Synthesis of structure of the onboard passive redundant subsystem with due consideration of the established tolerances

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Abstract: *The goal of this article is to select optimal parameters of various structures of the passive redundant subsystems of flying vehicles (hereinafter to be referred to as the "the aircraft") with due consideration of the established tolerances in respect of decrease in output parameters of these subsystems in the cases of failure of the aircraft components (in accordance with the required failure-free operation). It was demonstrated that synthesis of such structure is based on the solving the two-objective optimisation problem, which can be reduced to the one-criterion problem with the help of various methods. Authors of this article have selected the linear convolution method. Due to the absence of information on quantitative values of the prescribed tolerances and on the required failure-free operation of actual subsystems, authors have performed calculations of relevant dependences within the wide range of the realisable tolerances of the first and second levels. Authors have investigated influence of values of the required failure-free operation and values of tolerances upon the optimal structure of the passive redundancy subsystems of the aircraft.*

Key Words: *failure-free operation of subsystems, multipleness of redundancy, prescribed tolerances, realisable tolerances.*

1. INTRODUCTION

One of the important problems in the course of modernisation of the existing systems, as well as in the course of development of the future-oriented technical equipment is compliance with the ever-increasing requirements, which were established in order to ensure failure-free operation of the aircraft subsystems in the cases of sudden failures. As concerns many aircraft subsystems, which do not permit even short-time work stoppages, these requirements can be only complied with the help of the passive redundancy. In these circumstances, the neighbouring subsystem, which is connected with the relevant subsystem in accordance with its output parameter W, can exert essential influence upon the structure of redundancy (which is designated by any two of the following three parameters: quantity of the main elements m, quantity of the reserved elements r, and total quantity of all elements n). This influence is exerted in the cases, where such neighbouring subsystem permits deviation of the parameter W from its nominal value (nominal) upward or downward without failure of its working capacity. Situation, which is connected with deviation of the output parameter W downward, is the most important problem for the aerospace equipment [1], [2], [3]. This problem is analysed in the present article. In the following, we will designate structure of redundancy with the help of two parameters n and m, because of these parameters determine not only structure of subsystems. They determine multipleness of redundancy as well [4], [5]. The following subsystems are the subject matter of analysis within this article: onboard electric power supply subsystem (AC/DC; electricity consumers) [6]; fuel supply subsystem (aircraft engines) [7]; power amplifiers (actuators) [8], other actuators – actuating devices of the aircraft control circuits [9], and so on.

The goal of this article is to select optimal parameters of the structure of the aircraft passive redundant subsystems, which are designated in accordance with the best proportions between the failure-free operation parameters of these subsystems and elements of these subsystems, with due consideration of the requirements in respect of their failure-free operation, as well as the acceptable deviations of their output parameters.

2. DISCUSSION OF THE SYNTHESIS PROBLEM IN RESPECT OF THE STRUCTURE OF THE PASSIVE REDUNDANCY OF THE AIRCRAFT SUBSYSTEMS

Tolerable deviation ΔW_n of parameter W of the neighbouring subsystem in absolute terms is determined in accordance with the difference between its nominal W_{nom} and minimal W_m values as follows:

$$\Delta W_n = W_{nom} - W_m \quad (1)$$

This deviation is designated by designer of the neighbouring subsystem in accordance with the continuous scale of possible values. It determines the limiting quantity of the elements (r) (hereinafter to be referred to as the “the reserved elements”), which is acceptable for a failure within the passive redundant subsystem (which is connected with its neighbouring subsystem) without disturbance of working capacity of the neighbouring subsystem. Parameter r determines the realisable deviation ΔW_p of the output parameter of the passive redundant subsystem in absolute terms as follows:

$$\Delta W_p = W_{nom} \cdot r = W_{nom} \cdot n - W_{nom} \cdot m \quad (2)$$

It is obvious that the following condition must be met:

$$\Delta W_p \leq \Delta W_n \quad (3)$$

It would be handler to use the prescribed and realisable tolerances as relative units:

$$dW_p = \frac{w_{nom} \cdot n - W_{nom} \cdot m}{W_{nom} \cdot n} \cdot 100\% = \frac{r}{n} \cdot 100\% \quad (4)$$

Once again, the following condition must be met:

$$dW_p \leq dW_n \quad (5)$$

The prescribed tolerance dW_n (in relative units) is designated along the continuous scale of tolerances as values from 0 to 100%. The realisable tolerance dW_p is designated by values r and n. This tolerance determines the discrete scale of possible structures of the passive redundancy. The realisable tolerance is the most important parameter for solving the synthesis problem. One and the same realisable tolerance can be ensured by various structures, as a rule, by the structures of the aliquant passive redundancy [10]. These structures are determined by the line (multitude) of the increasing multiplenesses hereinafter to be referred to as K_i (in the aggregate), beginning from the minimal value of the respective index i:

$$K_i = \frac{n_i}{m_i} \tag{6}$$

$$dW_p = \frac{r_i}{n_i} \cdot 100\% = \frac{n_{i-m_i}}{n_i} \cdot 100\% \tag{7}$$

The realisable tolerances are varied in the levels depending on the quantity of the reserved elements, which are included to the minimal multipleness of redundancy. In the following, we will analyse the discrete scales of the realisable tolerances of the first and the second levels only because they are the most important figures for practical calculations. In these circumstances, we will further understand “tolerance” as the realisable tolerance without word “realisable”, if it is clear in context.

It is the most suitable to present scale of tolerances of the first level at minimal values of parameters of the structures n_m and m_m, which are designated as $r = 1$ in the course of variability of the total quantity of elements n = 2, 3, 4, 5, …:

$dW_p=\frac{1}{2} \cdot 100\% = 50\%$ $(n_m = 2, m_m = 1, r = 1)$; $dW_p=\frac{1}{3} \cdot 100\% = 33.3\%$ $(n_m = 3, m_m = 2, r = 1)$; $dW_p=\frac{1}{4} \cdot 100\% = 25\%$ $(n_m = 4, m_m = 3, r = 1)$ and so on.

