Analysis of isogrid reinforced cylindrical vessels in the case of axially symmetric buckling

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Abstract: *Isogrid structures are a configuration of stiffeners of different sections, which make up a lattice placed on thin plates, with the aim of increasing the buckling factor of the overall structure. Because of the major benefits of isogrid structures when applied to increase the buckling factor and to decrease the mass of the structures the isogrid is placed on, its use has intensified until it has become a complete design technique for building cylindrical vessels under high compressive forces. Unfortunately, the detailed geometry of isogrid structures cannot be easily modelled or computed using FEM software, due to the high number of elements required to reliably mesh such a structure and the large amount of time taken to compute the results. This paper attempts to mitigate this problem by considering an analytical approach of sectioning the cylindrical vessel into component modules. These modules, consisting of a thin plate with isogrid stiffeners attached to it, are approximated as an overall thin plate with modified properties. The analytical algorithm is then implemented in a computed algebra system, which will quickly compute approximate values for the buckling factor and mass of the structure.*

Key Words: *isogrid, numerical applications, stiffener, buckling, total mass, cylindrical vessels*

1. INTRODUCTION

This paper shall present an analytical solution to the problem of axisymmetric static and buckling analysis of an isogrid stiffened cylinder under axial load, followed by an implementation of the solution in a computer algebra system called Maxima.

The analytic solution is obtained by modifying the classical static and buckling equations of an axially symmetric loaded cylinder.

The result will be a generalised method of computing the elastic properties of a thin plate which approximates the type of stiffened plate of the cylinder.

The final formulas will then be implemented in Maxima, in order to make the computation faster and cleaner.

2. STATE OF THE ART

Isogrid stiffened thin structures are common in the aerospace industry. They are primarily used for increasing the buckling factor because of the advantage of also reducing the mass of the structure. The thickness of the cylindrical vessel will be reduced because a significant part of the forces and moments will be taken on by the stiffeners which have a higher bending stiffness than the thin plate itself. This is due to the thin radial geometry of the stiffeners, because, in axial bending, the stress is varying linearly in the radial direction. Therefore, the mass of the entire structure is predominantly distributed along the radial direction of the cylinder. Thanks to this design, higher buckling factors can be obtained.

This type of reinforced structure, having a reduced mass, was adopted by the aerospace industry in the construction of high-speed aeroplanes, rockets and even in the construction of satellites.

In recent times, a large number of analytical or semi-analytical methods were published that calculate the behaviour of these isogrid structures [4], [5], [6]. NASA's analytical method has to be mentioned, from the manual 'NASA Isogrid Design Handbook' [3], where an approximation to the isogrid reinforced thin plate with a thin plate with modified properties is used.

Unlike the approach used in this paper, the NASA approach uses a modified neutral axis, the Young modulus and the plate thickness. The method used in our paper doesn't need to use a modified neutral axis and the properties are computed in a different manner. The paper borrows from the literature the technique of dividing the structure into modules and the hypothesis on which the analytical method is based.

3. PROBLEM DESCRIPTION

The problem uses the method described in the paper: 'BUCKLING ANALYSIS OF GRID STIFFENED COMPOSITE STRUCTURES' by Samuel Kidane [2], of breaking down the structure into identical modules.

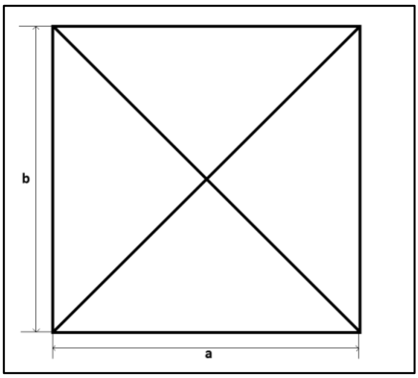

Fig. 1. Stiffened module

Fig. 2. FEM model for module

These elementary modules are a reduction of the design of the stiffeners attached to the vessel. This technique greatly simplifies the calculations. Fig. 1 represents an example module with two stiffeners on the diagonals.

The problem calculates the stress on the structure and the buckling factor. For this we will need the stiffness matrix of the module.

