Development of pre-flight planning algorithms for the functional-program prototype of a distributed intellectual control system of unmanned flying vehicle groups

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Abstract: *In article presents algorithmic for a reconnaissance and combat unmanned flying vehicle (UFV) group pre-flight planning. The algorithms, which we elaborated, includes: algorithm, involved for mission target decision, membership selection. Discussing algorithm is based on analytical probabilistic models, providing evaluation of UAF group efficiency operation, considering UAF reliability as well as on board navigation and weapon facility performances; algorithm of UAF routes at the stages of group motion to meeting point and further movement to target area. Developed algorithm provides definition both direction and velocity of UAF, considering actual environment condition and dangerous of UAF recognition by radar, acoustic or infrared facilities of enemy; algorithms of UAF group operation scheduling at the mission target area.*

Key Words: *reconnaissance-combat unmanned flying vehicle, pre-flight planning, Delaunay triangulation, Pontryagin’s maximum principle*

1. INTRODUCTION

In the frames of our previous work on this subject, we validated the architecture and algorithms [1], [2], [3], [4], [5], [6] of a reconnaissance-combat unmanned flying vehicle (UFV) group’s distributed intellectual control system. It comprises two components: onboard and extravehicular. The extravehicular component includes the hardware and software facilities of the ground control post and is a part of the automated complex of UFV application and flight task planning. The onboard component includes a complex of onboard hardware and software for estimation of the flight situations and solution of the problems in the interests of the UFV group control on all flight stages. Within the frames of this architecture, it becomes possible to implement the control structure, comprising three levels, distinguished by the capacity of the ground control post to participate and by the volume of information, available for the control purposes.

On the upper (the first) level of the proposed 3-level structure, the current target and the corresponding typical flight situation are assigned. Here we have to take into account that the UFV group actions suppose consequent execution of a number of stages, with a typical flight situation related to each of them. These stages are:

1) “the group’s take off from the base area and gathering for further flight to the target application area”. As a rule, the group flight at this stage is performed under command of the ground control post;

2) “the group flight to the target application area”. Taking into account the requirement to keep the group actions secret at the stage of flight to the target application area, it is feasible to provide the UFV group motion in a tight formation with minimum distances and intervals between the UFV’s. In this case the capacity of the enemy’s radar equipment to detect and identify the UFV’s are substantially limited. The particular values of the formation parameters will be specified at the following stages of the project, taking into account the methods and algorithms of the UFV flight in formation control. The group motion at this stage is also performed under control of the ground control post.

3) “action in the target application area”. At this stage, the group acts in the target application area, which is usually under the enemy’s control. Let us specify the significant factors, which affect the UFV group actions in the target application area: presence of the electronic countermeasure equipment which restricts (makes impossible) the UFV group support by the ground control post; presence of the UFV detection devices and weapons. The group actions in the conditions of antagonistic environment require implementing the control methods, allowing prompt reconfiguration of the control tasks, taking into account the possible UFV losses as a result of the enemy’s organized actions;

4) “the group gathering after the task execution and its return to the base area”. Just like at the stage of take-off, at this stage the group moves under command of the ground control post.

On the second level of the three level UFV group control structure in consideration, a control method is chosen for each of the typical flight situations, assigned at the first level. Solution of the control problems on the second level, depending on the assigned flight situation, can be provided by the ground control post operator or by the onboard algorithms, implemented within the onboard component.

On the third level, implementation the goal achievement method, chosen at the second level, is provided.

Execution of the tasks, corresponding to this level, is provided by the UFV onboard control algorithms.

In this article, we present algorithmic and software provision for a reconnaissance and combat UFV group pre-flight planning, which are based on the publications [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20].

These algorithms are the basis of the extravehicular component of the functional-program prototype of the system in question and allow resolving the following problems: definition of the group composition and the UFV types, involved in operation; UFV route planning at takeoff and group gathering stages; UFV route planning at the stage of approaching the target area; i.e. planning of the reconnaissance and combat UFV actions in the target area [21], [22], [23]. This article contains the essence of the approaches, taken as the basis of the above listed algorithms, and demonstration of their potentials.

2. METHODOLOGY

2.1 Defining the group composition and the types of UFV, involved in operation

The most important problem, resolved at the stage of pre-flight planning, is to specify the required UFV group composition for searching and destroying the objects, located in the target area. Its results define the efficiency of all further group actions. This intends that the number and types of the UFV's, involved in the action, should be defined, taking into account their reliability, capacities of their onboard optoelectronic observation instruments and their weapons. Solution of this problem supposes obtaining preliminary evaluations of the UFV joint actions efficiency within the frames of the scenario in consideration, and defining the requirements in respect of the UFV composition and their parameters, meeting which we can expect the maximum effect. It is obvious that precise estimation of the UFV group actions efficiency can be obtained based on a complex of math models, relevantly describing the time and space status of all UFV's, the conditions and results of the weapons application, the interference circumstances, the enemy's radio-electronic and firepower counteraction instruments.

