Relativistic Contact-Wall Effects at Start-up

Horia DUMITRESCU[1], Vladimir CARDOS*,[1], Radu BOGATEANU[2], Alexandru DUMITRACHE[1]

*Corresponding author
[1]"Gheorghe Mihoc – Caius Iacob" Institute of Mathematical Statistics and Applied Mathematics of the Romanian Academy,
Calea 13 Septembrie no. 13, 050711 Bucharest, Romania,
dumitrescu.horia@yahoo.com, v_cardos@yahoo.ca*, alex_dumitrache@yahoo.com
[2]INCAS – National Institute for Aerospace Research "Elie Carafoli",
B-dul Iuliu Maniu 220, Bucharest 061126, Romania,
unquradu@gmail.com, www.incas.ro

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Abstract: *General relativity describes the physical force of gravity and its relation with the space-time structure. The fundamental physical insight behind the theory of general relativity is that the effects of acceleration cannot be distinguished from those of gravity. The presence of matter changes the geometry of space and acceleration is experienced in relation to that geometry. In particular, the geometry of space and time is dynamically governed by physical laws that are not independently given. Such a case is the fluid turbulence which remained unexplained for over fifty years within the framework derived from the classical Navier-Stokes theory. The physical and mathematical problem of the genuine turbulence remains an unsolved one. This paper considers the turbulence as a manifestation of non-Newtonian behavior induced by the gravity-like accelerations at the beginning of motion. The fluid particles with high accelerations exhibit in wall-bound flows with large Reynolds number a sort of structure-less, background turbulence. It is shown that the relativistic viewpoint may solve the present paradigmatic crisis of turbulence.*

Key Words: *Laminar-turbulent transition, shear turbulence, coherent hydrodynamic structures, vortex dynamics, gravitational-shear waves*

1. INTRODUCTION

Historically, the velocity of light, c, expanding in empty space, independent of frame of reference, has almost always had a central role in previous expositions of relativity [1], [2]. Relativity, however, is not a branch of electromagnetism and the subject can be developed without any reference to light [3]. Since this is not the conventional approach, I should emphasize that by asserting that, relativity can be developed (without light) when a Galilean empty space has the acceleration of gravity with the value g, and independent of frame of reference. For the Galilean space-time structure, the Euclidian space is a fiber bundle (or

simply, a bundle) which can be either a vector space *V* or a Lie group *G* preserving the volume element. If the natural frequency of any form of matter or energy, that can actually propagate at the invariant velocity, is less than the inverse of rotation frequency of Earth, $f_E^{-1} \approx 10^5$, then the Euclidian space is an inertial 2-D vector space, while for frequencies exceeding f_E^{-1} the non-inertial 3-D space forms a Lie group, the motions of matter being dominated by the gravitational contact waves, called in engineering language torsional/twist vibrations. More information on the space-time structure and Galilean relativity can be found in [4].

The starting of any motion of both solid and fluid bodies involves aspects connected principally to matter-inertia changes and return forces, which have opposite tendencies and produce an intrinsic pre-compressing state of matter, this many times being ignored in the successive development of motion after start.

This pre-compressed state of matter during the onset of motion is the outcome of the law of equal action and reaction for internal stresses, which for the frequencies of matter around the inverse of Terrestrial frequency $f_E^{-1} = 10^5$ this generates its self-oscillating behavior, i.e. torsional vibrations. Generally, the start-up is an impact/ contact problem connected to the statistical mechanics in Gibbs's phase space based on a rather complicated mathematical theory. Thus, the starting impact can be either a weak interaction (molecular-thermal microstructure preserving), so-called "idle" inertial starting described by the classical Newtonian dynamics along with the frictional shearing resistance, or a strong interaction (with molecular microstructure changes), so-called "whirlwind"/ballistic starting involving the frictional rolling resistance, and relativistic effects, described by the quantum mechanics along with the fast gravitational-shear waves.

The suddenly compressed mass expands in the Galilean space, independent of frame of reference, filling up regions of Euclidian space ergodically. This rapid wavy microscopic process of collision type, at the contact surface of bodies, is called structured or Lagrangian turbulence which produces macroscopic/visible phenomena of fluids ("random turbulence") and solids ("spelling"/ fracture).

The standing gravitational-shear waves embedded in a moving body represent the reaction to starting impact producing rotatory dispersion of tensed matter (i.e. concentrated boundary vorticity of flowing fluids followed by the rapid restoration of homogeneity, isotropy and smoothness [5]).

The problem of turbulence is devoted to a subject that, though has been largely debated, at first by the engineering community and then mathematicians and physicists, over a hundred years ago, it has been rather controversial and often misunderstood. The subject is concerned with the description of a fluid under conditions such that the well-established set of hydrodynamic equations, the Navier-Stokes (NSE) equations ceases to be valid. Herein the turbulence is considered a manifestation of non-Newtonian behavior induced by the gravity-like accelerations during at the beginning of motion, where at high Reynolds numbers the Earth-frame of reference becomes a non-inertial one and, for frequencies around of f_E^{-1}, the relativistic effects in the form of gravitational-shear waves are produced.

The turbulence easily observed as an artificial/non-understood "structure-less" flow field in wall-bounded flow, is the outcome of the perturbation/excitation of gravity by the point-starting shock, that produces a visual Doppler/stroboscopic effect depending on the ratio of flow-bound to the flow-free frequencies, $\mathrm{Re}_l / \mathrm{Re}_{cth}$.

