tau} = \left(\log \mathrm{Re}_l\right)^{-1} && \text{for } \mathrm{Re}_l \geq R_{bcr} \text{ (strong coupling)}\end{aligned}, \quad (15)$$

Now once the Reynolds number Re_l is fixed, the solutions of the Blasius soliton can be easily computed by a standard shooting technique. Figure 8 shows the solution of transverse vorticity waves propagating under the form of three wave packets: shear stress wave, *f''*, elastic wave, *f'''*, and sinusoidal dispersion wave, f^{iv}. The super-imposing of vorticity waves and mean velocity *f'* suggest the λ shape for the envelope of vorticity waves (in turbulent flows). Figure 8 makes evident that the wave packet is embedded in the boundary layer as a λ – like wave.

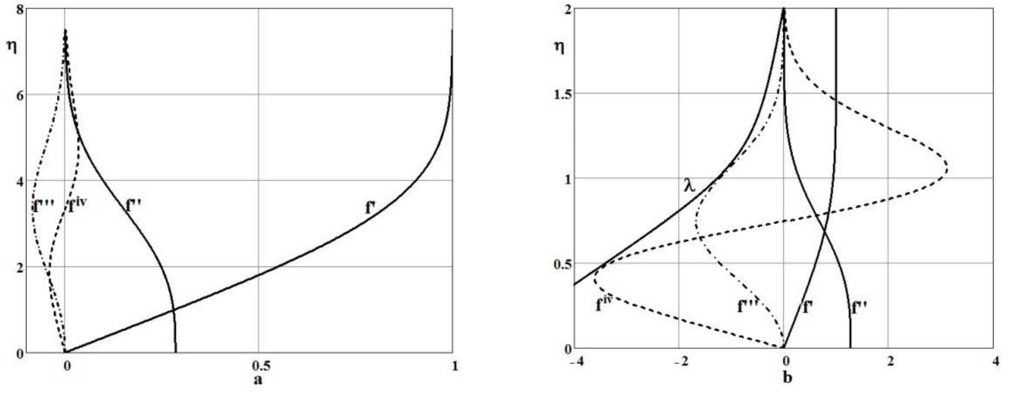

Figure 8. – Transverse vorticity waves in Prandtl flow (*f'* – mean velocity profile): *f''* - shear wave, *f'''* - elastic wave, i^{1v} –dispersion wave for a) $\alpha_4 = e$ (laminar) and b) $\alpha_4 = e^{-2}$ (turbulent)

B. Couette flows

Traditionally, the Couette flow is separately approached as "start-up" flow and its internal flow [17]. In contrast with these distinct approaches we propose a unitary approach, where the time scale is included in the Reynolds number as an initial condition: small Reynolds numbers for the start-up of the laminar flows, $\mathrm{Re}_h \leq \mathrm{Re}_{cs} = 2e\nu_0^{-1/2} \approx 1420$ and the start-up of the turbulent flows at supercritical Reynolds numbers. The ODE of the Blasius soliton is exactly the same as Eq. (13) with different boundary conditions

$$\begin{aligned}&\eta \to +\infty: \quad f' = 1, \\ &\eta = 0: \quad f = 0, \\ &\eta \to -\infty: \quad f' = 0,\end{aligned} \quad (16)$$

Figure 9 shows the solutions for the vorticity field, where the wave packet at solid walls contains only shear stress wave (*f''*) and elastic wave (*f'''*) because of the stronger restriction on the double bounded flow. The suppression of the dispersion wave leads to a pressure increase in the macroflow field as the Reynolds increases. For the critical Reynolds Re_{cs}, the concentrated vorticity (e^2) at the moving wall is broken down in two contra-rotating fragments, there is a rotation center of CBV on each solid wall (O', O'') and the slipping center S of velocity is approximately little shifted towards the moving wall. The adjacent layers at a distance *h*/2 from each other oscillate in opposite phase and the velocity field is regularized as Re_h increases according to $\mathrm{Re}_h^{1/2}\left(\dfrac{\tau_{ss}}{\rho U^2}\right) = \dfrac{1}{\pi}$, Fig. 10. The onset of turbulence is the vorticity jump itself when $\mathrm{Re}_h > \mathrm{Re}_{cs}$ the CBV at the moving wall is halved and the energy storage at impact as torsion boundary pressure is recovered as dynamic pressure of flow.

