Algorithm of probabilistic and guaranteed assessment of the current technical condition of the aircraft’s on-board systems based on the method for testing statistical hypotheses

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Abstract: *Methods for improving the efficiency of aircraft operation by the transition to on condition operation technology are considered. Traditional methods for assessing the technical condition of the aircraft's on-board systems are based on methods of tolerance control and they use average statistical indicators of reliability. With regard to the above mentioned, in the process of operation of the aircraft, large amounts of data are accumulated which describe the processes occurring in various aircraft systems. Such data characterize the individual state of a particular aircraft and can be used for continuous monitoring and assessment of the condition of its systems. The scientific paper proposes methods for the formation of assessments of the technical condition of the aircraft's on-board systems based on the methods for testing statistical hypotheses. Examples are given confirming the effectiveness of using the proposed methods for solving problems with the assessment of the condition of aircraft systems.*

Key Words: *operation of aircrafts, assessment of technical condition, on condition operation, failure forecast*

1. INTRODUCTION

A promising area of promotion of flight safety, the need for which is recognized by all aeronautical specialists, is the transition to the technology of aviation equipment operation on technical condition. We are talking about the transition from the traditionally used average reliability indicators of aviation systems and methods of tolerance control to individual assessments of their condition during operation, taking into account the current service life. The use of this technology will make it possible to significantly improve the reliability and safety of flights, reduce operating costs, promptly diagnose defects and malfunctions, and most importantly – make reasonable decisions regarding the possibility of further use of a particular aircraft. Such technology is based on models and algorithms that allow us to obtain reliable assessments of the current condition of components, units and on-board systems of specific aircrafts during operation as the service life deteriorates. The most common approaches to solving the problem of obtaining such condition assessments today are approaches that use the methods of identification theory [1-3] and statistical methods [4-7]. In scientific papers [8-12], a probabilistic and guaranteed approach to assessing the condition of an aircraft during operation was proposed and implemented [13-14], which is based on a system of probabilistic criteria characterizing the condition of the aircraft and its individual on-board systems from the point of view of their ability to fulfill their targets determined by the flight program.

The attractiveness of this approach is due to its universal nature, since for its implementation it is enough to fulfill the following conditions:

– registration and continuous accumulation of data reflecting the condition of the on-board systems is provided on the board of the aircraft. In other words, there are facilities on the board of the aircraft that allow the formation of a flight information database that is expanding during the operation of a specific aircraft;

– known operational limitations that determine the range of permissible values of the parameters controlled by a given on-board system.

Despite the obvious attractiveness of the approach developed in [8–14] for assessing the current technical condition of the aircraft's on-board systems, it has unsatisfactory features that limit its practical application. Let us choose the most significant of them:

1) a common disadvantage of the algorithms developed in the framework of the probabilistic and guaranteed approach is that the condition of the aircraft is assessed at fixed points of the trajectory corresponding to the moments of change of typical flight modes. According to this circumstance:

firstly, it significantly limits the amount of flight data available for analysis, since each new flight adds one single implementation of the state vector of the on-board system being under control. For this reason, in order to obtain assessments of probabilistic criteria with the required accuracy, a significant amount of flights is required. It should be noted that in the course of performing a typical flight mode, the flight information registration on-board system [15-17] provides for obtaining values of the condition parameters of the controlled on-board system with a given frequency. This information, which is not used in any way within the framework of the mentioned probabilistic and guaranteed approach, is important because it reflects the dynamics of the system's operation in the course of performing the typical flight mode;

secondly, the use of data on the condition of the aircraft's on-board systems only at discrete points of the trajectory, the number of which is limited, makes it impossible to use algorithms based on the traditional probabilistic and guaranteed approach for the timely assessment of the condition of the aircraft's on-board systems during flight.

2) obtaining probabilistic assessments of the condition of aircraft systems in the framework of the algorithms [8-14] requires a prior knowledge for each characteristic point of the trajectory corresponding to the time of the change of the typical flight mode, the reference area, which is determined by setting its boundary. Since the reference region can have an arbitrary shape, an adequate functional description of its boundary in the parameter space of the condition of the system being under control is a serious problem. In works [8-14], various versions of approximations of reference regions are proposed on the basis of their parametric representation in classes of confidence ellipsoids, spheres, parallelepipeds. However, it is not always possible to prove the validity of such a parametric representation in real conditions.

3) the existing algorithms based on the probabilistic and guaranteed approach use as an information basis the database of flight information that is expanding during operation. Considering the fact that modern aircraft combines a large number of on-board systems, the condition of which is characterized by a large number of analog and attribute parameters, the volume of such a database that is expanding during operation is constantly increasing, which may lead to the need to use huge computing resources to store all the accumulated information. If the algorithms are used only for the purpose of post-flight analysis of the condition of the aircraft's on-board systems, this problem is not fundamental, however, its urgency increases many times, if we consider the prospect of using such algorithms for the purpose of operational assessment of the condition of the aircraft's on-board systems during the flight while using on-board computer systems. In such a situation, there is an urgent need for a compact representation of the accumulated information.