Thus, the image texture of turbulent flow, easily recognized but very difficult to describe, could be quantified (see Sect. 3).

2. HEURISTIC RESULTS BASED ON THE GALILEI'S RELATIVITY PRINCIPLE

Around the contacting area the motion of matter (here fluid) during at motion startup is periodically induced, compressed and expanded (or discharged). Consequently, the temperature of the moving matter (fluid) varies between given limits, depending on the starting velocity. For small velocities, since the thermal capacity of the flow guiding walls is usually large compared with that of the working fluid, and since heat is transferred between them and the adjacent fluid at a relatively slow rate, the wall-bounded fluid assumes a low temperature along with a molecular thermal bulk effect given by the frictional shearing stresses, which delays restoring the initial state of fluid. The wall-slow motions for damping down/relaxing the inertial starting shock, embrace the quiet smooth flows known as "laminar" boundary layers, and the fast intrinsic oscillating motions in the form of invisible gravitational-shear waves on the fluid-solid interface. The opposite of laminar flows are the turbulent flows, easily observed through their non-understood structure-less, background turbulence in the case a ballistic starting. The ignorance of such wall-intrinsic relaxing wave motion and their cause (starting shock/excitation) led to the long-lasting and continuing wrong approach of turbulence.

Our local wavy description from the Galilei's relativity principle blends the law of equal action and reaction for the internal stresses, and a heuristic analysis of starting shocks. Herein arise three thresholds for a local minimum of the potential (measured in bits of *g*–acceleration) to overcome to escape to free motion. The wall complicated flow fields permanently change oppositely oriented gravitational field, in cascade with *g*–quanta, concomitantly with inertia decrease and proper entropy increase. The three thresholds of *g*–potential decline are closely connected with the stability of impacted fluids and a discussion of the critical Reynolds number on the stable flow conditions follows below.

How large Reynolds numbers must be along with the role of viscosity/dissipation are the big obsessions for the mathematical community concerning the Navies-Stokes equations [6], the mostly used paradigmatic and computational tool for the research in turbulence.

The critical Reynolds. The outcome of the universe creation or the big bang is the Eigen rotation of the Earth with a near constant angular velocity $\omega_E = 7.27 \cdot 10^{-5}$ or $f_E^{-1} \approx 10^5 \,\mathrm{Hz}$, and the presence at its surface of the atmosphere and oceans.

The fluids like air and water are special fluids with small viscosity, called Newtonian/linear fluids, their flows being of interest in many application and distinct from the rheological fluids with non-linear behavior [7]. According to the Galilei's relativity principle, (the physical laws cannot depend on time in inertial frames of reference), the Earth is an approximate inertial system, where the slow common motions of elastic bodies with their natural frequency $f_B = f_E^{-1}$ (accelerations = g) the Earth system with the additional Newton's law of inertia can be an empirical reference system; f_E^{-1} is the frequency of an unitary material point ($\rho = 1$ Kg/m^3) free of the Earth-bundle (escaped from the Newtonian constraint). The ratio of minimum continuous flow ($Vl = 1\ m^2/s$) to Earth circulations defines unambiguously the critical Reynolds number as

$$\mathrm{Re}_{cr} \equiv \frac{Vl}{l^2 f_E} = \frac{1m^2/s}{1m^2 f_E} = \frac{1}{\nu_0} \approx 10^5, \tag{1}$$

Where the dimensional quantity $\nu_0 \equiv 1m^2 \cdot f_E$, called kinematic viscosity, is the ratio of absolute/dynamic viscosity to density μ/ρ in an equilibrium thermodynamic state. In the Earth/Terrestrial frame the **kinematic viscosity ν_0 is a universal constant** of gravitational nature, while the dynamic viscosity and density are physical properties of real fluids, of molecular-thermal nature, depending on velocity. In the case of the small viscosity fluids (air and water-like fluids), all flows remain attached to solid surfaces for $\mathrm{Re}_l \le \mathrm{Re}_{cr}$ and the frictional shearing stresses which arise in fluid during its motion obey the empirical Newton's law of friction. For $\mathrm{Re}_l > \mathrm{Re}_{cr}$ the relativistic contact effects engendering gravitational-shear waves dominate the intermittent microscopic flow fluid relaxing in a macro-flow field which transports matter in the form of vorticity residues, so-called dust-vorticity flow [8].
The g – correction of Re_{cr} gives the static and dynamic stability limits

$$\mathrm{Re}_{cs} \ge \frac{g}{3}\mathrm{Re}_{cr}\left(\approx 3\cdot 10^5\right) \text{ and } \mathrm{Re}_{cth} > g\,\mathrm{Re}_{cr}\left(\approx 10^6\right), \tag{2}$$

