Experimental validation of Tsai-Wu failure criteria

Ion FUIOREA*[,1], Dumitrita GABOR[1]

*Corresponding author
*[,1]Institute for Theoretical and Experimental Analysis of Aeronautical Structures
STRAERO, B-dul Iuliu Maniu 220, Bucharest, 061126, CP 76/175
fuiorea@straero.ro

DOI: 10.13111/2066-8201.2012.4.1.5

Abstract: *The paper continues recent research of the authors, consisting in experimental determination of Tsai-Wu failure criteria coefficients as well as criteria validation considering a certain composite. The tested composite, polyester resin reinforced with 1x1 fiber glass fabric was in house manufactured by using VARTM technology. Both elastic and ultimate tests were performed in order to define the constitutive equation of the composite and the ultimate answer under complex loading cases. Special test procedures were considered in order to realize a definite complex plane stress state where the components of stress tensor were clear known until the failure moments. The specimens subjected to the tests were flat and cylindrical ones.*

Key Words: *failure criteria, Tsai-Wu failure criteria, composite materials*

1. INTRODUCTION

The validation of the Tsai-Wu failure criteria stems in some physical tests performed until the sample failure moments and recording the ultimate stress tensor components that will be posteriori followed by locating in the σ_1 - σ_2 – τ_{12} stresses space and analyzed with respect to so-called failure surface (or failure envelope) shown in figure 1.1. The measured results deviation from the prescriptions of the criterion will be considered the quantification of the Tsai-Wu prediction errors. The main problems in these experiments were to achieve a complex plane state perfectly controlled in the moment of material failure, to locate the correct point in the failure state space and quantify the deviation from the prescriptions of the criterion.

For the study two types of experiments were considered:

- Tensile tests on flat specimens that were cut on different angles except the direction of decoupling effects of the composite;
- Tests on cylindrical specimens complexly loaded.

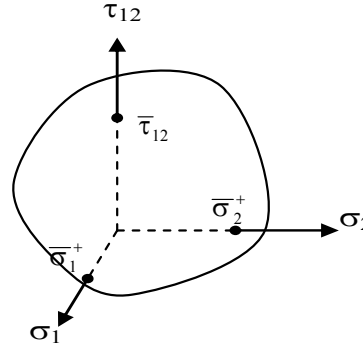

Fig. 1.1 Failure surface in the stress space

2. COMBINED TESTS ON PLANE SPECIMENS

The main problem of failure tests made on flat specimens cut on different directions with respect to the principal anisotropic directions consists in the difficulty to define the complex stress state.

Therefore a combined method was considered including both numerical and physical tests allowing a clear description of the complex stress state inside the specimen as a result of the parallel numerical tests.

As the Tsai-Wu failure criteria hypothesis is that the material has an elastic behavior up to failure, in the numerical calculus the same assumption was considered.

Experimental technology proposed is shown in the following sequences:

- specimens for tensile test are cut in different direction with respect to the anisotropic principal directions, but no identical with decoupling effects direction. The choice of angles was considered to cover a wide a range area of the failure surface.
- loading with low speed and carefully noting the maximum axial force where appeared the failure, and noting the zone on the specimen where the failure phenomenon started (usually in the vicinity of jaws clamping tensile machine). For an accurate location of failure point, a high speed camera was used for filming the failure phenomenon and then the exact place and time of the specimen failure moment was identified by image processing. All the made experiments showed that the failure phenomenon is preceded by changes in the appearance of the specimen where the failure occurs, making it significantly mat.
- the following step consists in a numerical test simulating closely the boundary conditions on displacements during the experiment. The boundary conditions on forces are set, corresponding to ultimate value of failure force determined during destructive tests.

The stress tensor component values are identified for the zone where the failure process started. Figures 1.2 and 1.3 are frames during the failure phenomenon on a specimen cut at 85^0 with respect to warp direction.

In fig. 1.3 the location of failure area can be noticed.

Fig. 1.2 85^0 specimen failure tensile tested

Fig. 1.3 Localization of failure area

Figure 1.4 shows an aspect of the σ_x tensor component map corresponding to the failure loading moment of the specimen.