4. CALCULATING THE STIFFNESS MATRIX

We start by defining the expressions for the strains and forces on the thin plate. This is a modified reiteration of the formulas for calculation the buckling factor for medium sized cylinders for the book: 'Buckling of Bars, Plates and Shells' de Robert Millard Jones [1].

$$
\begin{array}{lll}
\overline{\epsilon_x} = u_x + \dfrac{1}{2} w_x^2 & k_x = -w_{xx} & \epsilon_x = \overline{\epsilon_x} + z k_x \\
\overline{\epsilon_y} = v_y + \dfrac{w}{r} + \dfrac{1}{2} w_y^2 & k_y = -w_{yy} & \epsilon_y = \overline{\epsilon_y} + z k_y \\
\overline{\gamma_{xy}} = v_x + u_y + w_x w_y & k_{xy} = -2 w_{xy} & \gamma_{xy} = \overline{\gamma_{xy}} + z k_{xy}
\end{array}
\tag{1}
$$

The strains marked with an upper bar are the strains of a flat plate. These have been written in nonlinear form because they are necessary for the following calculations. The next three expressions are for the curvature of the plate, which represent the deviation for the flat plate deformation.

The last three expressions are the final forms of the strains. Since we are studying thin curved plates, the strains vary linearly along the thickness of the plate, represented in Fig. 3 by the z axis.

The x axis is along the axial direction of the cylinder, the y axis in the circumferential direction and the z axis in the radial direction.

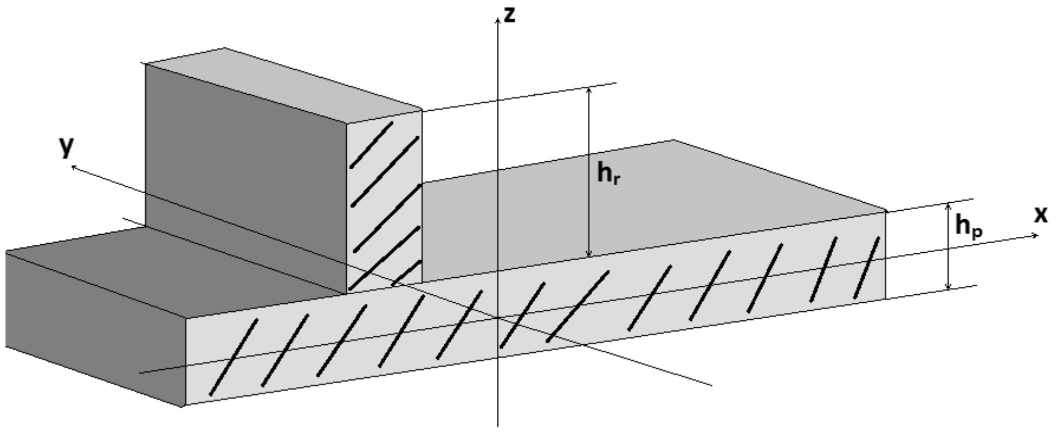

Fig. 3. Plate and stiffener

Since we are interested only in symmetric axial deformations, all the y derivatives will cancel out and the circumferential elongation v will be zero. Therefore, expressions (1) simplify to:

$$
\begin{array}{lll}
\overline{\epsilon_x} = u_x + \dfrac{1}{2} w_x^2 & k_x = -w_{xx} & \epsilon_x = u_x + \dfrac{1}{2} w_x^2 - z w_{xx} \\
\overline{\epsilon_y} = \dfrac{w}{r} & k_y = 0 & \epsilon_y = \dfrac{w}{r} \\
\overline{\gamma_{xy}} = 0 & k_{xy} = 0 & \gamma_{xy} = 0
\end{array}
\tag{2}
$$

The formulas for the distributed forces and moments are:

$$
\begin{aligned}
N_x &= \int_h \sigma_x \, dz = A_{11}\overline{\epsilon_x} + A_{12}\overline{\epsilon_y} & \qquad M_x &= \int_h \sigma_x \, z \, dz = A_{44} k_x \\
N_y &= \int_h \sigma_y \, dz = A_{21}\overline{\epsilon_x} + A_{22}\overline{\epsilon_y} & \qquad M_y &= \int_h \sigma_y \, z \, dz = 0 \\
N_{xy} &= \int_h \tau_{xy} \, dz = A_{33}\overline{\gamma_{xy}} & \qquad M_{xy} &= \int_h \tau_{xy} \, z \, dz = 0
\end{aligned}
\tag{3}
$$

The formulas for distributed forces and moments will be linear functions of strains and curvatures:

$$
\begin{pmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{pmatrix}
\begin{pmatrix}
A_{11} & A_{12} & 0 & 0 & 0 & 0 \\
A_{21} & A_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & A_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & A_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
=
\begin{pmatrix} \overline{\epsilon_x} \\ \overline{\epsilon_y} \\ \overline{\gamma_{xy}} \\ k_x \\ k_y \\ k_{xy} \end{pmatrix}
\tag{4}
$$

The matrix A_{ij} is the stiffness matrix of the module. The majority of the terms are negligible because they represent the coupling values of the stiffeners, which are insignificant.