The Earth–bound non-inertial systems. The motion of material bodies on the Terrestrial surfaces is possible due to their bundle with the Earth achieved via the gravity and shearing force, i.e. the interaction between the gravitational energy potential and internal/intrinsic energy of bodies for their integrity/existence. If the kinetic energy of motion for a material body ($E_k(t) = \frac{mV_{CM}^2}{2} + \frac{1}{2}\boldsymbol{\omega}^T \mathbf{I}\boldsymbol{\omega}$, V_{CM} - the velocity of mass center, $\boldsymbol{\omega}$ - the angular velocity, $\mathbf{I}$ – the diagonal inertia tensor) is much less gravitational force ($G = mg$) and internal energy (E_{th} – energy of matter in an equilibrium thermodynamic state), any motion obeys the empirical Newton's law of inertia, i.e. the Newtonian determinism principle and formalism, where the successive development of motion is determined from some initial conditions at some boundary conditions to an ultimate (statistical) state. For the dynamic systems, where $E_k(t)$ doesn't exceed the internal threshold energy (the minimum energy of matter below which the change of its thermodynamic equilibrium cannot take place, $M_\infty \le 2/3$ for air and $Fr \le 2/3$ for water), the Earth-bound reference frame is approximately inertial, and the Newtonian classical formalism often led to misinterpretation of some physical phenomena (d'Alembert's paradox (no drag) and nature of turbulence (unknown structure)). In this case, the statistical state of macro-flow field (random turbulence easily observed, but non-understood) is less influenced by the relativistic effects of the Terrestrial reference frame and this can be determined by the main relativistic parameters: Mach number (air), Froude number (water) and Reynolds number, Fig. 1.

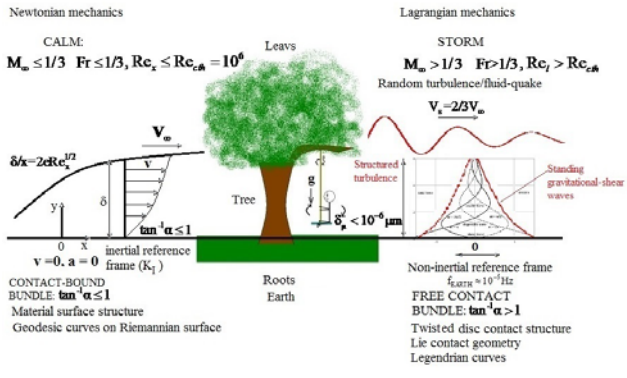


Figure 1. – The $M_\infty, Fr, \mathrm{Re}_l$ numbers – main relativistic parameters of fluid – flow state

In contrast to the macro-flow field, the microscopic wall-bounded flow creating the structured turbulence along with its self-sustaining mechanism (torsion pendulum like mechanism) are the outcome of the gravitational-shear waves produced by a rather complicated/twisted motion of the Earth, when it is strongly excited at the Re_{cth} number.

Fast kinematic-dynamic processes. The fast intrinsic contact twisted motions triggered off by pre-compressed matter during the ballistic starting (a really starting-shock without loss of the integrity of bodies) will be illustrated in the next section by two typical cases. Avoiding analytical and numerical results (though crucial for this field), we address below some topological considerations based on the law of equal action and reaction and contact Lie geometry involved in starting processes.

The rest-motion state change is a thermodynamic shock, more or less power-full, where the rapid contraction/twist of matter, inclusively air and water, produces fast blended kinematic-dynamic processes associated with the high accelerations of O(g). The high accelerations changing the frame of references are quantified by g – quanta (1/3, 1/2, 2/3) which represent the thresholds for minimum potential to restore the initial thermodynamic state a priori shock, Fig. 2 [9].