Taking into account the above advantages and disadvantages of algorithms based on the probabilistic and guaranteed approach, it seems attractive to develop and modify them, which will, while preserving their inherent advantages, eliminate these unsatisfactory features. There is an algorithm below that is a development of the probabilistic and guaranteed approach to assessing the current technical condition of on-board systems, but it significantly expands the capabilities of previously implemented algorithms by using the entire array of flight data obtained by on-board registration systems in the course of the typical flight mode.

2. RESEARCH METHODS

2.1 The reference pattern of the aircraft's on-board system being under control as the basis for assessing its current technical condition

The basis of the proposed post-flight integral assessment algorithm of the aircraft's on-board system is its reference pattern. This reference pattern is formed in the space of state parameters of the on-board system for each typical flight mode (run-on operation on the flight strip, take-off, climbing, landing, run-on operation on the flight strip, etc.) based on the data accumulated during the test flights and during the normal operation of a specific aircraft. It represents a set of points in the state parameter space of the on-board system being under control, corresponding to the condition of its normal operation on the selected typical flight mode.

Based on the above definition of the reference pattern of the aircraft's on-board system, let us give a formalized description of it. On the analogy of [8-14], we will assume that typical modes can be distinguished on the motion trajectory of the aircraft, the start and end times of which do not have a rigid timing, but can be localized by appropriate combinations of values of the state parameters of the aircraft. There are following examples of such typical modes: taxiing operation on the flight strip, accelerating run, take-off, climbing, straight level flight (en-route flight), descending, landing approach, landing, run-on operation on the flight strip.

For each of the typical flight modes listed above, signs indicating the moments of their beginning and end are accurately known. In this case, the duration of the regime may vary. Let us consider some typical flight mode. We introduce a vector of parameters X, the components of which X_i, $i=1, ..., n$ in the aggregate characterize the condition of the on-board system being under control in the selected typical mode. We consider that implementations X^j, $j=1, ..., N$ of the vector of parameters X corresponding to the normal mode of operation of the on-board system being under control are known. Each implementation x^j in the state parameter space of the on-board system being under control corresponds a point, the set of such points X^j, $j=1, ..., N$ forms a set, hereinafter referred to as the reference set E_X or the reference pattern of the on-board system (aggregate, unit or individual element) for the selected typical flight mode. The specified reference set forms the basis for assessing the condition of the aircraft's on-board system in the selected typical mode.

In the course of the flight, the standard flight information registration on-board system provides, with a certain frequency, obtaining parameter values reflecting the current condition of the on-board system being under control. Thus, the data that comes from the flight information registration on-board system, represent the increasing temporal sequence $X(t_j)$, $j=1, ..., m$ of vectors, each of which for a specific point in time t_j contains the values of parameters characterizing the current condition of the on-board system being under control. The set of vectors $X(t_j), j=1, ..., m$, forms the current pattern of $T_X = \{X(t_j), j=1, ..., m\}$ of the on-board system being under control, reflecting its actual condition during the execution of the selected typical mode.