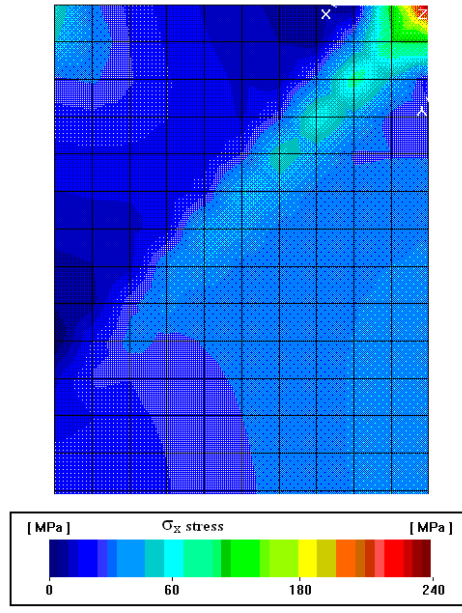

Fig. 1.4 Tensor component map for failure test simulation

Performing a coordinate transformation operation by rotating the axes, stress tensor components are determined for the principal directions of anisotropy, resulting $\sigma_1, \sigma_2, \tau_{12}$, that are posteriori positioned on the Tsai-Wu failure surface corresponding to an offset plane given by shear stress, σ_{12}.

3. COMBINED TESTS ON CILINDRICAL SPECIMENS

An experimental technology that assures a perfect control of stress tensor components until the failure for complex cases is provided by a combined test performed on cylindrical specimens. As the cylinder wall thickness, δ, is quite small, about 1%, it can be considered that a pure plane stress state took place in the specimen wall, far away from the clamping region.

Experiments performed on cylindrical specimens ensure the control of each component of the stress tensor throughout the test, including the failure phase due to the possibility of controlled loads. Referring to figure 1.5, by applying controlled tensile force (or compressive) $F_{tensile,}$ and an internal controlled pressure p_{int} it can be calculated stress value σ_1 (axial) inside the specimen wall:

$$\sigma_1 = \frac{F_{tensile}}{2\pi r_{med}\delta} + \frac{p_{int}\, r}{2\delta} \tag{1}$$

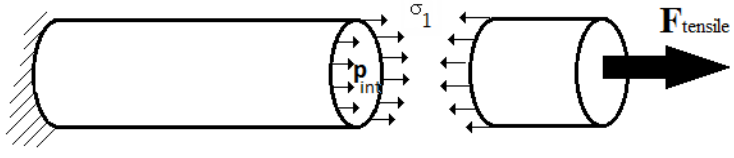

Fig. 1.5 Calculating scheme of axial stress

Similarly, for the applied controlled pressure inside the cylindrical specimen, p_{int}, the stress value σ_2 (tangential) in the specimen wall can be calculated (fig. 1.6).

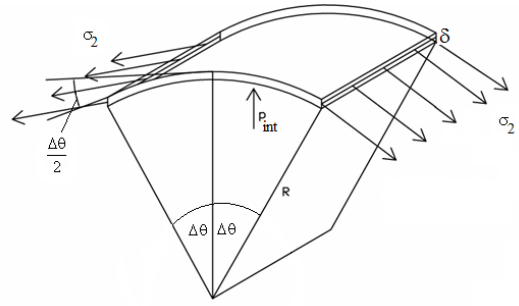

Fig. 1.6 Calculating scheme of tangential stress

$$\sin\frac{\Delta\theta}{2} \approx \frac{\Delta\theta}{2}. \tag{2}$$

And writing the equilibrium equations of a unit tube element along the normal:

$$2\sigma_2 \cdot 1 \cdot \delta \cdot \sin\frac{\Delta\theta}{2} = p_{\text{int}} \cdot R \cdot \Delta\theta \cdot 1 \tag{3}$$

Or:

$$\sigma_2 \cdot \delta = p_{\text{int}} \cdot R \Rightarrow \sigma_2 = \frac{p_{\text{int}} \cdot R}{\delta} \tag{4}$$

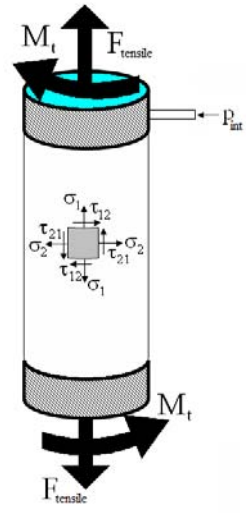

Fig. 1.7 Complex loading scheme of cylindrical specimen

And, similar by applying controlled torque moment M_t the value of sharing stress τ_{12} may be imposed in the specimen wall:

$$\tau_{12} = \frac{M_t}{2\pi R_{med}\delta} \tag{5}$$

According to this technology for testing is created in the cylinder wall (thin wall) a plane state of tension (fig. 1.7), under the assumptions of the definition in the failure criteria (and present in most thin aircraft composite structures).

Experiments were performed by maintaining constant the loading on two directions and increasing on the third one until specimen failure.

For accurate reading of the experimental data some strain gauges were mounted (as in fig. 1.8) for a parallel loading data acquisition.