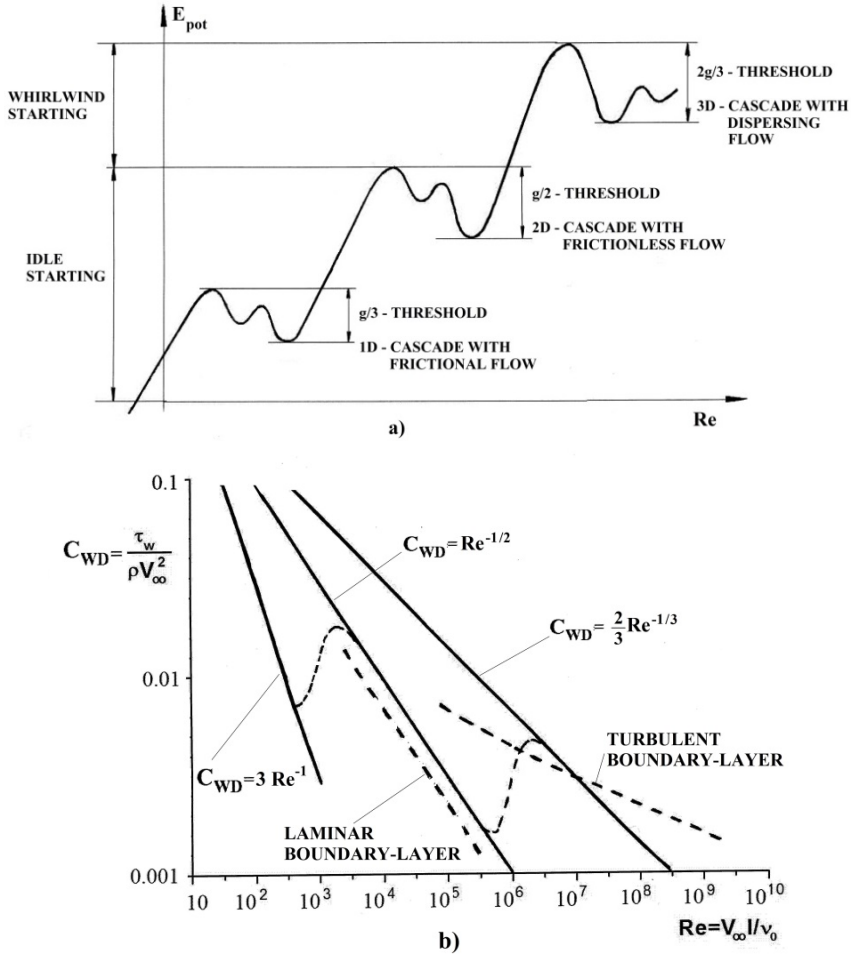


Figure 2. – The thresholds for a local minimum potential and ergodically filling up of 3D space: a) starting class; b) wave drag of flat smooth plates (air flow): ——— starting impact theory, ------ boundary-layer theory [9]

In the case of idle starting, i.e. (1/3 – 1/2)g – slow inertia rate, the 2-D motions may be described by the Navier's equations (solids) and the Navier-Stokes equations (fluids) derived from the framework of the classical Newtonian mechanics. In contrast to the smoothly starting, the ballistic (impulsively) starting, i.e. (1/2 – 2/3)g – fast inertia rate, is associated with the wall fluctuating heat flow, entropy increase, and frictional rolling shears, where it engenders fast momentum-kinetic energy-power exchanges between colliding wall-fluid. Through these fast processes of relativistic/quantum nature, the energy of impacted fluids is recovered by means of self-sustaining/standing gravitational-shear waves, known also as lee waves of gravitational nature [8]. The fast kinematic-dynamic processes may be described within of the framework of Lagrangian-Hamiltonian mechanics by means of hybrid computational models [10 - 12], derived from the updating of previous ones or new future developments.

The elastic collision-like starting-shock is a mechanical impulse-matter contraction/twist interaction, where the law of equal action and reaction for the twist stresses is self-contained and the dynamic processes obey the Ω – shape momentum-kinetic energy or power invariance. This term comes from the shape of intrinsic constant length-curved paths/orbits in the phase plane (at the fluid-solid interface) and their mutual linking, mathematically described by the contraction of the curvature tensor: 2-D curvature tensor called the (kinetic) energy-momentum tensor, and 3-D curvature tensor (Ricci tensor) or the power-kinetic energy-momentum tensor (see Sec. 3).

The fast intrinsic contact processes triggered off by pre-compressed matter during the starting shock (without loss of the integrity of molecular structure) are summarized below by three "laws":

1) **route of inertia** with two possible scenarios, $(2e+\pi)\approx(e+2\pi)=g$, $\left[L^2T^{-2}\right]$ preserving length-curved path and kinetic energy [KL^2T^{-2}], where g is potential gravitational energy per 1 Kg of mass and 1 m of length, e [KT^{-1}] is the rate of inertia and π [L^2T^{-1}] is the internal circulation, Fig. 4.

2) **Ω – linear shape of contact energy-momentum invariance** with reversible route, $(2e+\pi)$, and **Ω – non-linear shape of contact energy-momentum invariance** with irreversible/ergodic route -$(e+2\pi)$, Fig. 5 a, b, c, d, where the contact/wall is the energy of the boundary between oppositely oriented gravitational fields.

3) **The torsion pendulum – like mechanism**, called "**Poisson dynamical structure**" associated with a contact free motion-point/knot and the equation

$$\ddot{\tau}+k_{\lambda}^{2}\sin\tau, \tag{3}$$

where τ – twist angle, k_λ – wave number and π – azimuthally wavelength (Fig. 6).

The starting shock/perturbation is an initial localized disturbance which takes the form of groups/packets of fast gravitational-shear waves, perceived/seen in the wall-bounded flows as so-called turbulent wakes.

Such a shock is a molecular thermal shock embedded in both the flow and gravitational field, which is self-offset by viscous diffusion and/or dispersion processes. These rather complicated self-damping down processes are achieved by means of the gravitational-shear waves or lee waves illustrated in Sec. 3.

Mathematically, the gravitational-shear waves caused by a weak starting shock are standing non-linear waves like-solitons which offset the initial disturbance by viscous thermal diffusion and/or weak quantum dispersion of the mutual energy or wall energy.