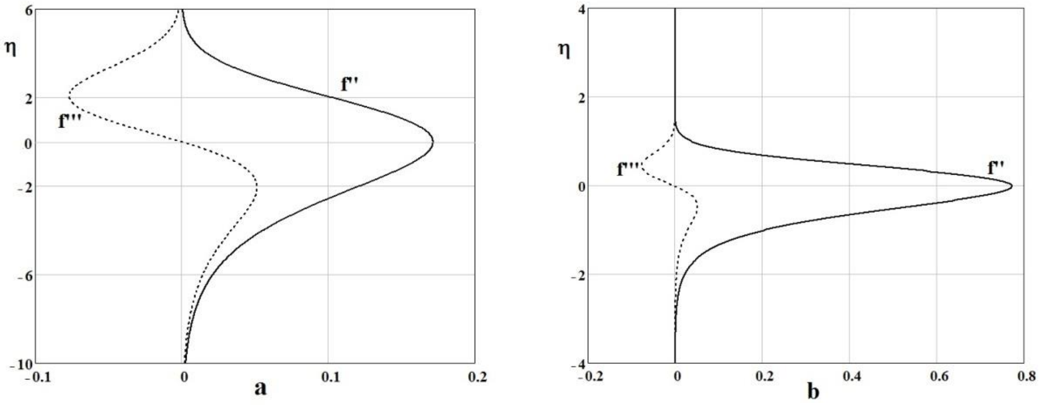

Figure 9. – Transverse vorticity waves in Couette flow: *f''* - shear wave, *f'''* - elastic wave, for a) $\alpha_4 = e$ and b) $\alpha_4 = e^{-2}$

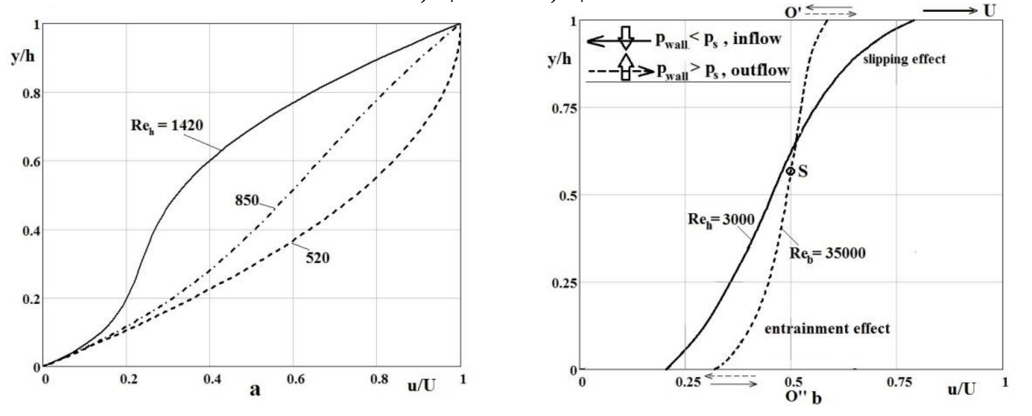

Figure 10. – Start-up of a Couette flow, a: laminar, $U^2/h \le g$, b: turbulent, $U^2/h > g$

It is worth noting that the turbulent flow produces a slipping effect at the moving wall and an entrainment effect in the region of fixed wall. The different behavior of the flow near walls is a consequence of the rotational relativity induced by the phase difference between translation and rotation motions at a fluid-solid interface.