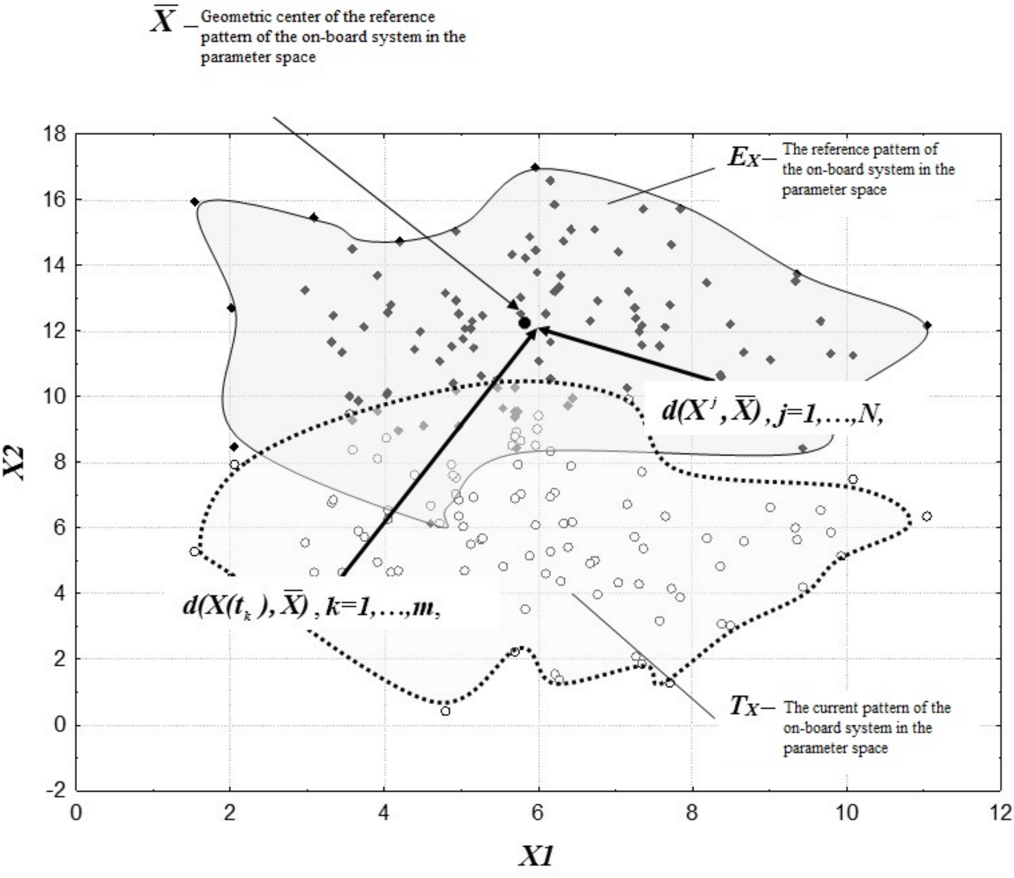

Fig. 1 – An example of the presentation of the reference and current patterns of the on-board system being under control in the plane of its state parameters

In the state parameter space of the on-board system being under control, the current pattern T_X, like the reference pattern E_X, is represented by a set of points the coordinates of which are given by the vectors $X(t_j)$, $j=1, ..., m$. In Figure 1, as an example, the reference pattern E_X of the on-board system (the points belonging to it are indicated by diamonds) and the current pattern E_X (the points forming it are marked with circles) are presented. Within the framework of such a presentation, the problem of assessing the current condition of the on-board system being under control is reduced to the choice of a scalar measure characterizing the proximity of the reference and current patterns. With regard to the above mentioned, since both the current and reference patterns objectively combine implementations of random vectors, some statistical characteristic should be used as such a measure.

The reference pattern of the on-board system being under control will be described using distances $d(X^j,\overline{X})$, *j=1, ..., N*, quantitatively expressing the scatter of the values of the system state parameters relative to the geometric center $\overline{X}$ of the set E_{X}. As a distance $d(X^j,\overline{X})$ Mahalanobis distances are used:

$$d(X^j,\overline{X}) = (X^j - \bar{X})^T K_X^{-1} (X^j - \overline{X}), \mathrm{j} = 1,\ldots,N \tag{1}$$

where $\bar{X}$– vector, the components of which determine the coordinates of the geometric center of the reference set, calculated on the basis of the expression:

$$\overline{X}_\iota = \frac{1}{N}\sum_{j=1}^{N} X_\iota^j \tag{2}$$

The geometric center of the reference set characterizes the average values of the state parameters of the on-board system being under control in the continuation interval of the typical flight mode, provided that it functions normally. K_X – covariance matrix describing the statistical relationships between the components of the vector *X* under the condition of the normal functioning of the on-board system being under control for the selected typical flight mode. Diagonal elements characterize the variability of the component values of the vector *X* relative to the geometric center $\overline{X}$. Elements of the covariance matrix K_X are calculated on the basis of the implementations X^j, *j=1, .., N* available for the analysis

$$\begin{gathered} K_{kl} = \frac{1}{N-1}\sum_{j=1}^{N}\left(X_k^j - \bar{X}_k\right)\left(X_l^j - \bar{X}_l\right), k \neq l; \\ K_{kl} = \frac{1}{N-1}\sum_{j=1}^{N}(X_k^j - \bar{X}_k)^2, k = l. \end{gathered} \tag{3}$$

The current pattern of the on-board system being under control during the execution of the selected typical flight mode will be described using the distances $d(X(t_k),\overline{X})$, *k=1, ..., m*, characterizing the scatter of the current values of the system state parameters relative to the geometric center $\bar{X}$ of the reference pattern in real conditions of system operation.

$$d(X(t_k),\overline{X}) = (X(t_k) - \overline{X})^T K_X^{-1} (X(t_k) - \overline{X}), k = 1,\ldots,m \tag{4}$$

Within the framework of such a presentation, the task of assessing the condition of the on-board system being under control in the process of performing a typical flight mode can be formulated as follows: there are two samples of random values $d(X^j,\overline{X})$, *j=1, ..., N* and $d(X(t_k),\overline{X})$, *k=1, ..., m*. It is necessary to determine whether there are statistically significant differences between these samples. The absence of such differences is a sign that the on-board system being under control was operating normally during the current flight. But if statistically significant differences between these samples will be revealed, it can be argued that there was an abnormal mode of operation of the on-board system being under control when the selected typical mode was performed during the current flight.

Methods for testing statistical hypotheses, the analysis of which is presented in the next section, can be used to answer the formulated questions.