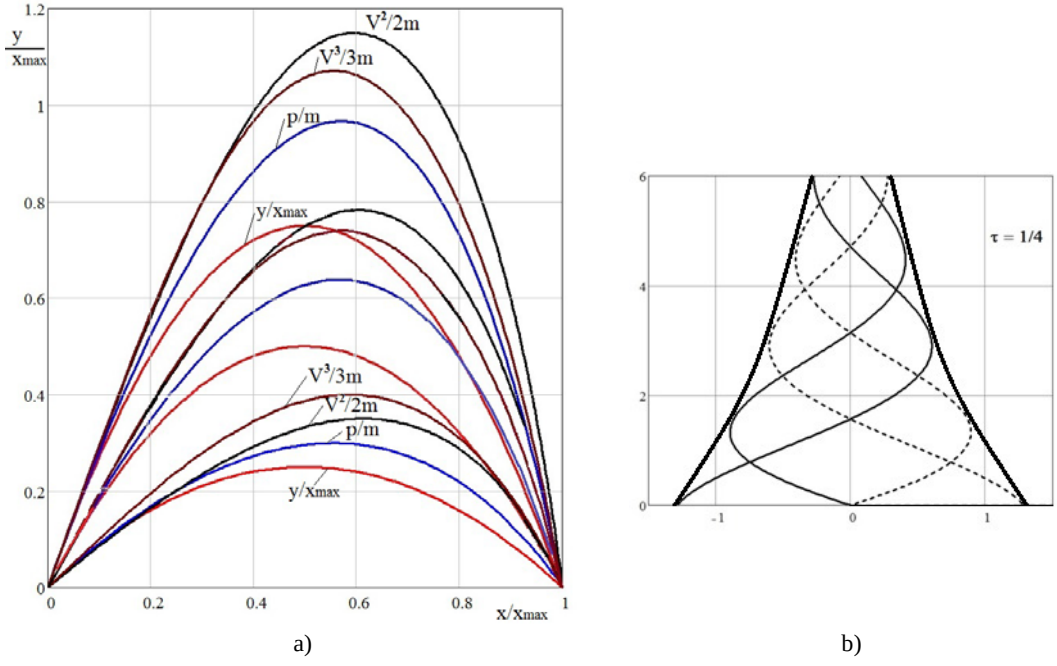
3. TWO TYPICAL RELATIVISTIC MOTIONS

The moving wheel/disc describing Legendre curves in space of contact elements of the plane with its natural contact structure, and the throwing of a heavy point mass with high resistance, $C_d = kV^n \left(n = 1,2,3\right)$ in the gravitational (g) field can illustrate, at the large scale, the relativistic relationships between geometry-acceleration changes induced by the intrinsic compressing processes caused at the start-up.

For the starting velocities sufficiently large $\left(Fr = \frac{V_\infty}{\sqrt{dg}} \geq 1/3 \right)$ the moving disc exhibits instabilities [5] like a water flow and when the Froude number touches the value 2/3, just above it the fracture of disc, respectively the water cavitation is produced. The water cavitation consists on the vaporization of a liquid due a decline the local pressure, when its molecular structure is destroyed.

In contrast to the water flows, in the case of gaseous flows for starting velocities exceeding $M_\infty \geq 1/3$, the pre-compressible flows $(1/3 \leq M_\infty < 2/3)$ exhibit the turbulence phenomena concomitantly with a little modified microstructure and hysteretic/ergodic damping.

The throwing of a heavy point mass with high resistance which changes its gravitational (g) field, is the macro-prototype of various compressing-expanding processes caused by the starting of flow, involving their self-sustaining mechanism through twisted microstructure along with kinetic energy release via gravitational-shear waves. This mechanism is illustrated below by the following figures.



a) b)

Figure 3. – The analogy between the ballistic curves and gravitational-shear waves: a) the twist effect of starting on charged ballistic curves for tan α = 1, 2, 3; b) the penetration of transverse waves packet into macro-flow field (wall structured turbulence or buffer zone)

Figure 3 shows comparatively the ballistic curves for different launching angles, α, $(\tan\alpha = 1,2,3)$, charged by momentum ($m\boldsymbol{V}$), kinetic energy ($mV^2/2$) and power ($mV^3/3$ – work

per unit time and length) and the penetration of invisible waves packet into the macro-flow field (structured turbulence).

The similarity of these behaviors is determined by the oppositely **g** large contact accelerations, O(*g*), involved in the two dimensional motions, for air $M_\infty \geq 1/3$ and water $F_r \geq 1/3$, at the microscopic scale, O($\rho_{Es}/\rho_{Ev} \approx 3r_E^{-1} < 1\mu m$).

Such large accelerations in the shape of gravitational-shear waves generate fast **non-perceptible and non-understood coherent fluid dynamic structures** [8].

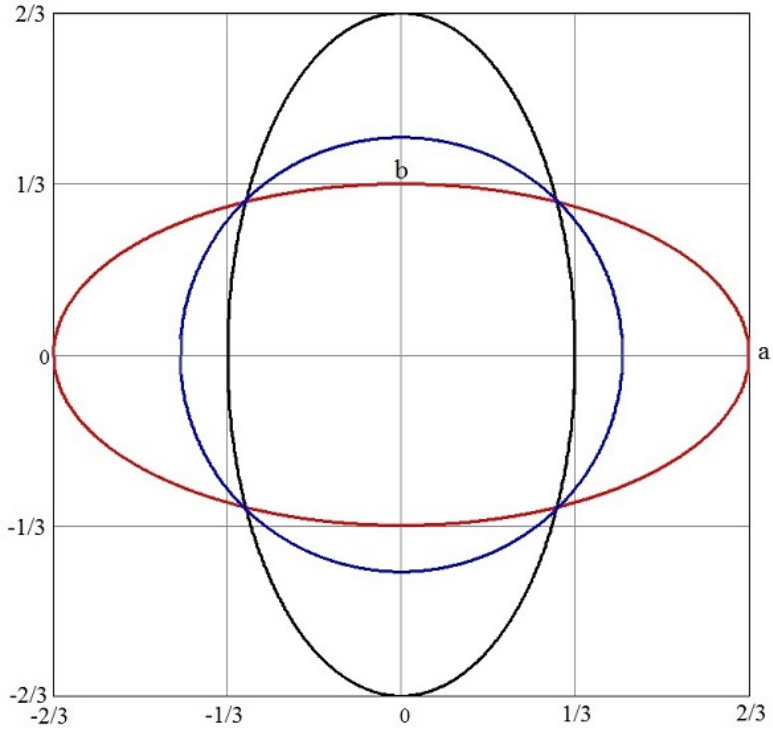


Figure 4. – The twist effect of starting on geometrical shape of length preserving curved paths/orbits

$\tau \equiv \frac{b}{a} \approx \tan^{-1}\alpha$, τ – twist index, α – throwing angle

Figure 4 shows **the twist effect** of starting on geometric shape of the length-preserving curved paths/orbits, where $\tau \equiv \frac{b}{a} \approx \tan^{-1}\alpha$ is a twist index.