Therefore, scale of tolerances of the first level is in the following form: 50%; 33.3%; 25%; 20%; 16.7%; 14.3%; 12.5%; 11.1%; 10% and so on. Similarly to the foregoing, it is the most suitable to present the scale of the second level tolerances at minimal values of parameters of the structures n_m and m_m, which are designated as $r = 2$, in the course of variability of the total quantity of elements n = 3, 4, 5, 6, …

$dW_p=\frac{2}{3} \cdot 100\% = 66.7\%(n_m = 3, m_m = 1, r = 2)$; $dW_p=\frac{2}{4} \cdot 100\% = 50\%(n_m = 4, m_m = 2, r = 2)$; $dW_p=\frac{2}{5} \cdot 100\% = 40\%(n_m = 5, m_m = 3, r = 2)$; and so on.

Scale of the second level tolerances includes the repeating (duplicate) values, which are the same as the values of the scale of the first level tolerances. Scale of the second level tolerances is in the following form: 66.7%. 50%; 40%; 33.3%; 28.6%; 25%; 22.2%; 20%; 18.2% and so on.

As it was mentioned earlier, each tolerance can be ensured by various structures, that is, by the realisable multitudes of the increasing multiplenesses, beginning from the minimal multipleness. For example, tolerance of the first level dW_p=50% is realised in the following manner: at $K_1 = \frac{2}{1}$ =2 (*n*-modular redundancy, $dW_p = \frac{1}{2} \cdot 100\% = 50\%$), at $K_2 = \frac{4}{2}$ (aliquant redundancy, $dW_p = \frac{2}{4} \cdot 100\% = 50\%$), at $K_3 = \frac{6}{3}$ (aliquant redundancy, $dW_p = \frac{3}{6} \cdot 100\% = 50\%$) and so on. Tolerance of the first level dW_p=33.3% is realised as follows: at $K_1 = \frac{3}{2}$ (aliquant redundancy, $dW_p = \frac{1}{3} \cdot 100\% = 33.3\%$), at $K_2 = \frac{6}{4}$ (aliquant redundancy $dW_p = \frac{2}{6} \cdot 100\% = 33.3\%$) and so on. Similarly to the foregoing, tolerance of the second level dW_p=66.6% is realised as follows: at $K_2 = \frac{3}{1}$ =3 (*n*-modular redundancy, $dW_p = \frac{2}{3} \cdot$

$100\% = 66.6\%$), at $K_3 = \frac{6}{2}$ (aliquant redundancy $dW_p = \frac{4}{6} \cdot 100\% = 66.6\%$), at $K_4 = \frac{9}{3}$ (aliquant redundancy $dW_p = \frac{6}{9} \cdot 100\% = 66.6\%$) and so on. In general terms, individual lines of multiplenesses are to be calculated using the following formula:

$$K_i = \frac{n_i}{m_i} = \frac{n_m \cdot i}{m_m \cdot i} \quad (8)$$

where $i \geq 1$ in respect of the 1 level tolerances and $i \geq 2$ in respect of the 2 level tolerances.

At this time, n_m and m_m are minimal values of parameters of the structures, which ensure formation of each tolerance. Therefore, there exists certain correspondence between each tolerance of any level and its own line of the increasing multiplenesses, which are designated (in the aggregate) as K_i. Let us assume that T is a random time to failure. For simplicity, we will use the probability of no-failure operation P_c during performance of the task t_3 [11] as the indicator of the failure-free operation of the redundant subsystem

$$P_c(t_3) = P(T > t_3) = P_c \quad (9)$$

while similar probability of no-failure operation of the redundant subsystem element p during performance of task t_3 would be assumed as the indicator of the failure-free operation of the non-redundant subsystem (of the relevant element of a redundant subsystem)

$$p(t_3) = P(T > t_3) = p \quad (10)$$

The conditions of functioning of elements of the passive redundant subsystems, which were accepted in this article, comply with the requirements of the Bernoulli theorem concerning repetition of trials/experiences [12]. Therefore, it is possible to use formula of the binomial distribution law in order to calculate indicator of the failure-free operation (9) as follows:

$$P_c = \sum_{i=m}^{n} C_n^i \cdot p^i \cdot (1-p)^{(n-i)} \quad (11)$$

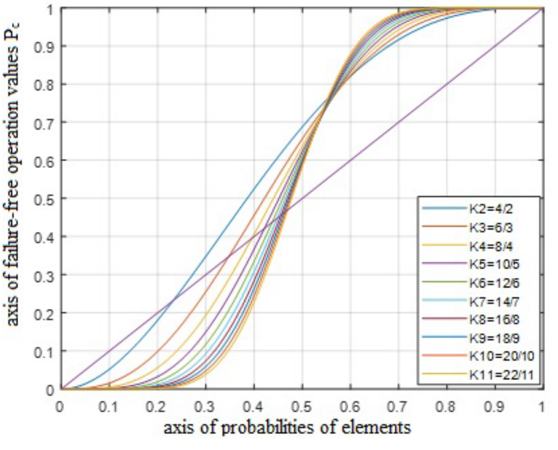

Fig. 1 – Diagrams of dependency of P_c from p for various multiplenesses, $dW_p = 50\%$

Parameters n and m in the formula (10) are determined by the multipleness of redundancy, while each tolerance (as it was mentioned above) is in correspondence with its multitude of the increasing multiplenesses. Therefore, there are good reasons to investigate dependence of P_c from p at various multiplenesses K_i, as well as at tolerances of the first and second levels with the help of this formula.

Figures 1, 2, 3, and 4 present sampling dependences of P_c $f$rom p in respect of 50% and 25% tolerances of the first level, as well as 40% and 18.2% tolerances of the second level.