We consider it obvious that analysis of even one variation of the UFV group actions in a search and destroy mission, using a complex of math models of relevant complication and volume, will require unacceptably high computation and time costs. This circumstance indicates that it is necessary to use rather simple models, allowing to obtain estimations of the UFV joint actions efficiency, taking into account the number of the UFV's involved and the potentials of their onboard observation instruments and weapons. Let us consider one of the possible alternatives of implementing such a model, based on the following suggestions: the group includes N monotype reconnaissance and combat UFV's, searching and attacking ground targets in a specified area. We shall also suppose that the UFV, when it finds the target, attacks it and after this its participation in the action finishes. We consider the probability p of the target object being destroyed as a result of such an attack, to be specified. It is known that T targets are located in the target area.

The duration t_1 of the interval, during which the UFV remains functionally operative, the moment t_2 of the target detection, as well as duration of the attack t_3 are random values with exponential distribution with parameters $\theta = 1/\bar{t}_1, \lambda = 1/\bar{t}_2, \mu = 1/\bar{t}_3$. (In these expressions, the average values of corresponding time characteristics are marked by overbars). Result, obtained at [24] shows that within the framework of these suggestions, we can obtain the analytical dependencies, defining:

1) the probability of the UFV loss by instant t:

$$P_1(t) = \frac{\lambda\theta}{\mu+\theta}\int_0^t e^{-(\lambda+\theta)s}\left(1-e^{-(\mu+\theta)(t-s)}\right)ds + \theta\int_0^t e^{-(\lambda+\theta)s}\, ds \tag{1}$$

2) the probability that the UFV will successfully complete its mission, i.e. will attack the target, by instant t:

$$P_2(t) = \frac{\lambda\mu}{\mu+\theta}\int_0^t e^{-(\lambda+\theta)s}\left(1-e^{-(\mu+\theta)(t-s)}\right)ds \tag{2}$$

Using relations (1) and (2), in its turn, allows calculating the estimations of the UFV group actions efficiency, namely:

- the probability that by an instant t all N UFV's will complete their participation in the action either due to their damage or following a successful attack on the target:

$$P_3(t) = (P_1(t) + P_2(t))^N \tag{3}$$

- the mathematical expectation of the number of target objects, destroyed by the time instant *t*:

$$M_T(t) = T(1 - (1 - \frac{P_2(t)p}{T})^N) \tag{4}$$

In expression (4), p is the probability of the target object rejection, which is also supposed to be known. At the stage of pre-flight planning, based on estimations (3) and (4), for the specified guaranteed probability, taking into account the UFV group composition, the parameters of onboard optoelectronic instruments (λ) and weapons (p,μ), the UFV operation reliability parameter (θ), we can promptly define the following characteristics: duration of the action; the number of UFV's, required to destroy the specified number of targets within the specified time period; the requirements in respect of the observation instruments and weapons to ensure destroying the required number of targets within the minimum time, etc.

The above reliability estimations were obtained with an assumption that the UFV actions are independent. Therefore, these estimations can be considered as lower estimates, corresponding to the less favorable variation of the UFV joint application, from the point of view of the expected effect.

In this case, the algorithm provides obtaining the guaranteed estimates of the above listed efficiency parameters, calculated for the less favorable combination of the above mentioned mathematical expectation values.

2.2 UFV route planning at the stages of approaching the group gathering and the targeted application areas

The algorithm, which we elaborated, includes two blocks:

1) the UFV trajectory building-up at the stage from takeoff until arrival at the group gathering area;

2) the UFV trajectory building-up for further approaching the area of the target application.

The first block provides computation of the UFV direct motion trajectory with climbing to the defined altitude and proceeding to the group gathering area at this altitude with constant speed.

Further on, at the stage of approaching the target area, the group moves as a flying formation. So, taking this into account, each UFV must take a certain position with the required azimuth by a certain time instant.

The last condition arises due to the necessity of providing simultaneous appearance of all UFV's in the group gathering area.

The following items are used as input information for this block:

1) the UFV motion parameters (coordinates, velocity, course) after takeoff from the base airfield, specified in the coordinate system, the center of which coincides with the departure airfield;

2) the UFV motion parameters (coordinates, velocity, course) in the final point of the route, defined depending on the flying formation type, for further movement to the target application area;

3) the UFV turning radius, defined by its maneuvering potential limitations.