The incoming kinetic energy-gravitational potential interaction involves such microscopic motions describing Legendre curve-like paths in the plane of a contact structure.

The contact structures are based on a rather complicated mathematical theory of the **contact geometry** (with the contraction of the curvature tensor and Legendrien lines) radically different from the **Riemannian geometry** (with tangential curvature and geodesic lines).

In contrast to the classical **Newtonian mechanics**, where the mass is a measure of inertia and the momentum preserving of body collisions assumes change of their mass center, the **Galilean relativity principle of inertia** (i.e. impossibility of time-dependence in an inertial frame of reference), involves elastic body collisions with free contact points, fast inertia changes without mass flux, and the loss of internal stability and eventually integrity of bodies (i.e. the equilibrium thermodynamic of matter).

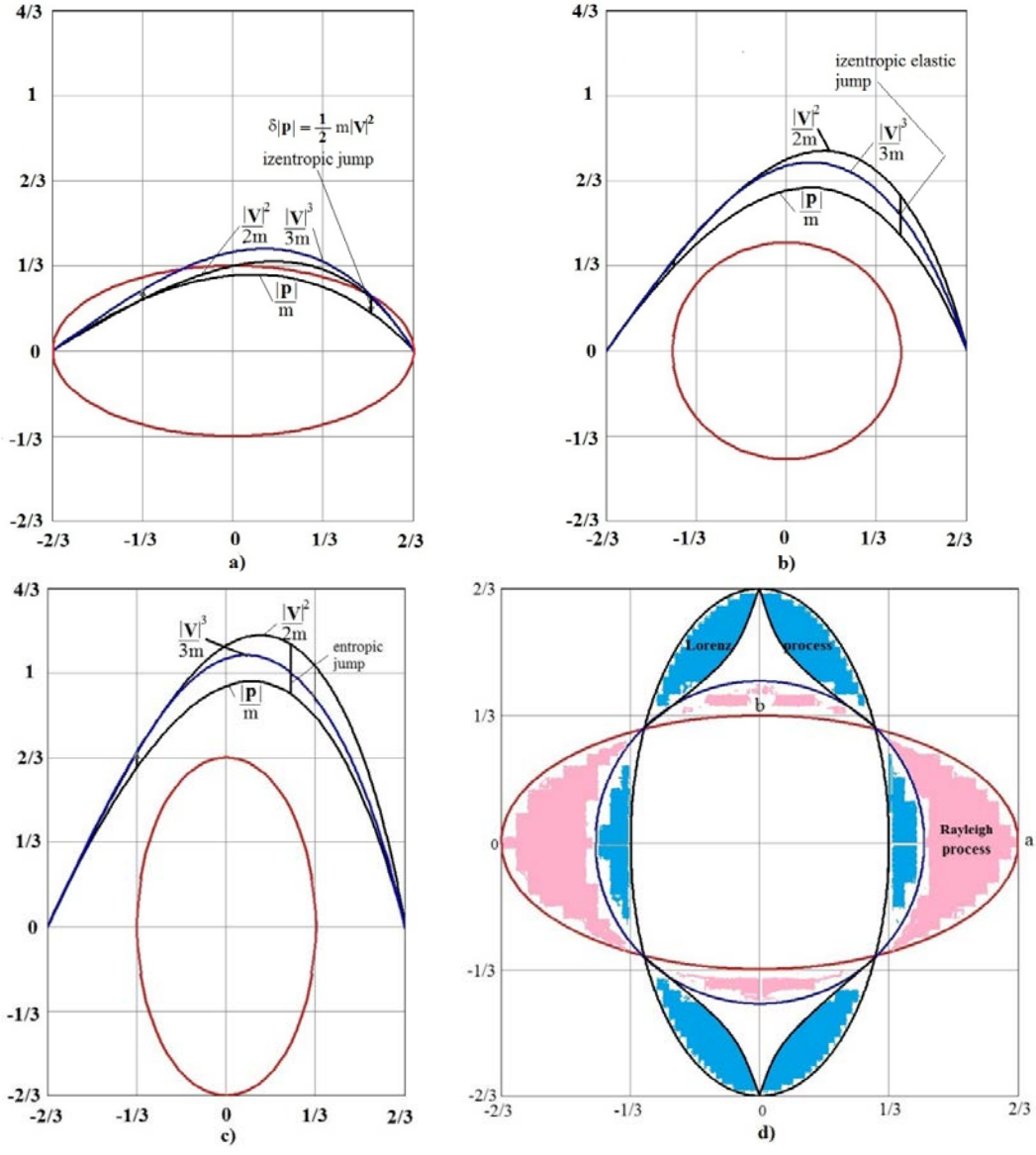


Figure 5. – The twisted load distributions on ballistic orbits: a) viscously delayed jump; b) elastic isentropic jump (reversible Rayleigh's stochastic process); c) entropic-ergodic jump (irreversible Lorenz stochastic process); d) the mutual linking of three intrinsic closed paths/orbits