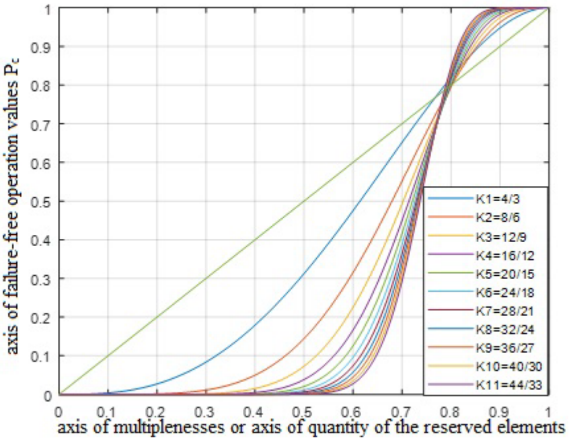

Fig. 2 – Diagrams of dependency P_c from p for various multiplenesses, $dW_p = 25\%$

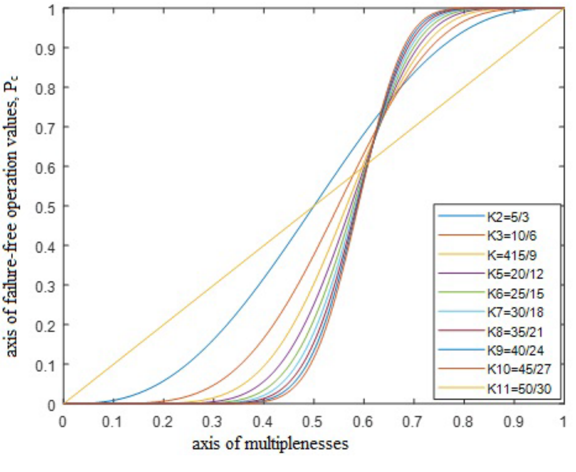

Fig. 3 – Diagrams of dependency of P_c from p for various multiplenesses, $dW_p = 40\%$

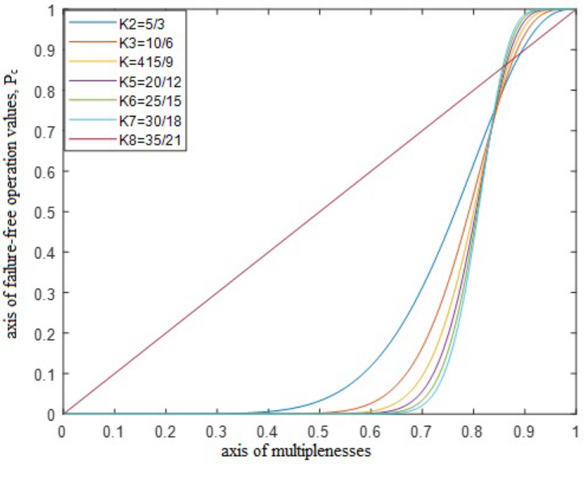

Fig. 4 – Diagrams of dependency of P_c from p for various multiplenesses, $dW_p = 18.2\%$

The above-presented diagrams make it possible to reveal the total regularities, which are inherent to all realisable tolerances:

1) In the case of the aliquant passive redundancy of the aircraft subsystems, there exist critical values of probabilities of their elements $P_{kr,}$, which are similar to the active redundancy [13] and which divide (0 – 1) interval into 2 areas: subcritical area (0 - $P_{kr,}$), within which failure-free operation of the redundant subsystem is lower than failure-free operation of the nonredundant subsystem, and supercritical area, within which this kind of redundancy is beneficial;

2) The supercritical area decreases along with decrease of the tolerance;

3) As concerns each tolerance for each multipleness of redundancy, there exists the extreme value of the failure-free operation of the redundant subsystem with respect to the failure-free operation of elements (of the non-redundant subsystem). These extreme values determine the best (in some specified sense) structures of redundancy;

4) As concerns each tolerance, the extreme values of the failure-free operation increase along with increase of the multipleness of redundancy.

The last two properties make it possible to formulate the synthesis problem, in other words, problem of selection of optimal parameters of the passive redundancy structure.

3. FORMULATION OF THE PROBLEM OF SYNTHESIS OF THE PASSIVE REDUNDANCY STRUCTURE OF THE AIRCRAFT SUBSYSTEMS

One of the problems facing the designers of the aircraft subsystems is as follows: development of the highly-reliable objects with the help of the objects, which are less reliable ones as compared with the highly-reliable objects and which consist of the inexpensive components, as a rule [14]. As concerns the task under investigation, this means that the optimal structure of the passive redundancy must comply with such values n and m, at which failure-free operation of the redundant subsystem would be as high as possible, while failure-free operation of the elements, which are included to this subsystem, would be as low as possible.

Therefore, we have the task of optimisation in respect of two indicators:

- indicator of the failure-free operation of the redundant subsystem

$$P_c = \sum_{i=m}^{n} C_n^i \cdot p^i \cdot (1-p)^{(n-i)} \rightarrow max \quad (12)$$

- indicator of the failure-free operation of the redundant subsystem elements:

$$p(p) \rightarrow min \quad (13)$$

It is necessary to perform convolution of two indicators and transform it into one complex criterion, because of the task of optimisation in respect of many indicators is incorrect from a mathematical standpoint.

As concerns the situation under consideration, the task of convolution of two partial indicators and subsequent transformation into one complex criterion is simplified by the fact that both indicators are dimensionless values, which have the same names. There exists a large set of possible methods of convolution, and there are two approaches, which are the best suited ones and which make it possible to find the compromise solution for the task of synthesis from a mathematical standpoint.

The first approach involves linear convolution of two indicators:

$$\Delta P_c = P_c - p, \quad (14)$$

where $P_c \to max$, $(-p) \to max$, $\Delta P_c \to max$.

The second approach involves formation of the ratio/proportion of two indicators. This proportion can be formed with the help of two methods:

$$dP_c = \frac{P_c}{p} \quad (15)$$

where $P_c \to max$, $p \to min$, $dP_c \to max$, or

$$dP_c = \frac{(P_c - p)}{p}, \quad (16)$$

where $\Delta P_c \to max$, $p \to min$, $dP_c \to max$.

Criteria (15) and (16) within the second approach provide the same result of synthesis. However, there is an essential difference between the results of synthesis in accordance with the criteria (14) and (15 or 16). In contrast to other well-known methods of convolution of partial indicators, these two approaches have obvious technical sense. Complex criterion (14) shows the absolute value of exceedance of the indicator of failure-free operation of the redundant subsystem over the indicator of failure-free operation of those elements, which are included to the redundant subsystem. Complex criterion (15 or 16) shows relative value of exceedance of the indicator of failure-free operation of the redundant subsystem over the indicator of failure-free operation of those elements, which are included to the redundant subsystem (in relative units). It should be noted that in the course of optimisation in respect of these complex criteria, no extremums are satisfied in respect of any partial indicators, which are included to these criteria. Instead of this, a new solution, which is a certain compromise between the partial extremums, is found. This compromise solution is determined (to a considerable degree) by the method of transformation of partial indicators into the single complex criterion and this solution depends on the goals and tasks, with which a designer is faced. Let us select the complex criterion (14) in order to use it in the course of the following investigations. In this case, mathematical statement of the problem of synthesis of the passive redundancy structure of the aircraft subsystems (with due consideration of the established requirements in respect of the failure-free operation and tolerances) will be written down as follows:

$$\Delta P_c^* = \max_{(p, K_i)} (P_c \left(p, K_i, dW_p, P_{\text{тр}}, dW_{\text{н}}, p_{kr}\right) - p), \quad (17)$$

provided that there exist limitations as follows:

$$p^* > p_{kr}, \quad (18)$$

$$dW_p \le dW_{\text{н}}, \quad (19)$$

$$P_c^* \ge P_{\text{тр}}, \quad (20)$$

$$K_i = \frac{n_i}{m_i}, n_i, m_i > 0, \text{ integer numbers.} \quad (21)$$

4. SOLVING THE SYNTHESIS PROBLEM OF THE PASSIVE REDUNDANCY STRUCTURE OF THE AIRCRAFT SUBSYSTEMS

Let us assume that the values of the prescribed tolerance $dW_{\text{н}}$, as well as values of the required failure-free operation $P_{\text{тр}}$ of the redundant of the aircraft subsystem are already prescribed. The prescribed tolerance $dW_{\text{н}}$ makes it possible to determine the nearest value

dW_p in accordance with the frame of the realisable tolerances of the first and second levels in accordance with the condition (19). As it was mentioned earlier, the found realisable tolerance is ensured by the multitude of the increasing multiplenesses K_i, beginning from the minimal multipleness.

Moreover, each multipleness specifies parameters n_i and m_i of the already determined structure of redundancy. Formula (17) for calculation of the complex criterion makes it possible to solve the so-called weakened problem of the synthesis, which will be written down as follows:

$$\Delta P_c^* = \max_{(p)}(P_c\left(p, K_i, dW_p, dW_{\text{н}}, p_{kr}\right) - p) \tag{22}$$

provided that there exist limitations as follows:

$$p^* > p_{kr}, \tag{23}$$

$$dW_p \leq dW_{\text{н}} \tag{24}$$

$$K_i = \frac{n_i}{m_i}, n_i, m_i > 0, \text{integer numbers} \tag{25}$$

The weakened task does not take into account the existing limitation in respect of the failure-free operation of the redundant subsystem (20) of the original task. Solution of this task makes it possible to find the dependences of optimal values of the complex criterion ΔP_c^* and partial indicators P_c^* and p^*, which create these values, from the redundancy multiplenesses K_i:

$$\Delta P_c^* = \Delta P_c^*(K_i); \tag{26}$$

$$P_c^* = P_c^*(K_i); \tag{27}$$

$$p^* = p^*(K_i). \tag{28}$$

As it follows from the diagrams, which are presented by Figures 1, 2, 3 and 4, dependence $P_c^*(K_i)$ has the growing nature along with increase of the redundancy multipleness K_i at any realisable tolerances.

This fact makes it possible to solve the original problem of the synthesis, in other words, to find such values of multiplenesses K_i^*, at which the following condition is fulfilled $P_c^* \geq P_{\text{тр}}$. It is very simple to solve the weakened task with the help of the enumerative technique.

If the value of the realisable tolerance dW_p, as well as the multitude, which ensures this tolerance of multiplenesses K_i, are already prescribed, it is necessary to perform enumeration of the probability values of the respective elements p within the supercritical area (p_{kr} – 1), and then calculate the complex criterion (24).

The step of such enumeration determines accuracy of calculation of the optimal value of the complex criterion ΔP_c^*. In addition, it determines accuracy of the relevant partial indicators P_c^* and p^*. The dependences $\Delta P_c^* = \Delta P_c^*(K_i)$; $P_c^* = P_c^*(K_i)$; $p^* = p^*(K_i)$, which would be determined in such a manner, make it possible to find the solution of the desirable synthesis problem.

Due to the absence of information concerning the quantitative values of the prescribed tolerances, as well as concerning the required failure-free operation of the prospective aircraft subsystems, which are now at the stage of development [15], let us calculate the dependences (26, 27, 28) for the wide range of the realisable tolerances of the first and second levels. Table 1 presents the results of calculations of the optimal values ΔP_c^*, P_c^*, and p^* for ten multiplenesses, as well as for the most important tolerances of the first level, while Table 2 presents similar results for nine multiplenesses, as well as for the important tolerances of the second level. These results were calculated with the accuracy to 0.001.

Table 1. – Optimal values of the complex criterion ΔP_c^* and of the partial indicators P_c^* and p^* for the realisable tolerances of the first level, as well as for 10 generalised multiplenesses