As a result of the studies, it was proved that the optimum trajectory, from the point of view of the minimum track length, represents a combination of direct motion sections and a turnaround maneuver by circle of a specified radius (taking into account the UFV maneuvering potentials) with constant angular speed.

Taking this into account, the analytical dependencies were elaborated and implemented in the block of the UFV route building-up at the stage of takeoff. The output of these dependencies are: the structure of the UFV route to the area of the group gathering, providing the required motion parameters in the final point.

The route structure is defined by the combination of the sections with zero roll angle and sections with maximum roll angle; the coordinates of the waypoints; the time of passing the waypoints. Thus, as a result of the algorithm's first block being executed for all UFV's within the group, the routes are computed, providing the time alignment of their appearance in the gathering area and ensuring the required configuration of the flight formation.

The second block is the block of the UFV route building-up at the stage of approaching the target application area. The most important requirement, applied to the UFV group at this stage, is providing its actions secrecy.

Taking this requirement into account, the preferred alternative of the group motion is moving in a flight formation with minimum intervals and distances between the UFV's. In this case, it becomes more difficult to detect the group by the enemy's radiolocation instruments and to identify separate UFV's, forming the group.

We have shown that motion as a flight formation with the specified intervals and distances, can be obtained by assigning the leading UFV within the group, which sets the group motion course and its velocity.

Within such a control system, the UFV group route planning problem reduces to building-up a route of approaching the border of the target application area only for the leading UFV, since the problem of the supporting UFV's control reduces to keeping the required intervals and distances with respect to the leader.

2.3 Planning of the reconnaissance and combat UFV group actions in the target application area

Analysis shows that the most complicated task in planning of the UFV group actions is arranging their coordinated interaction in the target application area. Choice of this or that planning method depends significantly on the level of prior awareness of the current situation.

The most simple situation arises when there is precise information about the number and location of the targets in the application area of the UFV group, about location of the flight restriction zones and about the potential hazard sources.

For these conditions, we propose the algorithm for pre-flight route to the target area planning, based on Delaunay triangulation [25] (Fig. 1), which provides building-up the optimal UFV routes, taking into account the above listed requirements, by resolving the mathematical programming problem, using criterion:

$$F(R,S) = W(S) + \mu_U(R) + \mu_C(R) \tag{5}$$

This criterion includes three components: $W(S)$ characterizes the amount of total loss, incurred during the motion of all UFV's in the group to the assigned targets by the routes, described by matrix S.

The total loss was evaluated from the point of view of the total energy expenditures for the route execution and the hazard of the UFV detection by the enemy's radiolocation instruments; $\mu_C(R)$ is the component, estimating the group actions, taking into account the number of the target objects engaged; $\mu_U(R)$ is the component, evaluating the group actions, accounting for the number of UFV's attending one target.

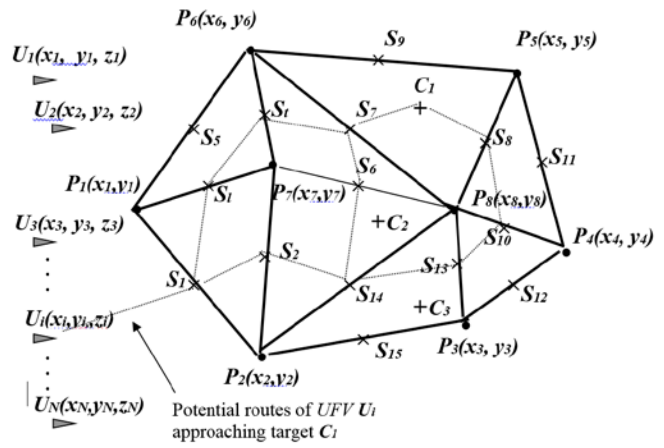

Fig. 1 – Building-up of Delaunay triangulation for the UFV group action pre-flight planning

Let us note that regarding the problem of the UFV route planning, the Delaunay triangulation can be considered as a geometrical structure, the edges of which connect the points, corresponding to location of potential hazard sources. The middles of the sections, forming the Delaunay triangulation, are the most distant from the corresponding hazard sources (the enemy’s air defense positions). Therefore, they can be considered as candidates for being used as waypoints.

The output of the algorithm is two matrixes:

- *R* is the matrix, defining the optimal variation of the target distribution, i.e. the option of directing each UFV to the corresponding target object;

- *S* is the matrix, defining the optimal routes of each UFV approaching the targets within a certain target distribution (the order of passing the middle points of Delaunay triangulation).

As a result, this algorithm provides building-up such routes of single UFV’s appearance near the selected target objects, which ensure reaching the maximum effect of the group actions.