C. Stokes's problems

The Stokes problems excluding the length scales from the analysis of starting process, i.e. the starting acceleration, and by this eluding the law of equal action and reaction these problems, ill defined, turn into only simple mathematical exercises without any physical signification. The misunderstanding of the starting mechanical process led to two different representations of the start up of a flow: suddenly accelerated (infinite) plate-Stokes's first problem (similar solutions), and vibrating plate-Stokes's second problem (without similarity). The second Stokes's problem in fact represents the impulsive start –up of a finite length plate when $U_0^2/l > g$. For $U_0^2/l \le g$, the slow/laminar start-up, the Navier-Stokes equations by introducing the dimensionless similarity variable

$$\eta = \frac{y}{2\sqrt{\nu t}} = \frac{y}{2l}\mathrm{Re}_l^{1/2}, \mathrm{Re}_l = \frac{U_0 l}{\nu_0} \qquad (17)$$

and the function $u/U_0 = f(\eta)$ reduce to the ODE

$$f'' + 2\eta f' = 0 \tag{18}$$

with the boundary conditions $f(0)=1$ and $f(\infty)=0$. The solution is $\frac{u}{U_0} = \text{erfc}\,\eta = 1 - \text{erf}\,\eta$ where the error function $\text{erf}\,\eta = \frac{2}{\sqrt{\pi}}\int_0^{\eta} e^{-\eta^2} d\eta$ and erfc is the complementary error function.

The correction on the velocity distribution of CBV, given by $\eta_{corr} = \frac{y}{2l}\text{Re}_l^{1/2} e^{\tau}$, $\tau \in [0,2]$ is shown in Fig. 11.

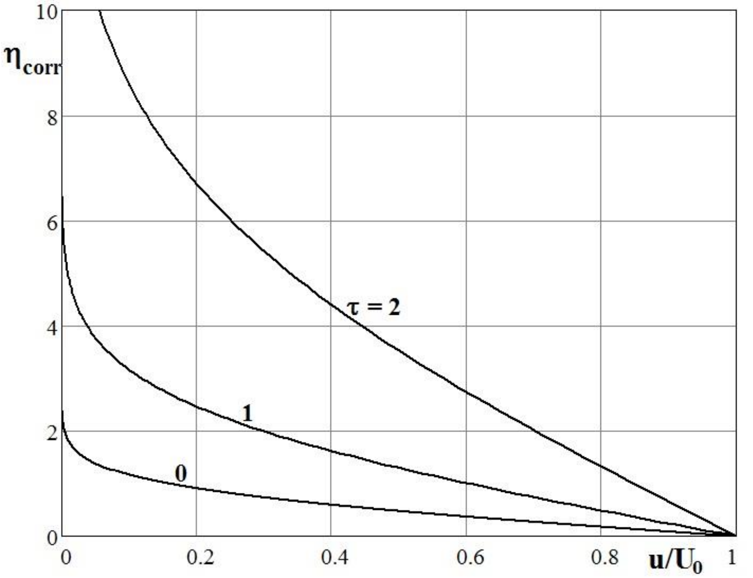

Figure 11. – Distributions of velocity in Stokes's first problem

For $U_0^2/l > g$, the penetrating process of vorticity waves from the fluid-solid boundary into the macroflow field is like that of Fig. 6, and can be described by the local-induction of CBV with constant circulation condition

$$U(y,t) = U_0 \frac{1}{\pi} e^{-k_t y}\left(\cos nt - k_t y\right) \tag{19}$$

where $k_t y = \left(\frac{2\pi}{\Gamma_0}\right) y$ and $\Gamma_0 = U_0 l$ is the circulation assumed constant across the boundary layer.