dW_p %	Para-meters	multiplenesses K_i									
		K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	K_{10}
50	ΔP_c^*	-	0.224	0.233	0.245	0.257	0.267	0.276	0.284	0.291	0.297
	P_c^*	-	0.863	0.898	0.915	0.926	0.934	0.940	0.944	0.948	0.951
	p^*	-	0.639	0.665	0.670	0.669	0.667	0.664	0.660	0.657	0.654
	$n^*.m^*$	2.1	4.2	6.3	8.4	10.5	12.6	14.7	16.8	18.9	20.10
33.3	ΔP_c^*	0.096	0.104	0.119	0.131	0.141	0.150	0.157	0.164	0.170	0.175
	P_c^*	0.885	0.933	0.947	0.954	0.958	0.962	0.964	0.967	0.969	0.970
	p^*	0.789	0.829	0.828	0.817	0.812	0.807	0.803	0.799	0.795	0.792
	$n^*.m^*$	3.2	6.4	9.6	12.8	15.10	18.12	21.14	24.16	27.18	30.20
25	ΔP_c^*	0.048	0.062	0.075	0.085	0.094	0.101	0.107	0.112	0.117	0.121
	P_c^*	0.944	0.961	0.967	0.970	0.973	0.974	0.976	0.977	0.978	0.979
	p^*	0.896	0.899	0.892	0.885	0.879	0.873	0.869	0.865	0.861	0.858
	$n^*.m^*$	4.3	8.6	12.9	16.12	20.15	24.18	28.21	32.24	36.27	40.30
20	ΔP_c^*	0.028	0.042	0.053	0.062	0.069	0.075	0.080	0.084	0.088	0.092
	P_c^*	0.968	0,975	0,977	0,979	0,980	0,981	0,982	0,983	0,984	0,985
	p^*	0.940	0.933	0.924	0.917	0.911	0.906	0.902	0.899	0.895	0.893
	n. m	5.4	10.8	15.12	20.16	25.20	30.24	35.28	40.32	45.36	50.40
16.7	ΔP_c^*	0.018	0.030	0.040	0.048	0.054	0.059	0.063	0.067	0.070	0.073
	P_c^*	0.979	0.981	0.983	0.984	0.985	0.986	0.986	0.987	0.987	0.987
	p^*	0.961	0.951	0.943	0.936	0.931	0.927	0.923	0.920	0.917	0.914
	$n^*.m^*$	6.5	12.10	18.15	24.20	30.25	36.30	42.35	48.40	54.45	60.50
14.3	ΔP_c^*	0.013	0.023	0.032	0.038	0.044	0.048	0.052	0.055	0.058	0.061
	P_c^*	0.986	0.986	0.987	0.987	0.988	0.988	0.989	0.989	0.989	0.990
	p^*	0.973	0.963	0.955	0.949	0.944	0.940	0.937	0.934	0.931	0.929
	$n^*.m^*$	7.6	14.12	21.18	28.24	35.30	42.36	49.42	56.48	63.54	70.60
12.5	ΔP_c^*	0.010	0.019	0.026	0.032	0.036	0.040	0.044	0.047	0.050	0.052
	P_c^*	0.990	0.989	0.989	0.990	0.989	0.990	0.991	0.991	0.991	0.992
	p^*	0.980	0.970	0.963	0.958	0.953	0.950	0.947	0.944	0.942	0.940
	$n^*.m^*$	8.7	16.14	24.21	32.28	40.35	48.42	56.49	64.56	72.63	80.70
11.1	ΔP_c^*	0.007	0.015	0.022	0.027	0.031	0.035	0.038	0.041	0.043	0.045
	P_c^*	0.992	0.991	0.991	0991	0991	0992	0992	0.992	0993	0.993
	p^*	0.985	0.976	0.969	0.964	0.960	0.957	0.954	0.952	0.950	0.948
	$n^*.m^*$	9.8	18.16	27.24	36.32	45.40	54.48	63.56	72.64	81.72	90.80
10	ΔP_c^*	0.006	0.013	0.019	0.923	0.027	0.030	0.033	0.036	0.038	0.040
	P_c^*	0.994	0.993	0.993	0.992	0.992	0.992	0.993	0.994	0.994	0.994
	p^*	0.988	0.980	0.974	0.969	0.965	0.962	0.960	0.958	0.956	0.954
	$n^*.m^*$	10.9	20.18	30.27	40.36	50.45	60.54	70.63	80.72	90.81	100.99

Table 2. – Optimal values of the complex criterion ΔP_c^*, values of the partial indicators P_c^*, and p^* for the realisable tolerances of the second level, as well as for 9 generalised multiplenesses

dW_p %	Para-meters	multiplenesses K_i								
		K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	K_{10}
66.7	ΔP_c^*	-	0.391	0.410	0.427	0.441	0.453	0.463	0.471	0.479
	P_c^*	-	0.880	0.908	0.924	0.934	0.941	0.946	0.950	0.954
	p^*	-	0.489	0.498	0.497	0.493	0.488	0.483	0.479	0.475

	n^*, m^*	3.1	6.2	9.3	12.4	15.5	18.6	21.7	24.8	27.9
50	ΔP_c^*	0.224	0.245	0.267	0.284	0.297	0.308	0.317	0.325	0.331
	P_c^*	0.863	0.915	0.934	0.944	0.951	0.956	0.959	0.962	0.964
	p^*	0.639	0.670	0.667	0.660	0.654	0.648	0.642	0.637	0.633
	n^*, m^*	4.2	8.4	12.6	16.8	20.10	24.12	28.14	32.16	36.18
40	ΔP_c^*	0.147	0.173	0.194	0.209	0.221	0.231	0.239	0.246	0.252
	P_c^*	0.907	0.939	0.951	0.957	0.962	0.966	0.968	0.971	0.972
	p^*	0.760	0.766	0.757	0.748	0.741	0.735	0.729	0.725	0.720
	n^*, m^*	5.3	10.6	15.9	20.12	25.15	30.18	35.21	40.24	45.27
33.3	ΔP_c^*	0.104	0.131	0.150	0.164	0.175	0.183	0.191	0.197	0.202
	P_c^*	0.933	0.954	0.962	0.967	0.970	0.972	0.975	0.976	0.978
	p^*	0.829	0.823	0.812	0.803	0.795	0.789	0.784	0.779	0.776
	n^*, m^*	6.4	12.8	18.12	24.16	30.20	36.24	42.28	48.32	54.36
28.6	ΔP_c^*	0.079	0.104	0.121	0.134	0.144	0.151	0.158	0.163	0.168
	P_c^*	0.951	0.963	0.969	0.973	0.975	0.977	0.979	0.980	0.981
	p^*	0.872	0.859	0.848	0.839	0.832	0.826	0.821	0.817	0.813
	n^*, m^*	7.5	14.10	21.15	28.20	35.25	42.30	49.35	56.40	63.45
25	ΔP_c^*	0.062	0.085	0.101	0.112	0.121	0.128	0.134	0.139	0.143
	P_c^*	0.961	0.970	0.974	0.977	0.979	0.980	0.982	0.983	0.983
	p^*	0.899	0.885	0.873	0.865	0.858	0.852	0.848	0.844	0.840
	n^*, m^*	8.6	16.12	24.18	32.24	40.30	48.36	56.42	64.48	72.54
22.2	ΔP_c^*	0.050	0.072	0.086	0.097	0.105	0.111	0.116	0.121	0.126
	P_c^*	0.969	0.975	0.978	0.981	0.982	0.983	0.984	0.985	0.985
	p^*	0.919	0.903	0.892	0.884	0.877	0.872	0.868	0.864	0.861
	n^*, m^*	9.7	18.14	27.21	36.28	45.35	54.42	63.49	72.56	81.63
20	ΔP_c^*	0.042	0.062	0.075	0.084	0.092	0.098	0.103	0.107	0.110
	P_c^*	0.975	0.979	0.981	0.984	0.985	0.986	0.987	0.987	0.987
	p^*	0.933	0.917	0.906	0.899	0.893	0.888	0.884	0.880	0.877
	n^*, m^*	10.8	20.16	30.24	40.32	50.40	60.48	70.56	80.64	90.72
18.2	ΔP_c^*	0.035	0.054	0.066	0.075	0.082	0.087	0.092	0.096	0.099
	P_c^*	0.978	0.982	0.984	0.985	0.987	0.987	0.988	0.989	0.989
	p^*	0.943	0.928	0.918	0.910	0.905	0.900	0.896	0.893	0.891
	n^*, m^*	11.9	22.18	33.27	44.36	55.45	66.54	77.63	88.72	99.81

In order to ensure usability of the above tables in the course of finding solutions of the synthesis problem, these tables present not only values ΔP_c^*, P_c^*, and p^*, but parameters of the redundancy structures n^*, m^* (which were calculated using the formula (8)) as well.