2.2 Integral post-flight assessment of the condition of the aircraft's on-board systems based on methods for testing statistical hypotheses

At present, various statistical methods and criteria for assessing differences between independent samples of random values are known [18, 19]. In particular:

– Student's test, effectively working in situations where the distribution $d(X^j,\overline{X}), j = 1,\dots,N$ and $d(X(t_k),\overline{X}), k = 1,\dots,K$ complies with the normal law;

– Mann-Whitney test [19], considered as a non-parametric analogue of Student's criterion, which does not require confirmation of compliance of the distribution of values $d(X^j,\overline{X}), j = 1,\dots,N$ and $d(X(t_k),\overline{X}), k = 1,\dots,K$ normal law, but effectively working on a set of shear distributions, that is, in situations where the forms of distributions are similar, but shifted on the number axis;

– the Kolmogorov-Smirnov two-sample test [18, 20], which uses sample distribution functions as a basis for comparison.

A brief information on each of the above tests is presented below in terms of the features of their use for the purpose of an integrated assessment of the condition of the aircraft's on-board systems.

Student's test. The field of practical application of the Student's test requires the fulfillment of two mandatory conditions:

1) random variables $d_T = d(X^j,\overline{X})$ and $d_E = d(X(t_k),\overline{X})$ have a normal distribution with parameters: $d_E \in N(m_E,\sigma_E^2)$, $d_T \in N(m_T,\sigma_T^2)$, where m_E, m_T – mathematical expectations, and σ_E^2, σ_T^2 – dispersion of values d_E and d_T;

2) there is equality of general dispersions $\sigma_E^2 = \sigma_T^2$.

When the above conditions are fulfilled, the considered task of assessing the current technical condition of the aircraft's on-board system being under control can be interpreted as the task of comparing mathematical expectations m_E, m_T. Let us formulate the null hypothesis $H0$: $m_E = m_T$, which implies equality of mathematical expectation values d_E and d_T. If based on the analysis of values $d(X^j,\overline{X}), j = 1,\dots,N$ and $d(X(t_k),\overline{X}), k = 1,\dots,K$, the introduced assumption will be confirmed with a sufficient guarantee of probability, this suggests that the system being under control is functioning normally, since the differences in the values describing the reference and current patterns are due to random variations, the source of which are the errors of sensors of the flight information registration on-board system.

For testing the null hypothesis, the Student's statistics are used.

$$T = \frac{\overline{d_E} - \overline{d_T}}{\hat{\sigma}^2\sqrt{\dfrac{1}{N} + \dfrac{1}{K}}} \tag{5}$$

where $\overline{d_E}, \overline{d_T}$ – sample averages calculated by a set of values $d(X^j,\overline{X}), j = 1,\dots,N$ and $d(X(t_k),\overline{X}), k = 1,\dots,K$ forming the reference and current patterns, respectively:

$$\begin{aligned} \bar{d}_E &= \frac{1}{N}\sum_{j=1}^{N} d\left(X^j,\bar{X}\right), \\ \bar{d}_T &= \frac{1}{K}\sum_{k=1}^{K} d(X(t_k),\bar{X}) \end{aligned} \tag{6}$$

$\hat{\sigma}^2$ – assessment of value dispersion $d(X^j,\overline{X}), j = 1,\ldots,N$ and $d(X(t_k),\overline{X}), k = 1,\ldots,K$ in the combined sample:

$$\hat{\sigma}^2 = \frac{\hat{\sigma}_E^2(N-1) + \hat{\sigma}_T^2(K-1)}{N+K-2} \tag{7}$$

$$\hat{\sigma}_E^2 = \frac{1}{N-1}\sum_{j=1}^{N}(d(X^j,\bar{X}) - \bar{d}_E)^2, \tag{8}$$

$$\hat{\sigma}_T^2 = \frac{1}{K-1}\sum_{k=1}^{K}(d(X(t_k),\bar{X}) - \bar{d}_T)^2, \tag{9}$$

It is known that the above Student's statistic *T*, in the case when the null hypothesis is correct, has a theoretical *t* – distribution (Student's distribution) with *N*+*K*-2 degrees of freedom.

Therefore, the implementation of inequality

$$T_{\frac{1-\alpha}{2}} \leq T \leq T_{\frac{1+\alpha}{2}} \tag{10}$$

$T_{\frac{1-\alpha}{2}}$ – Student's reciprocal distribution with the number of degrees of freedom *N*+*K*-2, corresponding to the confidence probability $(1-\alpha)/2;$ $T_{\frac{1+\alpha}{2}}$ – Student's reciprocal distribution with the number of degrees of freedom *N*+*K*-2, corresponding to the confidence probability $(1+\alpha)/2$ suggests that the system being under control is functioning normally. The validity of this conclusion is guaranteed by the probability α. Thus, setting the confidence probability α close to one $\alpha \geq 0{,}95$), it is possible to make a reliable conclusion about the condition of the on-board system being under control.