Figure 12 illustrates the Stokes's second problem where the distribution of mean velocity and vorticity waves, including their λ envelope exhibits the rotational relativity effect of the impulsive motion of a finite plate as entrainment, (e^{-1} proportion at a large scale) and slipping (2/π proportion at a small scale) effects. Compared with the original Stokes's problems, these model problems are more realistic prototypes expressing respectively, the adherence (Fig. 11) and molecular slipping (Fig. 12) effects at a fluid-solid interface.

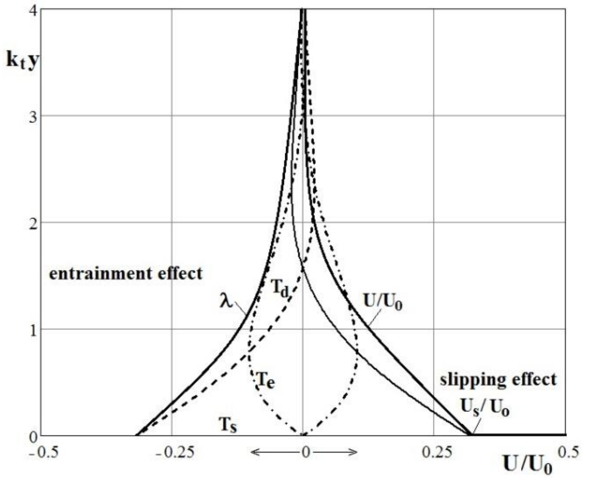

Figure 12. – Transverse vorticity waves in Stokes's second problem

5. CONCLUSIONS

The "start-up" of a flow from rest evolves mostly impulsively when the starting acceleration U_∞^2/l exceeds the acceleration of gravity g and the flow is strong disturbed becoming turbulent. The neglect of such large accelerations led to misconceptions and approaches failing to describe the essence of turbulence, i.e. the origin and the self-sustaining process of turbulence [13]. At the beginning most of the research in turbulence was conducted almost exclusively by the engineering world for solving some practical tasks. The last three decades have been marked by the increasing involvement of physicists and applied mathematicians though still with pretty limited foci. The less interest on the details of the mechanisms of turbulence production and sustainment, promoted particularly by Kolmogorov's ideas [16], led to what now is known as the crisis/ paradox of turbulence. The main paradigmatic misconceptions of turbulence such as Newtonian viscous fluid, boundary vorticity flux, vorticity-rate of strain tensor mutual mechanism of turbulence, eluding the law of equal action and reaction and circulation-preserving near-wall flow, and whereby fail to describe most of issues/ difficulties/ features of turbulence. In the present paper a new flexible paradigm for the turbulence phenomenon is proposed according to the dual concept of concentrated boundary vorticity-thixotropic fluid that can remove the above drawbacks. The suddenly change of the motion state of bodies, with the fluid bounded by a solid surface, involves large accelerations that can be less or more than g. The latter introduces deviant behavior from continuity hypothesis of both motion/flow and fluid microstructure called turbulence and thixotropy, respectively. This localized perturbation case has been approached in Fourier sense by a continuum of modes, that is by packets of waves: hydrodynamic instabilities as disturbances to the velocity field (transverse shear waves) and instabilities of fluid-microstructure as a thixotropic effect able to adjust itself continuously with the flow state. The perturbation of the translation motion U_e of the fluid at a fluid-solid boundary interface breaks off the continuity of motion, that can be restored smoothly as laminar flows with low frequency U_e/l, acceleration $U_e^2/l \le g$, large scales (small wave numbers) and the discontinuity of wall-bounded flow (u = 0), or impulsively as turbulent flow involving large accelerations $U_l^2/l > g$, high frequency (≈ 100 kHz), small scales (large wave number proportional to frequency and the molecular slip of velocity at wall $U_s/U_l \approx 2/\pi$). The high frequency – large wave number proportionality ($2\pi f_l/U_s = k_l$) for turbulent wall-bounded flows known as Taylor's hypothesis is a consequence of the invariance of rotation motion where any incompressible plane flow disturbed by the boundaries guiding a flow is restored by rotation oscillations with high frequency on account of the molecular potential energy of fluid, exhibiting a rotational relativity. The present paradigm using the concentrated vorticity – thixotropic fluid concept has allowed finding the mechanisms of turbulence phenomenon at molecular scale that physically represents early compressibility effects ($M \le e^{-1/2}$). As a general rule, any plane turbulent flow can be restored as a smooth (in statistic sense) plane flow in ($2/\pi$) proportion carrying distributed vorticity/dipoles in (e^{-1}) proportion, without energy loss. The new vision on the turbulence phenomenon has been illustrated by some model problems.