Minimum values n_m and m_m of the first level tolerances, which are presented in Table 1, are included to the first column, which corresponds to the generalised multipleness K_1. As concerns the second level tolerances, which are presented in Table 2, values n_m and m_m are included to the first column, which corresponds to the generalised multipleness K_2.

Values n_m=2 and m_m=1 of the tolerance 50% in Table 1, as well as values n_m=3 and m_m=1 of the tolerance 66.7% in Table 2 correspond to the multiple redundancy. Therefore, parameters ΔP_c^*, P_c^*, and p^* for these structures are not presented.

In order to illustrate results of optimisation, which are presented in Tables 1 and 2, Figures 5, 6, 7, 8, 9, and 10 present dependences of the desirable optimal values from the generalised multipleness K_i.

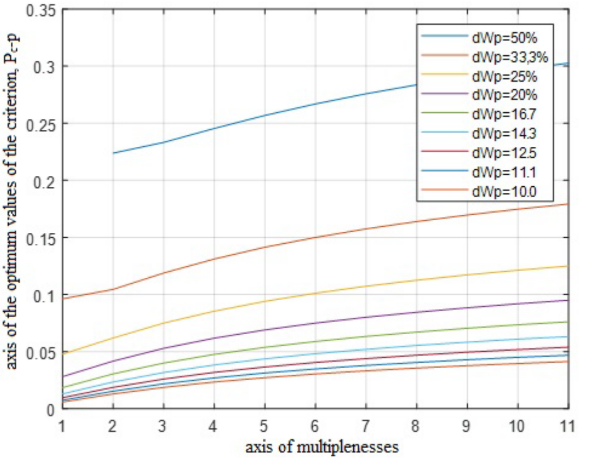

Fig. 5 – Diagrams of dependency $\Delta P_c^*(K_i)$ in respect of the 1 level tolerances

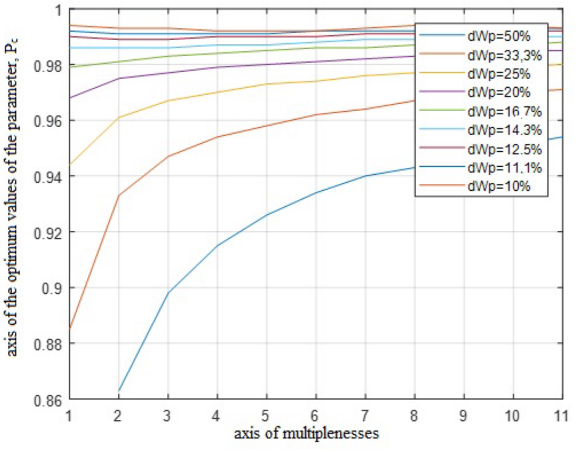

Fig. 6 – Diagrams of dependency $P_c^*(K_i)$ in respect of the 1 level tolerances

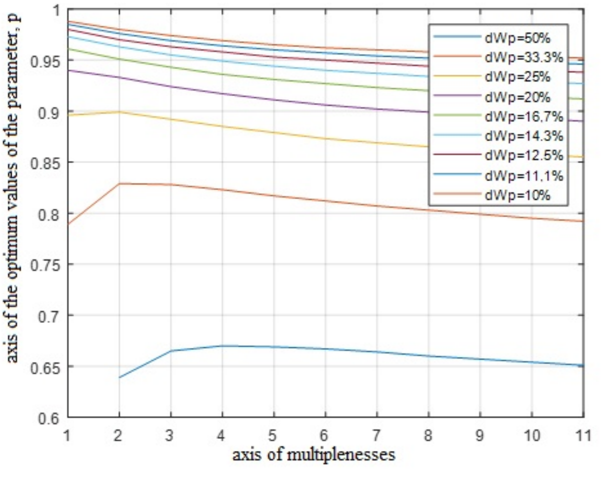

Fig. 7 – Diagrams of dependency $p^*(K_i)$ in respect of the 1 level tolerances

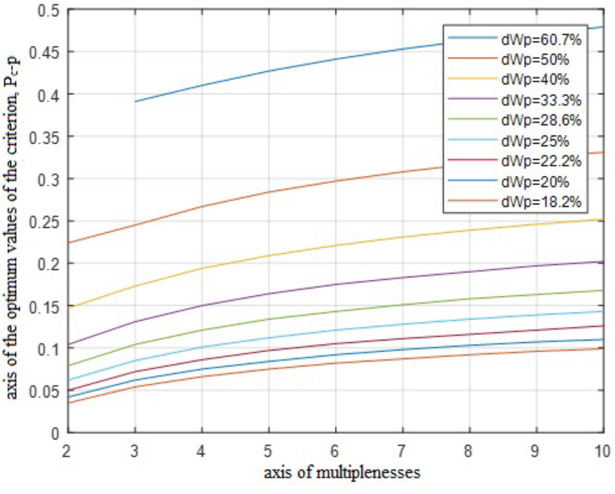

Fig. 8 – Diagrams of dependency $\Delta P_c^*(K_i)$ in respect of the 2 level tolerances