The fast intrinsic contact processes triggered off by a starting impact without lost integrity of body obey the three laws describing the physical processes through which the starting shock is offset/damped down and the fluid state is restored to the initial conditions. Figure 5 (a, b, c, d) shows the twisted load distributions on the ballistic orbits and the mutual linking of three charged orbits. There two internal transfer processes from the gravitational potential disturbed during the starting to the dynamic/flow field: the reversible route formed by the oscillating wall pressure (wall energy) to 2-D momentum flux involving a viscous thermal diffusion process (Rayleigh process), and the irreversible route formed by oscillating wall pressure-kinetic energy blend (wall energy) to 3-D momentum flux carrying kinetic energy (concentrated vorticity [5]) residues.

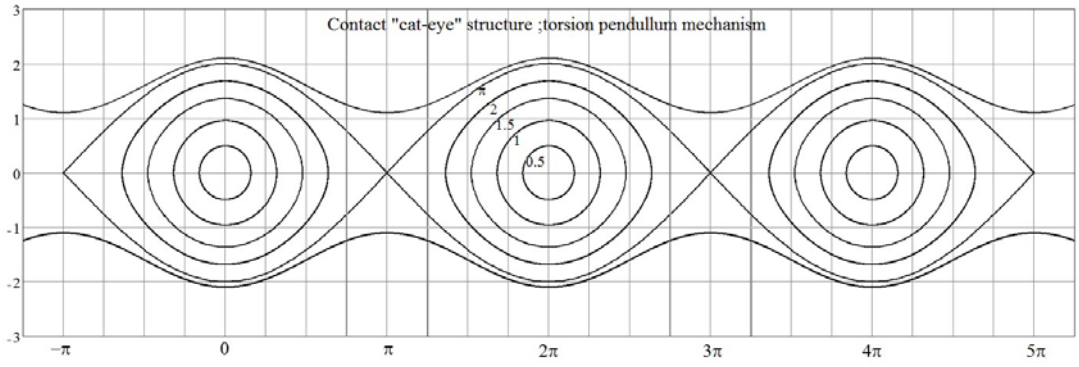
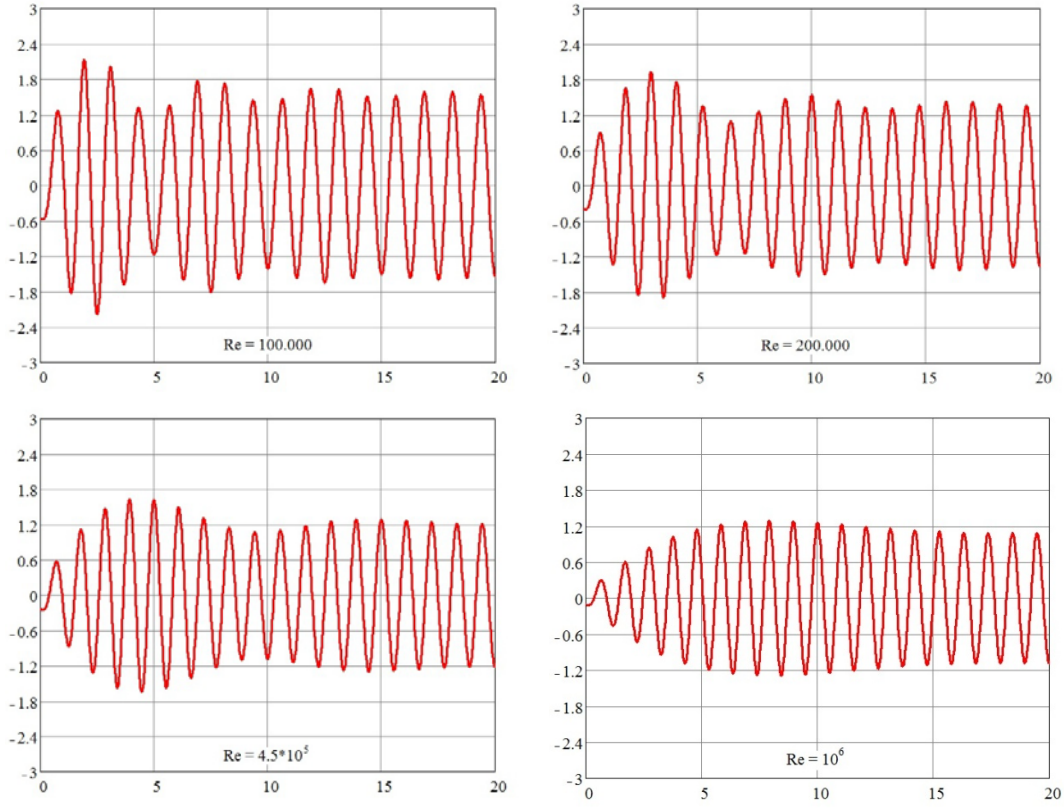
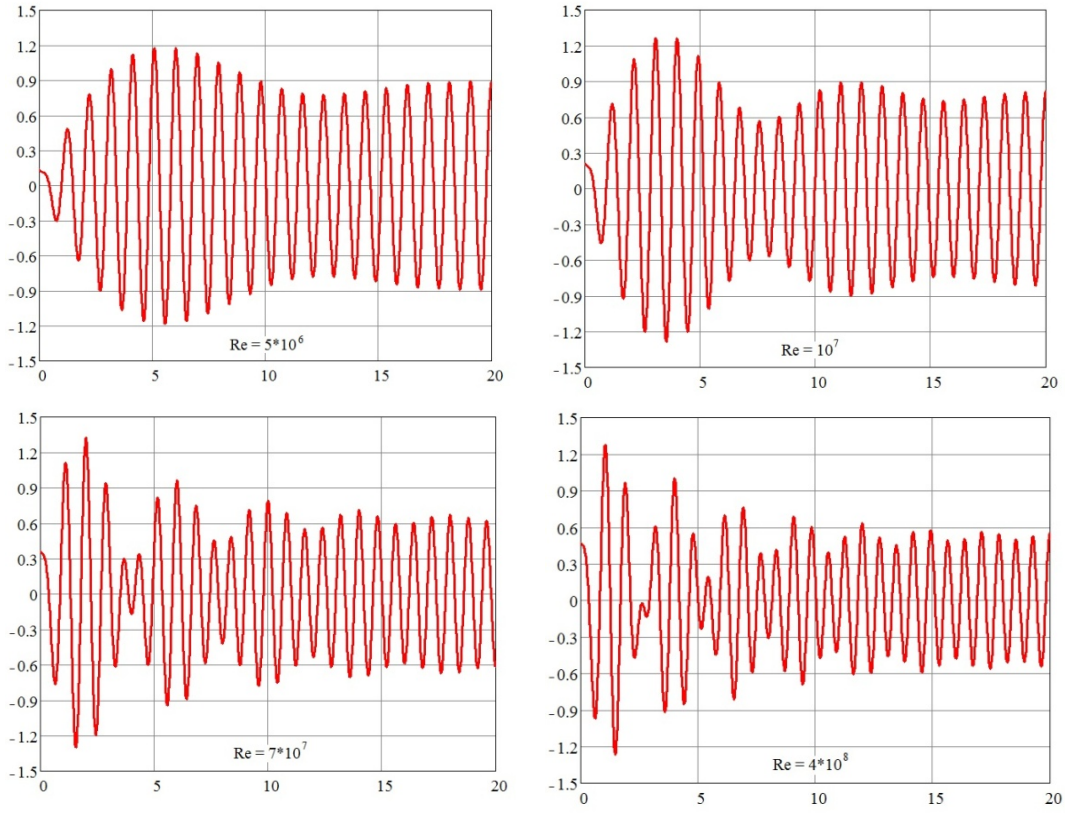