Mann-Whitney test. The Mann-Whitney test is based on the procedure of pairwise comparisons of quantities $d(X^j,\overline{X}), j = 1,\ldots,N$ and $d(X(t_k),\overline{X}), k = 1,\ldots,K$. The result of comparing two specific values $d(X^j,\overline{X}),d(X(t_k),\overline{X})$ encoded by the indicator function μ_{jk} determined in accordance with the following rule:

$$\mu_{jk} = \begin{cases} 1, d(X^j,\bar{X}) > d(X(t_k),\bar{X}); \\ 0, d(X^j,\bar{X}) < d(X(t_k),\bar{X}). \end{cases} \tag{11}$$

If among the compared values there are the equal values, they are excluded from processing. The Mann-Whitney statistic is known as a scalar value:

$$U = \sum_{j=1}^{N}\sum_{k=1}^{K}\mu_{jk} \tag{12}$$

Obviously, in this way a certain quantity *U* takes values within a segment:

$$0 \leq U \leq NK \tag{13}$$

and the limit values indicate obvious differences between the reference standard E_X and the current T_X patterns of the on-board system being under control. $U = NK$ is valid if all the values that belong to the reference pattern E_X exceeds any value belonging to the current pattern T_X.

The case $U = 0$ takes place in the reverse situation. In a situation where the reference and current patterns of the on-board system being under control do not demonstrate significant differences, it can be expected that the random variable

$$U \to \frac{NK}{2} \tag{14}$$

It is known that with sufficiently large sample sizes N, K there is a fair transition from the Mann-Whitney statistic U to a random value Z determined on the basis of the ratio:

$$Z = \frac{U - \frac{NK}{2}}{\frac{NK(N+K+1)}{12}} \tag{15}$$

This value has the following proven property, which in terms of the initial problem of assessing the condition of the on-board system being under control can be formulated as follows: if samples $d(X^j, \overline{X}), j = 1, \dots, N$ and $d(X(t_k), \overline{X}), k = 1, \dots, K$ do not show differences, that is, the current condition of the system fully corresponds to its reference pattern, the random variable has a normal distribution with zero expectation and unit variance.

A sign of the normal functioning of the system is the fulfillment of inequality:

$$Z_{\frac{1-\alpha}{2}} \le Z \le Z_{\frac{1+\alpha}{2}} \tag{16}$$

$Z_{\frac{1-\alpha}{2}}$ – equivalent deviate of standard normal distribution for confidence probability $(1 - \alpha)/2$, accordingly $Z_{\frac{1+\alpha}{2}}$ – equivalent deviate of standard normal distribution for confidence probability $(1 + \alpha)/2$.

The values of equivalent deviates (or critical levels) for the standard normal distribution are given in statistical tables [18]. In particular, for the probability $\alpha = 0.95$, equivalent deviate values are:

$$Z_{\frac{1-\alpha}{2}} = -1{,}96, Z_{\frac{1+\alpha}{2}} = +1{,}96$$

For the probability $\alpha = 0.99$:

$$Z_{\frac{1-\alpha}{2}} = -2{,}56, Z_{\frac{1+\alpha}{2}} = +2{,}56$$

Thus, the implementation of the above inequality allows with guaranteed probability α close to one, to state that the aircraft's on-board system being under control functions normally. The probability that this conclusion is erroneous does not exceed $1 - \alpha$.

Kolmogorov-Smirnov two-sample test. The test is based on the verification of the assumption (null hypothesis) $H0: F_T(\mu) = F(\mu)$ against alternative $H1: F_T(\mu) \ne F(\mu)$.

Confirmation of the null hypothesis indicates the proximity of the statistical properties of the reference and current patterns, which suggests a normal operating mode of the on-board system being under control.

In fact, the validity of the null hypothesis means that the set of values $d(X^j, \overline{X}), j = 1, \dots, N$ and $d(X(t_k), \overline{X}), k = 1, \dots, K$ are samples of the same population.

The differences between them are due only to the effect of random errors in measuring state parameters. Sample analogues $\hat{F}_T(\mu), \hat{F}(\mu)$ of distribution functions $F_T(\mu), F(\mu)$ are used to test the introduced assumption (see Figure 2).