The next group of algorithms is used in the presence of statistical uncertainty in respect of the hazard locations and sources in the area of a UFV group targeted application.

For such a situation, we suggest an algorithm of pre-flight planning of the combat-reconnaissance UFV group actions, which is an adaptation of the approach, based on Delaunay triangulation, considered above, taking into account the statistical uncertainty in respect of the air defense complexes in the target application area [26]. In this article, we shall limit ourselves to the general information about this algorithm.

The following information is used as initial data for the algorithm execution:

1) the UFV group composition, involved in the task of hitting (reconnaissance) of a group of ground objects;

2) the coordinates, defining each UFV position at the moment of appearance in the target area, in the inertial reference system, the initial point of which coincides with the departure airfield;

3) the specified altitude (level) of each UFV trajectory;

4) the specified constant speed of each UFV;

5) weight coefficients, defining each UFV significance, taking into account its weapons, etc.;

6) the set of ground target objects;

7) each target object's coordinates;

8) weight coefficients, defining significance of each target as an object of attack;

9) the set of air defense complexes, located in the UFV group target application area;

10) the density function, characterizing the statistical uncertainty of the information on the enemy's air defense complex coordinates.

The logic of this algorithm is as follows. Using the known density functions, describing the joint distribution of coordinates of the hazard source locations, the implementations are generated, which define a specific variation of their location. For this specific variation, the corresponding variation of Delaunay triangulation is formed, for which the problems of the target distribution and optimal UFV routes building-up are resolved in accordance with criterion (5). The solution is considered as optimal, when the mathematical expectation (on the set of implementations) of criterion function (5) reaches its minimum

The weakness of this algorithm is that the building-up of the optimal routes of the UFV approach to the targets, is performed on a finite-dimensional set of routes of a certain structure, induced be the Delaunay triangulation.

Taking this into account, an algorithm of building-up the UFV optimal routes, based on implementation of the Pontryagin maximum principle, was elaborated. It allows building-up the optimal routes of an arbitrary structure in the conditions when there is no credible information on the hazard source locations. This expands the pre-flight planning potentials, compared to the previous approach. The idea of this algorithm is based on representation of the target application area in the form of a grid with square cells (Fig. 2).

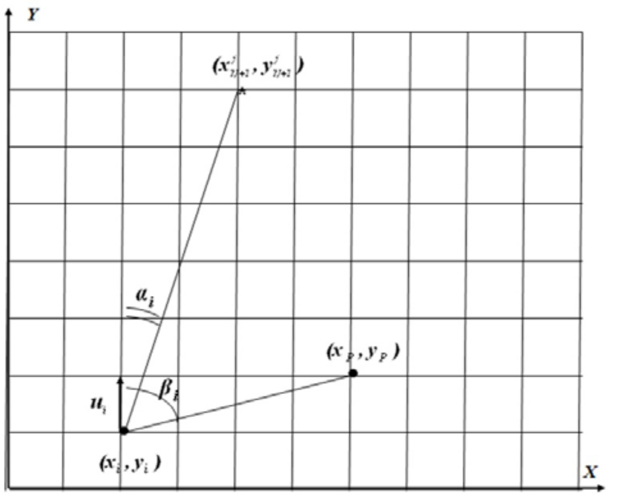

Fig. 2 – The algorithm of the UFV route planning, based on Pontryagin's maximum principle

The nodes of this grid are considered as the UFV waypoints. Here the UFV motion from one waypoint to another is performed only along the square cell sides. The grid size is chosen so that a UFV, taking into account its fuel range, could transfer from the specified initial position to any node. It was proved that within such a representation, the process of sequential waypoint bypassing by a UFV, can be described in the form of a discrete dynamic system, the control of which is presented by a discrete set of integer values.

$$z_{i+1} = z_i + Bu_i \tag{6}$$

where z_i is the current system state vector, the components of which are the coordinates (x_i, y_i) of the waypoints in the coordinate system OXY. Here, we assume that the UFV initial position (x_1, y_1) is defined and it coincides with one of the grid nodes; *B*- is a 2×2 matrix with components:

$$B = \begin{pmatrix} 0 & \Delta \\ \Delta & 0 \end{pmatrix} \quad (7)$$

Δ - the square cell side; u_i – the current control vector with components $u_{i,1}$, $u_{i,2}$.

Taking into account that the motion between the waypoints takes place only along the node sides, values $u_{i,1}$ = {-`1,0,1}, $u_{i,2}$ = {-`1,0,1} are used as controls $u_{i,1}$, $u_{i,2}$. Thus, on every step, controls $u_{i,1}$, $u_{i,2}$ can only take one value from the set of *U* = {-1, 0, 1}. It is obvious that combination $u_{i,1}$, $u_{i,2}$, with such a method of defining their admissible values, exhausts all possible alternatives of motion from some node of the grid (waypoint) to the next node, taking into account the motion along the square sides.