Figure 6. – Poisson dynamical structure of contact knots in the phase plane [8]

Figure 6 shows Poisson dynamical structure of contact knots (free points) which works like a regular torsion pendulum for the 2-D curvature tensor of the double mutual linking, and an inverted torsion pendulum for the 3-D curvature tensor of the triple mutual linking. This illustrates the mathematical notion of Poisson bracket [13], [14] of which physical meaning is less known. The intrinsic transient oscillating motions under harmonic excitations with starting frequency, Re (reduced frequency), assigned, are illustrated in Fig. 7 a, b for laminar ($\mathrm{Re} < \mathrm{Re}_{cth} = 10^6$) and turbulent ($\mathrm{Re} > \mathrm{Re}_{cth}$) flows, where from this visual description it is easily recognized the running as regular torsion oscillator for $\mathrm{Re} > \mathrm{Re}_{cr} = 10^5$ and as slightly damped inverted oscillator for $\mathrm{Re} > \mathrm{Re}_{cth} = 10^6$.



a)



b)

Figure 7. – The spectral description of wall-bounded flows (buffer zone): a) laminar texture/structure; b) turbulent texture/structure

4. CONCLUSIONS

The physical and mathematical problem of turbulence (fluids) and spelling (solids) phenomena appears when the onset of motion develops contact/wall circulations exceeding a critical value equal to that given by the eigen frequency of Earth $\left(1m^2 \cdot f_E \approx 10^{-5} m^2/s\right)$ and the frames of reference associated with the Earth become the non-inertial ones.

The relativistic non-inertial effects associated with high starting accelerations of order O (g) produce rapid loadings in the contacting area from which standing gravitational-shear/twist waves are emitted as structured turbulence and propagated with finite velocities through a macro-flow field as random turbulence, Fig. 1.

The true meaning of the critical Reynolds number, defined as an universal stability parameter connected with the non-inertial Earth-frame of reference, affects the processing and interpretation of numerical and experimental results (many times misinterpreted and/or non-understood, see the coherent hydrodynamic structures) by the lack and/or ignorance of some correction for the relativistic/stroboscopic effects.

The twisted curved paths of intrinsic near harmonic motions can be mathematically described by means of the contact geometry notions of the Poisson bracket and curvature tensor for the contraction/twist of the space-time structure at the start-up. The laws of basic physical processes induced by such fast motions, in an Earth-like non-inertial frame of reference, have

been identified in correlation with the contraction of the curvature tensor which changes the topology of 3-manifolds.

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