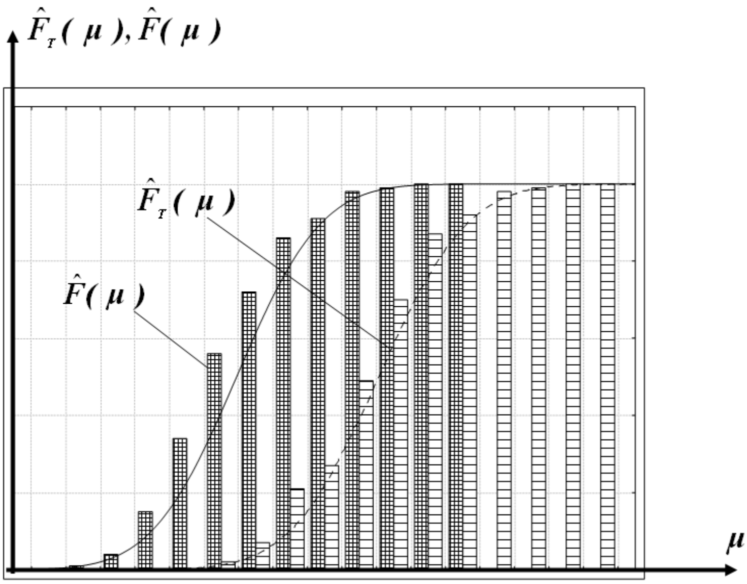

Fig. 2 – Illustration of the Kolmogorov-Smirnov two-sample test

As a measure of the proximity of functions $\hat{F}_T(\mu), \hat{F}(\mu)$ in the Kolmogorov-Smirnov two-sample test, a scalar value is used, which is called statistic, of the form:

$$D = \sup_{0<\mu<\infty} \left| \hat{F}_T(\mu) - \hat{F}(\mu) \right| \tag{17}$$

From the structure of this statistic, it follows that it take on greater significance, the more pronounced the differences are between the functions $\hat{F}_T(\mu), \hat{F}(\mu)$. In other words, the large value of this statistic is an indicator of the differences between the reference and current patterns.

The problem is that the statistic defined in such a manner is random, since it is calculated on the basis of the values of the sample distribution functions, which are random. The following property is known, which has a random variable *D* [18]: if the null hypothesis is true ($H0: F_T(\mu) = F(\mu)$) the random value

$$\sqrt{n_0} D = \sqrt{\frac{KN}{K+N}} D \tag{18}$$

has a stable statistical property, which are described by the Kolmogorov distribution. The Kolmogorov distribution function *F(X)* is described by the dependency type:

$$\Phi(X) = \left\{ \sum_{k=-\infty}^{\infty} (-1)^k exp\{-2k^2x^2), x > 0;\ 0, x \le 0 \right. \tag{19}$$

Using the well-known Kolmogorov distribution function, we can propose the following algorithm for the integral assessment of the condition of the on-board system being under control:

1) the value of the confidence probability α is set, determining the requirement for the reliability of the conclusion about the condition of the on-board system being under control.

2) using the Kolmogorov distribution function (19) for the selected value of the confidence probability $\boldsymbol{\alpha}$ the equivalent deviate value μ_α is determined:

$$\Phi\,(\mu_\alpha\,) = \alpha \tag{20}$$

where Φ ( ) – the Kolmogorov distribution function (19).

3) based on values $d(X^j, \overline{X}), j = 1, \dots, N$ and $d(X(t_k), \overline{X}), k = 1, \dots, K$, characterizing, respectively, the reference and current patterns of the on-board system being under control, the sample distribution functions $\hat{F}_T(\,\mu\,), \hat{F}(\,\mu\,)$ are calculated corresponding to them the observed value of statistic (19) D_H , as well as the observed value associated with it

$$Z_H\ = \sqrt{\frac{KN}{K+N}}D_H \tag{21}$$

4) the condition is checked:

$$Z_H\ \le\ \mu_\alpha \tag{22}$$

confirmation of which allows with guaranteed probability α to assert that the system being under control functioned normally in the process of performing the typical flight mode. Violation of the above inequality indicates the presence of an abnormal mode of operation of the on-board system in the process of performing the typical flight mode.

The use in the process of post-flight assessment of the condition of the aircraft's on-board systems of the considered algorithm can significantly increase the reliability of conclusions regarding the current condition of the on-board systems being under control.

3. RESULTS AND DISCUSSIONS

For an example demonstrating the operation of the algorithms, real data accumulated during the four-month operation of the aircraft for the period from January 6, 2013 to April 16, 2013, which characterize the condition of the air conditioning system, were used. The choice of air conditioning system for analyzing the effectiveness of the methods for testing statistical hypotheses considered above was due to the fact that during the operation of the aircraft it was detected an abnormal operation of this system, caused by overheating and subsequent destruction of the compressor bearing, confirmed by an inspection certificate of the air conditioning system dated March 27, 2013, which led to the need to replace it.

Since in the process of operation of the aircraft, the abnormal operation of the air conditioning system in the temperature control channel, the vector of parameters X unified a group of analog parameters characterizing the condition of the air conditioning system from the point of view of ensuring the desired temperature in the aircraft cabin: X_1 – temperature of the air supply to the cabin; X_2 – air temperature in the hot line of the air conditioning system; X_3 – bearing temperature; X_4 – temperature in the aircraft cabin. As a typical flight mode the landing mode was considered.