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REFERENCES

[1] H. Dumitrescu, V. Cardos, A wave theory of incompressible fluid, *INCAS BULLETIN*, vol. **8**, issue 2, (online) ISSN 2247–4528, (print) ISSN 2066–8201, ISSN–L 2066–8201, DOI: 10.13111/2066-8201.2016.8.2.4, pp. 41-51, 2016.

[2] H. Schlichting, *Boundary-Layer Theory*, (McGraw-Hill Book Company, New York, 1968).

[3] C. Truesdell, R. A. Toupin, *Enciclopedia of Physics. Flugge,* 5 Ed. Vol. **3**, Part I, Springer-Verlag, Berlin, 1960.

[4] M. Rutgers, http://www.martenrutgers.org/science/turbulence/gallery.html.

[5] T. J. Stuart, On finite amplitude oscillations in laminar mixing layers, *Journal of Fluid Mechanics*, **29**, part 3, pp. 417-440, 1967.

[6] S. Arendt, D. C. Fritts, O. Andreassen, The initial value problem for Kelvin vortex waves, *J. Fluid Mech*. **344**, pp. 181-212, 1997.

[7] K. T. Nijenhuis, G. H. McKinley, S. Spiegelberg, H. A. Barnes, N. Aksel, L. Heymann, J. A. Odell, Thixotropy, Rheopexy, *Yield Stress*, In Springer Handbook of Experimental.

[8] F. S. Crawford, Waves, *Berkeley Physics Course*, Vol. **3**, Education Development Center, Inc., Newton, Massachusetts, 1968.

[9] C. Knittel, W. D. Knight, M. A. Ruderman, Mechanics, *Berkely Physics Course*, Vol. **1**, Mc Grow-Hill, New York, 1973.

[10] W. K. George, Asymptotic effect on initial and upstream conditions on turbulence, *J. Fluid Eng.*, **134**/ 061203-1, 061203-27/, June 2012.

[11] M. J. Lighthill, *Introduction: Boundary layer theory, in Laminar Boundary Theory*, ed. by L. Rosenhead, (Oxford University Press, Oxford, pp. 46-113, 1963.

[12] O. Reynolds, Study of fluid motion by means of colored bands, *Nature,* **50**, pp 161-164, 1894.

[13] A. Tsinober, *The Essence of Turbulence as a Physical Phenomenon*, Springer Dordrecht Heidelberg, New York, London, 2014.

[14] H. Dumitrescu, V. Cardos, I. Malael, Boundary vorticity dynamics at very large Reynolds numbers, *INCAS Bulletin* vol. **7**, issue 3, (online) ISSN 2247–4528, (print) ISSN 2066–8201, ISSN–L 2066–8201, DOI: 10.13111/2066-8201.2015.7.3.8, pp. 89-100, 2015.

[15] A. Enciso, D. Peralta-Salas, Knotted vortex lines and vortex tubes in stationary fluid flows, *News letter of the European Mathematical Society*, No. **96**, pp. 26-53, 2015.

[16] A. Tsinober, *An informal Conceptual Introduction to Turbulence*, Springer Dordrecht Heidelberg, New York, London, 2009.

[17] H. Schlichting, K. Gersten, *Boundary Layer Theory*, Springer-Verlag Berlin, Heidelberg, New York, 2000.