Fig. 1.8 Instrumented cylindrical specimen

A device allowing simultaneous loading of the specimen with the three load components as shown in fig. 1.7. was imagined and designed for complex loading experiments The device can be mounted on tensile/compressive machine applying simultaneously: tensile force (or compression) along cylinder axis direction, torque and internal pressure. In this way the complex stress state is achieved inside the thin wall of the cylinder as theoretically case shown in fig. 1.7. Figures 1.9 and 1.10 illustrates both the design model and the real manufactured device, accomplished in the prototyping workshop of STRAERO.

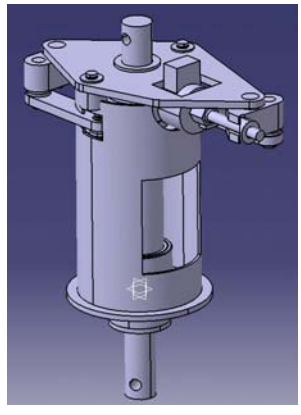

Fig. 1.9 The complex loading device (design)

Fig. 1.10 The complex loading device mounted on tensile machine

4. CRITICAL ANALYSIS. CRITERIA VALIDATION

In the figures 1.12 – 1.14 there are presented the results of the tests for failure criteria validation. Validation experiments were made considering three cases of constant shearing stress $\tau_{12} = 0$; $\tau_{12} = 0{,}2$; and $\tau_{12} = -0{,}2$ represented as curves resulted from intersection of the

Tsai-Wu failure surface (eq. 6) with the planes up mentioned (τ_{12} = 0; τ_{12} = 0,2; and τ_{12} = -0,2). In the figures 1.12 – 1.14 the actual curve is weighted presented as a difference to the other ones that are in a doted shape presented.

For an easy the interpretation of results, there were normalized in relation to ultimate stress corresponding to the decoupled simple stresses.

$$-\frac{\sigma_1}{101}-\frac{\sigma_2}{92}+\frac{\sigma_1^2}{23202}+\frac{\sigma_1\sigma_2}{6711}+\frac{\sigma_2^2}{18083}+\frac{\tau_{12}^2}{13228}=1 \tag{6}$$

The test results are presented both for cylindrical specimens and for the flat specimens.

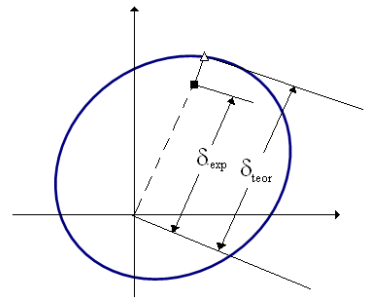

Fig. 1.11. Experimental evaluation according to Tsai-Wu failure criteria

The estimation of the difference between the Tsai-Wu criterion predictions and the real failure case was done by calculating the corresponding distances from the quadric center as presented in fig. 1.11 and using the formula (7).

$$\varepsilon=\left|\frac{\delta_{teor}-\delta_{\exp}}{\delta_{teor}}\right|\times 100 \tag{7}$$

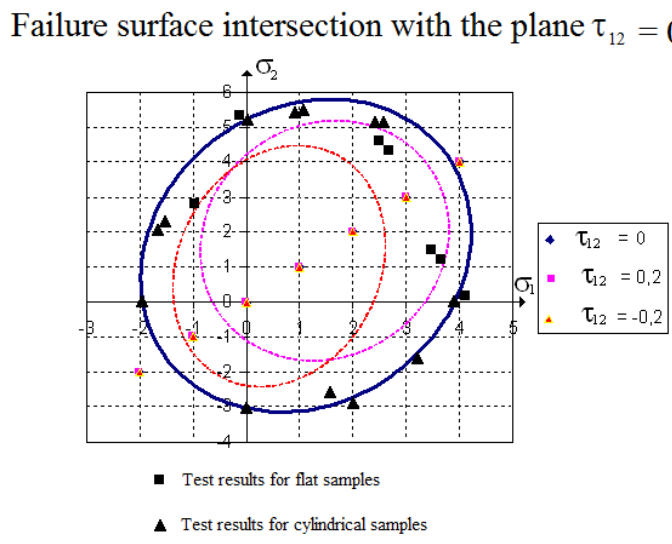

Fig. 1.12 Test results for equatorial curve
(the other two parallel curves are presented in a dotted shape)