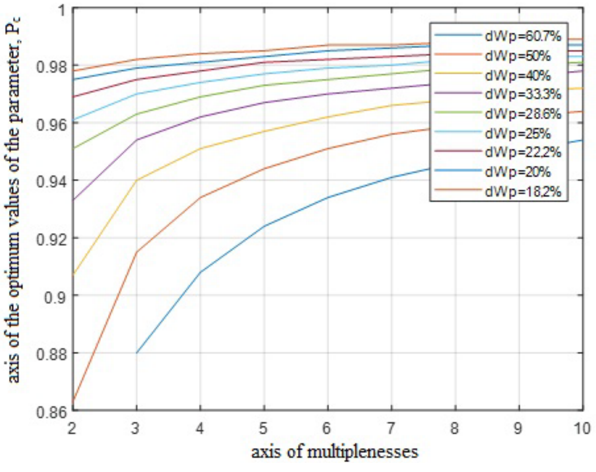

Fig. 9 – Diagrams of dependency $P_c^*(K_i)$ in respect of the 2 level tolerances

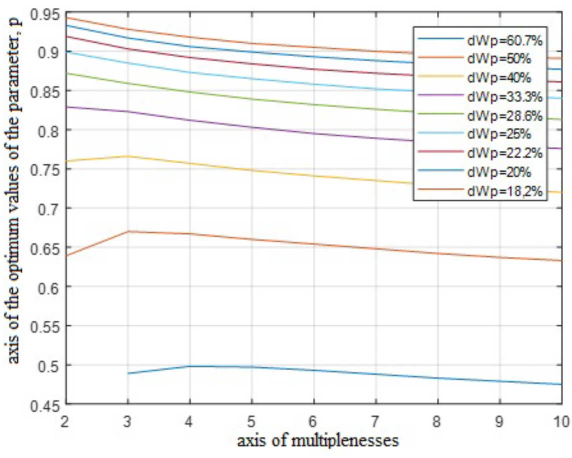

Fig. 10 – Diagrams of dependency $p^*(K_i)$ in respect of the 2 level tolerances

It should be noted that the lines (multitudes) of the increasing individual multiplenesses of the 1 level and 2 level tolerances have different natures. Therefore, these lines/multitudes do not match with each other. This is due to the fact that lines of the 1 level tolerances include many series of the increasing numerical values of the reserved elements, which are governed by the arithmetic progression law with the difference d=1. At the same time, lines of the 2 level tolerances include many series of the increasing numerical values of the reserved elements, which are governed by the arithmetic progression law with the difference d=2. For example, as concerns the 1 level tolerances, we have the following:

dW_p=50%, $K_1=\frac{2}{1}=2\ (r_1=1)$, $K_2=\frac{4}{2}\ (r_2=2)$, $K_3=\frac{6}{3}\ (r_3=3)$,…

dW_p=33.3%, $K_1=\frac{3}{2}\ (r_1=1)$, $K_2=\frac{6}{4}\ (r_2=2)$, $K_3=\frac{9}{6}\ (r_3=3)$,…

dW_p=25%, $K_1=\frac{4}{3}\ (r_1=1)$, $K_2=\frac{8}{6}\ (r_2=2)$, $K_3=\frac{12}{9}\ (r_3=3)$,…

and so on. Similarly to the foregoing, as concerns the 2 level tolerances we have the following:

dW_p=66.7%, $K_2=\frac{3}{1}=3\ (r_2=2)$, $K_3=\frac{6}{2}\ (r_3=4)$, $K_4=\frac{9}{3}\ (r_4=6)$,…

dW_p=50%, $K_2=\frac{4}{2}\ (r_2=2)$, $K_2=\frac{8}{4}\ (r_3=4)$, $K_4=\frac{12}{6}\ (r_4=6)$,…

dW_p=40%, $K_2=\frac{5}{3}\ (r_2=2)$, $K_3=\frac{10}{6}\ (r_3=4)$, $K_4=\frac{15}{9}\ (r_4=6)$,…,

and so on.

This regularity results in the different pattern of change of the desirable characteristics depending on the multiplenesses of the tolerances of various levels because of quantity of the reserved elements within the redundancy structures of the aircraft subsystems has the greatest influence upon the indicators of their failure-free operation.

Let us draw up the summary table with the help of the data, which are presented in Tables 1 and 2. This Table 3 makes it possible to find solution of the synthesis problem in respect of the aircraft subsystems for seven values of the required failure-free operation, which is used in the course of operation of the frame/multitude, which consists of the 1 level and the 2 level tolerances.

Table 3. – Optimal characteristics, which ensure achievement of the required level of failure-free operation of the aircraft subsystems

$P_{тр}$	Optimal characteristics $P_{тр}$	Values of the optimal characteristics $P_{тр}$					
0.93	$P_c^*\geq P_{тр}$	0.934	0.934	0.939	0.933	-	-
	dW_p%	66.7	50	40	33.3	-	-
	K_i^*	K_6	$K_6^1.K_4^2$	K_3	$K_2^{1.2}$	-	-
	n^*,m^*	15.5	12.6	10.6	6.4	-	-
0.94	$P_c^*\geq P_{тр}$	0.941	0.944	0.944	-	-	-
	dW_p	66.7	50	25	-	-	-
	K_i^*	K_7	$K_8^1.K_5^2$	K_1^1	-	-	-
	n^*,m^*	18.6	16.8	4.3	-	-	-
0.95	$P_c^*\geq P_{тр}$	0.950	0.951	0.951	0.954	0.951	-
	dW_p	66.7	50	40	33.3	28.6	-
	K_i^*	K_9	$K_{10}^1.K_6^2$	K_4	$K_4^1.K_3^2$	K_2	-

	n^*, m^*	24.8	20.10	15.9	12.8	7.5	-
0.96	$P_c^* \geq P_{тр}$	0.962	0.962	0.962	0.963	0.961	0.968
	dW_p	50	40	33.3	28.6	25	20
	K_i^*	K_9^2	K_6	$K_6^1.K_4^2$	K_3	$K_2^{1.2}$	K_1^1
	n^*, m^*	32.16	25.15	18.12	14.10	8.6	5.4
0.97	$P_c^* \geq P_{тр}$	0.971	0.972	0.973	0.970	0.975	0.979
	dW_p	40	33.3	28.6	25	20	16.7
	K_i^*	K_9	K_7^2	K_5	$K_4^1.K_3^2$	$K_2^{1.2}$	K_1
	n^*, m^*	40.24	36.24	28.20	16.12	10.8	6.5
0.98	$P_c^* \geq P_{тр}$	0.980	0.980	0.981	0.981	0.982	0.981
	dW_p	28.6	25	22.2	20	18.2	16.7
	K_i^*	K_9	K_7^2	K_5	$K_6^1.K_4^2$	K_3	K_2
	n^*, m^*	56.40	48.36	36.28	30.24	22.18	12.10
0.99	$P_c^* \geq P_{тр}$	0.990	0.990	0.991	-	-	-
	dW_p	14.3	12.5	11.1	-	-	-
	K_i^*	K_{11}	K_6	K_3	-	-	-
	n^*, m^*	77.66	48.42	27.24	-	-	-