As noted above, the basis of the post-flight assessment of the condition of the air conditioning system is the reference pattern, represented by the values $d(X^j, \overline{X}), j = 1, \dots, 374$ that are calculated based on implementations of temperature parameters $X_1^j, X_2^j, X_3^j, X_4^j, j = 1, \dots, 374$ corresponding to the normal functioning of the air conditioning system.

Value distribution histogram $d(X^j, \overline{X}), j = 1, \dots, 374$ forming the reference pattern of the air conditioning system, is shown in Figure 3, where the normal distribution density function is presented for comparison. For the purpose of the possibility of the subsequent use of the Student’s test for assessing the condition of the air conditioning system during operation, it was conducted an analysis of the compliance of sample distribution of values $d(X^j, \overline{X}), j = 1, \ldots, 374$ to the normal distribution law based on the Kolmogorov-Smirnov consent test. The value of the Kolmogorov-Smirnov statistic calculated here was $D=0.0829$. The obtained value of the Kolmogorov-Smirnov statistic indicates a significant (at the significance level $p<0.01$) difference in the distribution of values $d(X^j, \overline{X}), j = 1, \ldots, N$ from normal law. This circumstance makes it impossible to use Student's test in the further procedure for assessing the condition of the air conditioning system. Given this, nonparametric statistical criteria are the subject of further analysis.

First of all, we consider the reliability of assessments of the condition of the air conditioning system based on the Mann-Whitney test. In this case, in order to assess the condition of the air conditioning system, the values of temperature parameters are used $X_1(t_k), X_2(t_k), X_3(t_k), X_4(t_k), k = 1, \ldots, 32$ registered in the course of the landing mode during the flight, on the basis of which the values are calculated using (1)-(4) $d(X(t_k), \overline{X}), k = 1, \ldots, K$ forming the current pattern of the air conditioning system.

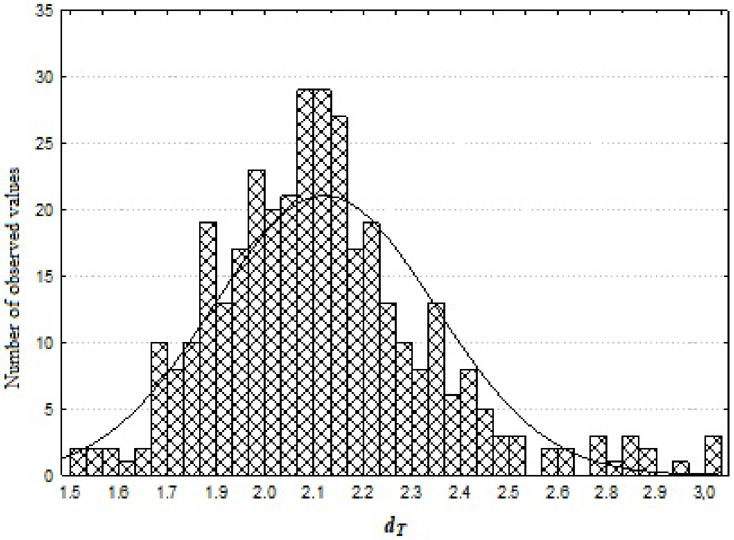

Fig. 3 – Value distribution histogram $d(X^j, \overline{X}), j = 1, \ldots, 357$ forming the reference pattern of the air conditioning system

The degree of proximity of the current pattern of the air conditioning system formed in this way with the reference pattern of the system was estimated on the basis of the Mann-Whitney statistic U (12), and the Z value (15) associated with this statistic. Efficiency area of the the air conditioning system defined by condition (16) is determined by equivalent deviate values $Z_{\frac{1-\alpha}{2}}$, $Z_{\frac{1+\alpha}{2}}$ standard normal distribution for confidence probability $\alpha = 0.95$.

Figure 4 shows the values of the Z statistic calculated using (15) on the basis of data accumulated during the landing mode for a particular flight within the considered time interval. We see that already after the completion of the flight on February 25, 2013, the use of the Mann-Whitney statistic revealed the presence of a hidden defect of the air conditioning system compressor, long before its failure, recorded by a technical inspection certificate. Note that after the replacement of the turbine bearing of the air conditioning system compressor, the system returned to a functionally operative condition, as evidenced by technical inspection certificates. A similar result is observed when using the Kolmogorov-Smirnov two-sample test as a basis for assessing the current technical condition of the air conditioning system. In this case, a sign that the system is functioning normally is the fulfillment of inequality (22). On the contrary, the violation of this condition indicates the presence of hidden defects, the presence of which leads to the development of a non-standard mode of the air conditioning system function.