Starting with corresponding description of the waypoints bypassing process in the form of a dynamic system control problem (6), the problem of the route building-up can be formulated as finding such a sequence of controls $u_{i,1}$, $u_{i,2}$, $i=1,\ldots,N$, which provides transferring the system from the specified initial state (a point with coordinates x_1, y_1) to the specified final state (a point with coordinates x_{N+1}, y_{N+1}) in the best, in the sense of some criterion, way. The optimality criterion has the form:

$$J(u) = \sum_{i=1}^{N} f_i(z_i, u_i) \underset{u_i \in U}{\longrightarrow} min \quad (8)$$

The following designations are used here (Fig. 2):

$$f_i(z_i, u_i) = k_1 f_i^1(z_i, u_i) + k_2 f_i^2(z_i, u_i), \quad (9)$$

$$f_i^1(z_i, u_i) = M\left[cos\beta_i(x_{p,}y_p)\right], f_i^2(z_i, u_i) = cos\alpha_i \quad (10)$$

$$cos\beta_i(x_{p,}y_p) = \frac{(x_p - x_{i)} u_{i,1} + (y_p - y_{i)} u_{i,2}}{\sqrt{(x_p - x_{i)}^2 + (y_p - y_{i)}^2}} \quad (11)$$

$$cos\alpha_i = \frac{(x_{N+!} - x_{i)} u_{i,1} + (y_{N+!} - y_{i)} u_{i,2}}{\sqrt{(x_{N+1} - x_{i)}^2 + (y_{N+1} - y_{i)}^2}} \quad (12)$$

For the formulated problem at $\Delta \to 0$ an analytical solution was obtained, represented by the following relations.

$$u_i = \begin{cases} u_{i,1} = 1, u_{i,2} = 0, H_i^1 = min\{H_i^1, H_i^2, H_i^3, H_i^4\} \\ u_{i,1} = 0, u_{i,2} = 1, H_i^3 = min\{H_i^1, H_i^2, H_i^3, H_i^4\} \\ u_{i,1} = -1, u_{i,2} = 0, H_i^2 = min\{H_i^1, H_i^2, H_i^3, H_i^4\} \\ u_{i,1} = 0, u_{i,2} = -1, H_i^2 = min\{H_i^1, H_i^2, H_i^3, H_i^4\} \end{cases} \quad (13)$$

where $H_i^1 = \bar{\bar{F}}_{x,i} - \Phi_{x,i}$, $H_i^2 = -\bar{\bar{F}}_{x,i} + \Phi_{x,i}$, $H_i^3 = \bar{\bar{F}}_{y,i} - \Phi_{y,i}$, $H_i^4 = -\bar{\bar{F}}_{y,i} + \Phi_{y,i}$

$$\bar{\bar{F}}_{x,i} = M\left[\frac{(x_p - x_{i)}}{\sqrt{(x_p - x_{i)}^2 + (y_p - y_{i)}^2}}\right], \quad (14)$$

$$\Phi_{x,i} = \frac{(x_{N+1} - x_{i)}}{\sqrt{(x_{N+1} - x_{i)}^2 + (y_{N+1} - y_{i)}^2}}, \quad (15)$$

$$\bar{\bar{F}}_{y,i} = M\left[\frac{(y_p - y_{i)}}{\sqrt{(x_p - x_{i)}{}^2 + (y_p - y_{i)}{}^2}}\right], \tag{16}$$

$$\Phi_{y,i} = \frac{(y_{N+1} - y_{i)}}{\sqrt{(x_{N+1} - x_{i)}{}^2 + (y_{N+1} - y_{i)}{}^2}}, \tag{17}$$

3. RESULTS AND DISCUSSIONS

Figure 3 demonstrates the evaluations of the UFV group actions efficiency, calculated using dependencies (3)-(4) for different group size N (Fig. 3a), UFV survival quotients $\theta = 1/\bar{t}_1$ (Fig. 3b), characteristics $\lambda = 1/\bar{t}_2$ of the electrooptical observation facilities, installed on the UFV's (Fig. 3c) and the weapons characteristics $\mu = 1/\bar{t}_3$ (Fig. 3d).