The generalised multiplenesses, which are presented in Table 3, can have both lower and upper indices. The upper index 1 determines that this multipleness is connected with the tolerance of the first level, while index 2 determines that this multipleness is connected with the tolerance of the second level. Presence of two values of multiplenesses within a single cell of this Table states that relevant structure of redundancy can be implemented with the help of two different generalised multiplenesses of two similar tolerances of different levels, provided that these tolerances ensure the same individual multipleness of redundancy. For example, the required failure-free operation P_{mp}=0.93 can be implemented with the help of the tolerance $dW_p = 50\%$ of the first level and the generalised multipleness K_6, as well as with the help of the tolerance dW_p=50% of the second level and the generalised multipleness K_4. In this case, individual multiplenesses in both situations are the same and they designate the optimal structure of redundancy with parameters: n^*=12, m^*=6. The optimal failure-free operation of the redundant subsystem, which can be achieved, is as follows: P_c^*=0.934>P_{mp}=0.93.

The Summary Table 3 makes it possible to find the optimal structures of redundancy for the prescribed multitude of the required values of the failure-free operation, as well as values of the realisable tolerances of the 1 and 2 levels. For example, the required value of the failure-free operation of the redundant subsystem, which is equal to 0.95, can be ensured in the situations that are described as follows: realisable tolerance at the level of 66.7% along with the structure n^*=24, m^*=8; realisable tolerance at the level of 50% along with the structure n^*=20. m^*=10; realisable tolerance at the level of 40% along with the structure n^*=15, m^*=9; realisable tolerance at the level of 33.3% along with the structure n^*=12, m^*=8; realisable tolerance at the level of 28.6% along with the structure n^*=7, m^*=5. Similarly, to the foregoing, with the help of Table 3 it is possible to find optimal structures of redundancy for other values of the required failure-free operation of the redundant subsystem of the aircraft.

In order to ensure the pictorial view of the results, which are presented in Table 3, Figure 11 presents dependences of the optimal quantity of the elements (which are included to the optimal structure n^*) from the values of the realisable tolerances for various values of the required failure-free operation of the redundant subsystem of the aircraft.

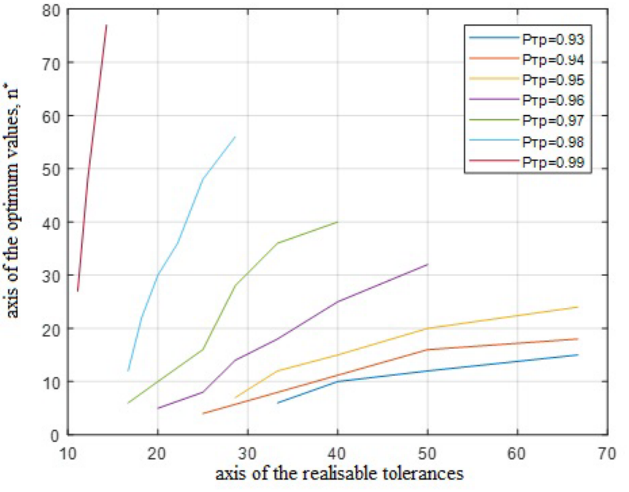

Fig. 11 – Diagrams of dependency n^* from values of the realisable tolerances in percentage points

5. CONCLUSIONS

Analysis of the results, which are presented in the tables and diagrams, makes it possible to establish the following regularities:

1.Optimal values of the complex criterion $\Delta P_c^* = P_c^* - p^*$ at any values of the realisable tolerances increase monotonically along with increase of the redundancy multipleness K_i. At any value of the redundancy multipleness K_i they decrease monotonically (tending to zero) along with decrease of the realisable tolerances.

2.Optimal values of the partial indicator p^*, which determines failure-free operation of elements, decrease monotonically along with increase of the redundancy multipleness K_i at the realisable tolerances of the 1 level, which are lesser than 25%, as well as at the realisable tolerances of the second level, which are lesser than 40%. At the tolerances, which exceed the above-stated values, these optimal values begin to increase (but no more than by 5% at the redundancy multiplenesses, which are lesser than K_4), and then they begin to decrease monotonically along with increase of the redundancy multipleness.

3.It is possible to increase the optimal values of failure-free operation of the redundant subsystems P_c^* by decreasing the realisable tolerances at any prescribed redundancy multipleness K_i or by increasing the redundancy multipleness K_i at the prescribed tolerance.

4.Optimal values of failure-free operation of the redundant subsystems P_c^* increase monotonically along with decrease of the realisable tolerances (tending to unity) at the approach of the realisable tolerance to zero at any multipleness of redundancy.

5.Optimal values of failure-free operation of the redundant subsystems P_c^* increase monotonically along with increase of the redundancy multipleness K_i (tending to unity) at the tending of the redundancy multipleness to infinity for the realisable tolerances of any level, which exceed 12.5%. As concerns the realisable tolerances which do not exceed 12.5%, indicator P_c^* does not change practically along with increase of the redundancy multipleness and it maintain the values, which are designated by the value of the realisable tolerance near the level of 0.99.

6.At the tolerances, which exceed 25%, as well as at high requirements to the failure-free operation of the redundant subsystems (more than 0.98), solution of the synthesis problem can be only found at big redundancy multiplenesses, where optimal quantity of elements of the redundant subsystem n^* is essentially higher than 100. As concerns the tolerances, which do not exceed 25% and at the same high requirements to the failure-free operation of subsystems, the optimal multipleness of redundancy K_i^* drops sharply, thus ensuring process of solving the synthesis problem at optimal quantity n^*, which does not exceed 20-30 elements.

In conclusion, it should be noted that realisable tolerances of two levels only were analysed in this article. Increase of the range of levels of the realisable tolerances would make it possible to present more detailed description of all possible structures of the passive redundancy, as well as all possible solutions of the synthesis problem depending on the prescribed tolerances and requirements in respect of the failure-free operation of the aircraft subsystems.

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