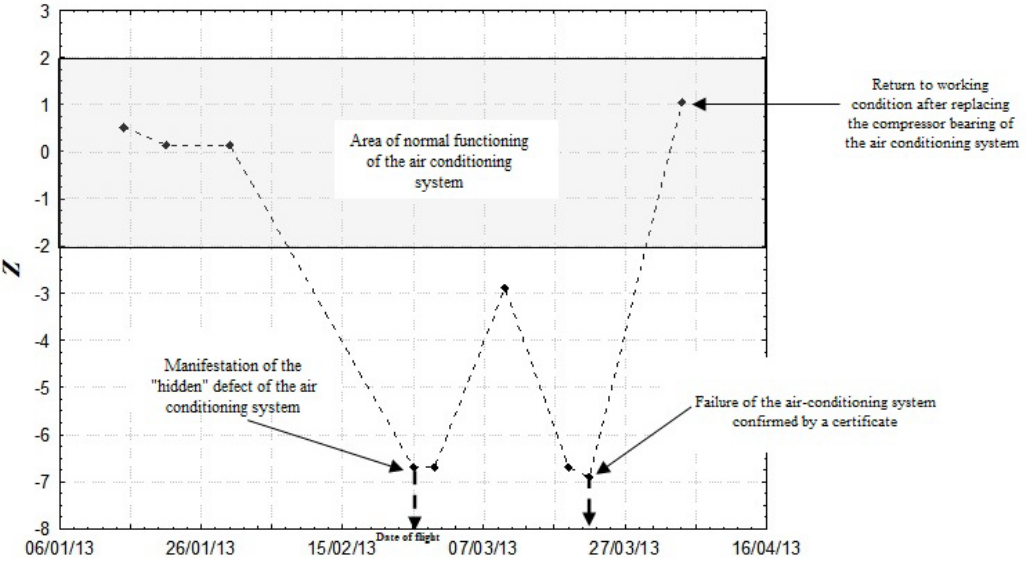

Fig. 4 – The results of the post-flight assessment of the air conditioning system during the operation of the aircraft based on the Mann-Whitney statistic

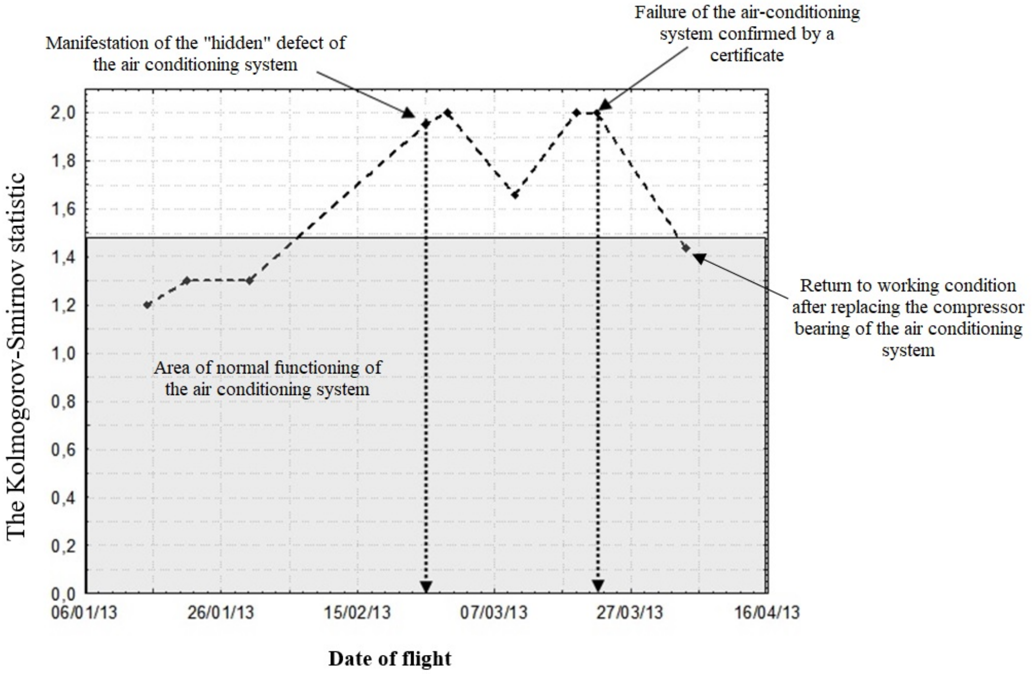

Fig. 5 – The results of the post-flight assessment of the air conditioning system during the operation of the aircraft based on the Kolmogorov-Smirnov two-sample test

4. CONCLUSIONS

Methods were proposed for assessing the technical condition of the aircraft's on-board systems, based on methods for testing statistical hypotheses about the properties of the distributions of parameters characterizing the condition of the on-board systems. Based on the processing of real flights data, the proposed methods were tested. The results obtained confirm the possibility of their effective use for solving the problem of assessing the condition of the aircraft's on-board systems.

Thus, the use of methods of testing the statistical hypotheses for the post-flight assessment of the condition of the aircraft's on-board systems considered in this scientific paper allows us to identify the danger of the occurrence of abnormal operation of the system before its obvious manifestation as a failure.

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