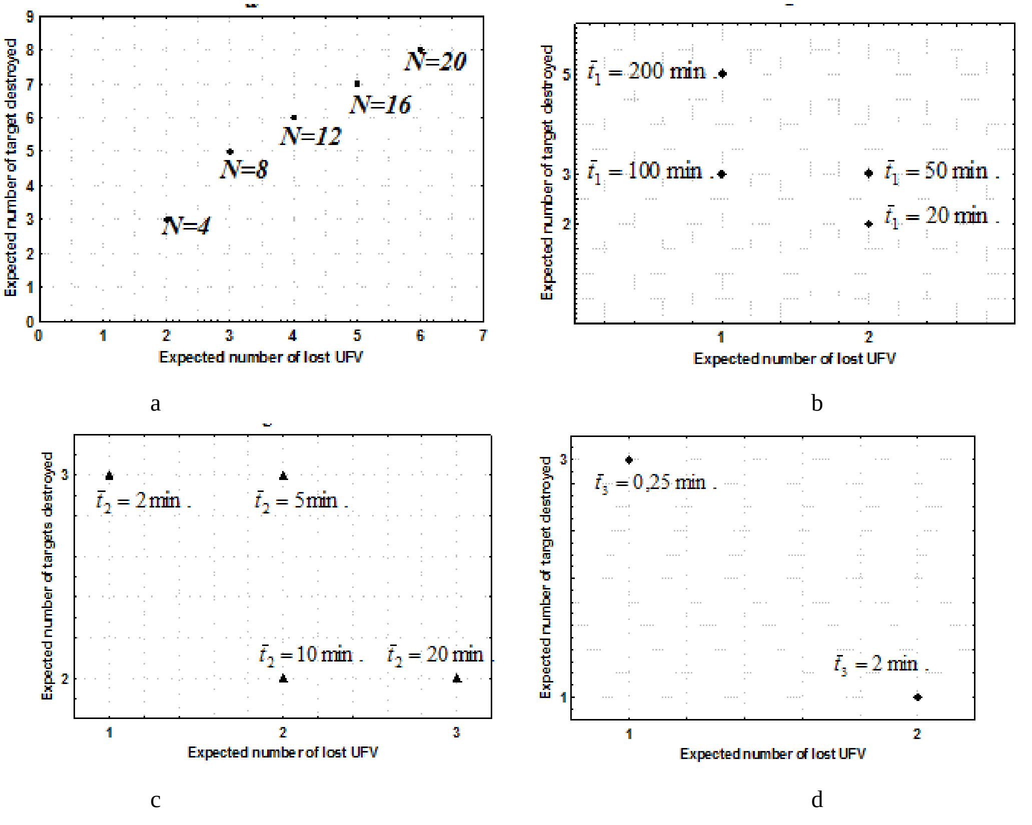

Fig. 3 – Evaluations of the UFV joint actions efficiency

The data provided confirms that with the existing UFV reliability values and the observation facilities and weapons characteristics, the UFV group is capable of detecting and destroying the target objects with minimal losses.

We have noted earlier that the problem of planning of the UFV's actions in the area of the target application is the most complicated one. In sections 2 and 3 we considered possible approaches to this problem solution, based on Delauney triangulation and Pontryagin's maximum principle.

As an example of the algorithm, based on Delaunay triangulation, we have considered the action planning problem for a group of five UFV's (***U_i, i=1,..,5***), participating in an action against a complex of four ground objects (***C_j, j=1,..,4***). The initial UFV's positions and positions of the target objects are demonstrated in Fig.4. It was assumed that the status (significance) of the UFV's within the group were different: $k_{U1} = k_{U2} = k_{U3} = k_{U4} = 0{,}1; k_{U5} = 0{,}6$ Significances of the targets as the attack objects are also different: $k_{C1} = 0{,}1;\ k_{C2} = 0{,}6; k_{C3} = 0{,}2; k_{C4} = 0{,}1$. It was also assumed that there are eight hazard sources in the target application area of the UFV group, information about which is restricted by availability of the Gaussian density functions of their coordinates' joint distribution $p(x_{Pk}, y_{Pk}), k = 1,..,8$. Using the Monte-Carlo method, in accordance with the density functions $p(x_{Pk}, y_{Pk}), k = 1,..,8$, we have modeled implementations $(x_{Pk}^t, y_{Pk}^t), k = 1,..,8; t = 1,...,1000$, defining the hazard sources positions. For each particular implementation set $(x_{Pk}^t, y_{Pk}^t), k = 1,..,8; t = 1,...,1000$ the algorithm [8] was executed, the results of which, as in the example above, are the following items:

- the optimal target distribution 5x4 matrix R_t^*, defining the best option of the UFV distribution by the target objects, taking into account criterion (5). Each element $R_{ij}, i = 1,...,5; j = 1,..,4$ of this matrix can take one of two possible values: $R_{ij} = 1$ if the j-th target is selected as the priority for the i-th UFV, $R_{ij} = 0$ otherwise;

- 5×15 matrix S_t^*, defining the optimal trajectory of the UFV's reaching the assigned target (the order of passing the Delaunay triangulation middle points) within the particular target distribution as defined by matrix R_t^*. I.e. elements $S_{ij}, j = 1,..,15$ of the i-th string of this matrix take values 0, 1, …. 15, where value 1 indicates that the corresponding Delaunay triangulation middle point corresponds to the 1st route waypoint, value 15 corresponds to the last one. Value 0 means that the corresponding Delaunay triangulation middle point is not included in the route.