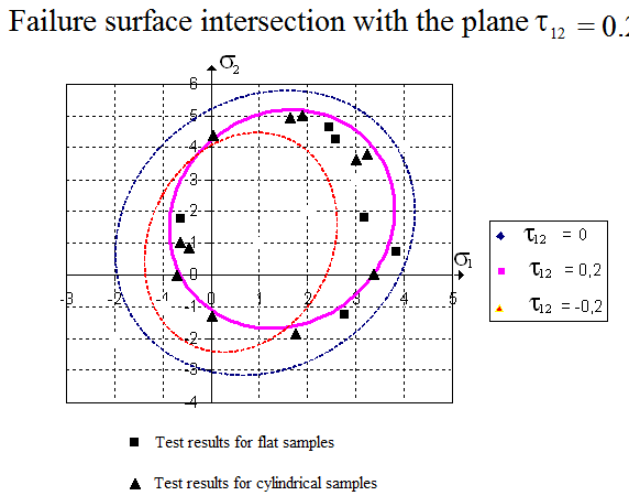

Fig. 1.13 Experimental results for τ_{12} = 0,2 parallel
(the other two parallel curves are presented in a dotted shape)

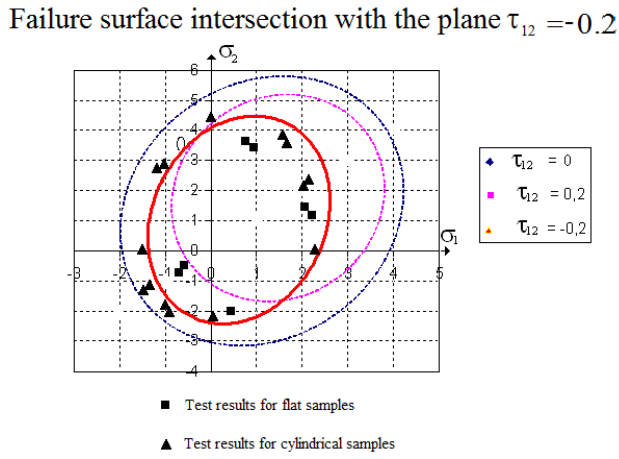

Fig.1.14 Experimental results for τ_{12} = - 0,2 parallel
(the other two parallel curves are presented in a dotted shape)

4. CONCLUSIONS

The analysis of the results presented in a graphical manner can lead to the following conclusions:

- Due to the small anisotropy of composite material studied (1x1 fabric impregnated with resin) the quadric eccentricity is relatively small, which may influence the conclusions of the study.
- Experimental results on cylindrical specimens are closer to the predictions of the criterion, with errors below than 6% for complex loads with positive or zero torque.
- The experimental results on cylindrical specimens are less close to the predictions of the criterion, with errors of up to 13% for complex loads with negative torque.

- The experimental results on flat specimens are less precise in relation to the predictions of the criterion, with errors of up to 20% for complex loads regardless of the torque sign.
- As an immediate consequence, a future work will focus on the study of the influence of torque sign and the corresponding terms of the criterion responsible for shearing stress sign.
- For a better estimation of the criterion and the influence of different defining factors it would be necessary to consider a composite material with a higher anisotropy, ideally to use unidirectional lamina.

REFERENCES

[1] Ion Fuiorea, *Materiale compozite Proiectarea răspunsului mecanic*, Editura Pan Publishing House, Bucureşti 1995

[2] Ion Fuiorea, s.a, Contract nr. 643/2008, "Mathematical Models for Complex Ultimate States in Composite Materials", Raport de cercetare 1, 2009.

[3] Laszlo P. Kollar, George S. Springer – *Mechanics of Composites Structures* (Failure criteria 411-434), ISBN-10: 0521801656 | ISBN-13: 978-0521801652, Cambridge University Press 2003.

[4] T. H. G. Megson, *Aircraft Structures for Engineering Students* , Third Edition, ISBN-10: 0340705884 | ISBN-13: 978-0340705889, Arnold, London, 1999.

[5] G. H. Staab, *Laminar Composites* , ISBN 0750671246, 9780750671248, Butterworth Heinemann, 1999.

[6] Robert M. Jones, *Mechanics of Composite Materials*, Second Edition, ISBN 156032712X, 9781560327127, Taylor & Francis, 1999.

[7] Gabriel Gh. Jiga, *Noțiuni fundamentale în mecanica materialelor compozite*, Editura Atlas Press, Bucureşti 2004

[8] Valery V. Vasiliev, Evgeny V. Morozov, *Mechanics and Analysis of Composite Materials*, ISBN 0080427022, 9780080427027, Elsevier 2001.

[9] Daniel Gay, *Materiaux composites*, ISBN 2866012682, 9782866012687, Hermes 1991, Critere de rupture de Tsai – Hill, pg. 335 – 345.

[10] Ion Fuiorea, Wiliam Tiu, *Decoupling Effects – An Interesting Way in Composites Characterization*, Proceedings of The Romanian Academy, Series A, Volume **12**, Number 1/2011, pp. 55–62, ISSN: 1454-9069.

[11] Nicolae Cristescu, *Mecanica materialelor compozite*, Ed. Universității Bucureşti, 1983.