The described procedure, corresponding to the method of statistical testing (the Monte Carlo method), was repeated 1000 times, and as a result, 1000 implementations of the criterial function and corresponding matrixes R_t^*, S_t^* were obtained. Analysis of the obtained implementations has shown that in 90% of the cases, the best solution of the pre-flight planning was obtained in the pair of matrixes R^*, S^* with components $R_{14}^* = R_{23}^* = R_{32}^* = R_{42}^* = R_{51}^* = 1;$ $S_{14}^* = 1,\ S_{15}^* = 2,\ S_{15}^* = 2,\ S_{19}^* = 3;$ $S_{23}^* = 1,\ S_{24}^* = 1,\ S_{25}^* = 2,\ S_{29}^* = 3,$ $S_{212}^* = 4;\ S_{31}^* = 1,\ S_{41}^* = 1,\ S_{58}^* = 1$. The results of the statistical tests are shown in Figure 4. Here the dark areas correspond to the areas of hazard sources location uncertainty, the UFV's are marked by triangles, and the targets are marked by crosses. Let us note that the matrix pair R^*, S^* defines only the optimal variant of the target distribution and the order of passing the middle points of the sections, connecting the Delaunay triangulation knots.

In the process of the statistical tests for each implementation $(x_{Pk}^t, y_{Pk}^t), k = 1,..,8; t = 1,...,1000$ the coordinates $(x_{Sl}^t, y_{Sl}^t), l = 1,..,15; t = 1,...,1000$ of the triangulation middle points were defined. On the XOY subspace these implementations together form the points, limited by some areas (in Fig.4 these areas are designated as G_1,G_4,G_5, G_9,G_{12}. Coordinates $(x_{Sl}, y_{Sl}), l = 1,..,15$ are correlated with coordinates $(x_{Pk}, y_{Pk}), k = 1,...,8$, defining the positions of the Delaunay triangulation knots, as a non-linear function. This is why, despite the fact that the densities $p(x_{Pk}, y_{Pk}), k = 1,..,8$ were assumed to be Gaussian, the values $(x_{Sl}, y_{Sl}), l = 1,..,15$ distribution is not normal. This fact complicates building up the areas G_1, G_4, G_5, G_9, G_{12}. Taking this into account, we used the approximation of the said areas by confidence ellipses. I.e. the confidence ellipses were used as areas G_1, G_4, G_5, G_9, G_{12}, to which the corresponding middle points belong with a 0,95 probability. The coordinates of the confidence ellipses centers were used as the coordinates of the UFV route waypoints within the optimal variant of the target distribution.

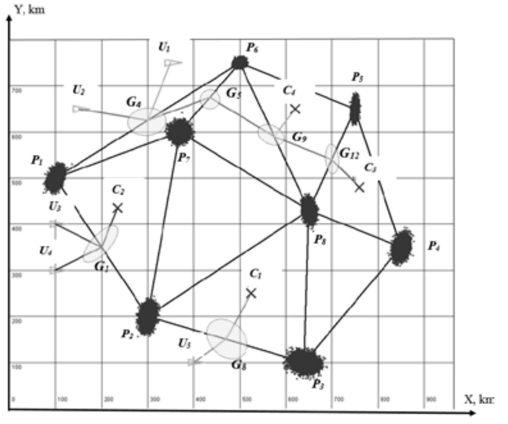

Fig. 4 – UFV optimal routes planning based on the Delaunay triangulation

As an example, illustrating the application of Pontryagin’s maximum principle, we considered the problem of pre-flight route planning for a single UFV, performing reconnaissance or combat tasks in a specified target application area. The purpose of the preflight planning is forming the UFV route in the horizontal subspace, in which the OXY coordinate system is set up, providing its positioning in any point of some specified area Ω (the area of interest), within which the targets are located. It is assumed that the observation instruments, installed on the UFV, provide detecting the target with the required accuracy from any point within area Ω irrespective of the UFV movement direction.

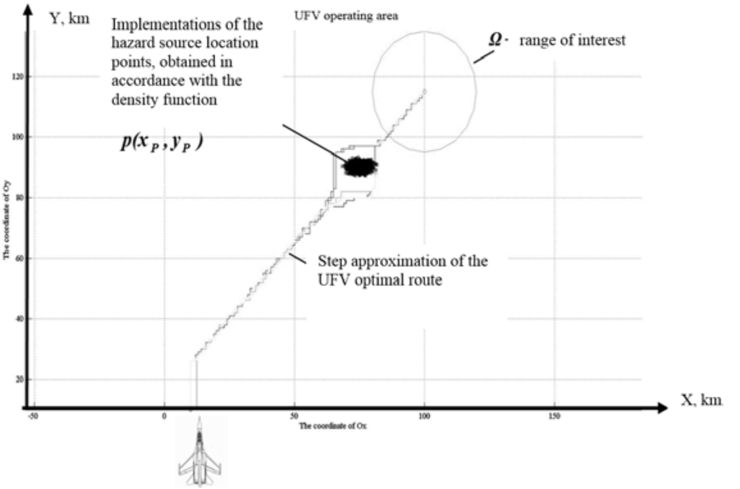

Fig. 5 – An illustration of the UFV route planning algorithm, based on Pontryagin’s maximum principle

Further on, we shall assume that the area of interest is a circle with specified center coordinates. Let us note that representation of the area of interest Ω as a circle, in no way restricts the generality of further presentation.

The problem is complicated by the fact that there is a hazard source in the area of target application, the exact coordinates (x_P, y_P) of which are unknown and are defined by the probability density distribution function $p(x_P, y_P)$. Figure 5 illustrates the formulated problem description.

The algorithm of the UFV route to the area of interest Ω calculation included the following actions:

1. For the specified initial UFV position, the implementations of coordinates $x_P^j, y_P^j, j = 1,...,100$, defining the probable position of the hazard source, were generated in accordance with the density function $p(x_P, y_P)$.

2. Based on implementations $x_P^j, y_P^j, j = 1,...,100$, using (14)-(17), mathematical expectations $\bar{F}_{x,1}, \bar{F}_{y,1}$ and values $\Phi_{x,1}, \Phi_{y,1}$ were obtained. Here, we assumed that the coordinates of the center of the area of interest Ω are: $x_C = 100\ km., y_C = 170\ km$

3. Using (13), the values of the Hamiltonian operator $H_1^1, H_1^2, H_1^3, H_1^4$ were calculated, and based on them, the controls $u_{1,1}, u_{2,1}$ were obtained.

4. The controls were used to calculate the next waypoint based on (6) for Δ = 100m, after which the above actions were repeated for the new waypoint with coordinates x_2, y_2 and so on. The route building up was terminated when condition $(x_{N+1} - x_c)^2 + (y_{N+1} - y_c)^2 \leq \varepsilon, \varepsilon = 50$m. held true.

Figure 5 demonstrates several route options, built using this algorithm. All the routes are of equal value from the point of view of the optimality criterion (8). Their diversity is explained by the fact that calculation of the Hamiltonian values $H_1^1, H_1^2, H_1^3, H_1^4$ was performed on a limited number of implementations $x_P^j, y_P^j, j = 1,...,100$.

4. CONCLUSIONS

The algorithms, described in this article, can be considered as an algorithmic basis of prior action planning for a diverse UFV group within the framework of a distributed intellectual control system. These algorithms are realized as functionally completed program modules, forming the program-mathematical support of the extravehicular component of the UFV group distributed intellectual control system within an automated complex of UFV application planning.

Algorithm of UFV group, involved in the mission target decision, membership selection is based on analytical probabilistic models, providing evaluation of the UFV group operation efficiency, considering the UFV reliability as well as onoard navigation and weapon facility performances. Utilization of such model gives an opportunity at pre flight scheduling stage to define both the number and types of UFV’s in order to reach the required value of the UFV group efficient operation criteria.

Algorithm of the UFV routes at the stages of group motion to meeting point and further movement to target area. The most important requirement in respect of the UFV motion at the mentioned above mission stages, consists in providing the UFV’s stealth while flying in formation with minimal values of both intervals and distances between the UFV’s. The developed algorithm provides definition of both direction and velocity of the UFV, considering the actual environment conditions and the hazard of the UFV’s being recognized by radars, acoustic or infrared facilities of the enemy.

The algorithm of the UFV group operation scheduling at the stage of the group return to the base area after the mission is completed. The algorithm provides generation of the route of each UFV group member, considering various requirements, such as minimization of the danger of the UFV discovery and destroying by the enemy as a result of the UFV motion in formation with minimal values of both intervals and distances between the UFV’s as well as an opportunity of practical implementation of the generated UFV routes, considering the environment conditions and the actual fuel stock.

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