The role of the stiffeners in the analytical method is to modify the properties of the thin plate. All other effects are ignored.

The computation of the distributed forces and moments integral will be divided into two steps: one for the stiffeners and one for the thin plate.

$$
\begin{aligned}
N_x &= \int_{-\frac{h_p}{2}}^{\frac{h_p}{2}} \sigma_x \, dz + \int_{\frac{h_p}{2}}^{\frac{h_p}{2}+h_r} \sigma_x \, dz & \qquad M_x &= \int_{-\frac{h_p}{2}}^{\frac{h_p}{2}} \sigma_x z \, dz + \int_{\frac{h_p}{2}}^{\frac{h_p}{2}+h_r} \sigma_x z \, dz \\
N_y &= \int_{-\frac{h_p}{2}}^{\frac{h_p}{2}} \sigma_y \, dz + \int_{\frac{h_p}{2}}^{\frac{h_p}{2}+h_r} \sigma_y \, dz & \qquad M_y &= \int_{-\frac{h_p}{2}}^{\frac{h_p}{2}} \sigma_y z \, dz + \int_{\frac{h_p}{2}}^{\frac{h_p}{2}+h_r} \sigma_y z \, dz \\
N_{xy} &= \int_{-\frac{h_p}{2}}^{\frac{h_p}{2}} \tau_{xy} \, dz + \int_{\frac{h_p}{2}}^{\frac{h_p}{2}+h_r} \tau_{xy} \, dz & \qquad M_{xy} &= \int_{-\frac{h_p}{2}}^{\frac{h_p}{2}} \tau_{xy} z \, dz + \int_{\frac{h_p}{2}}^{\frac{h_p}{2}+h_r} \tau_{xy} z \, dz
\end{aligned}
\tag{5}
$$

In Fig. 3 the heights of the plate and stiffeners were represented as h_p and h_r. The expresions for the stress components are:

$$
\begin{aligned}
\sigma_x &= \frac{E}{1-\nu^2}\left(\epsilon_x + \nu\epsilon_y\right) \\
\sigma_y &= \frac{E}{1-\nu^2}\left(\epsilon_y + \nu\epsilon_x\right) \\
\tau_{xy} &= \frac{E}{2(1+\nu)}\gamma_{xy}
\end{aligned}
\tag{6}
$$

In Fig. 4 is shown a plate with a stiffener at an angle along with the coordinate system of the plate, XoY.

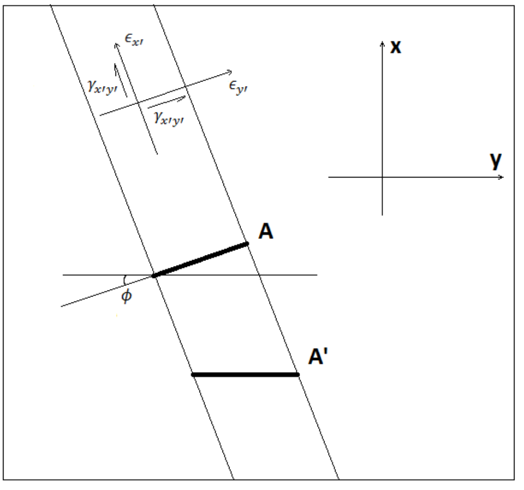

Fig. 4. Plate and rotated stiffener

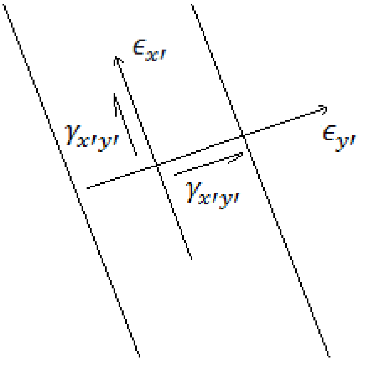

Fig. 5. Rotated strains

The effect of the stiffeners is to increase the bending resistance of the underlying plate. To find the expressions in the global coordinate system, the strains along the stiffener have to be rotated by the angle ϕ, illustrated in Fig. 5. We will write this relationship as a matrix equation, as follows, where $c = \cos(\phi)$ iar $s = \sin(\phi)$:

$$\begin{pmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} c^2 & s^2 & sc \\ s^2 & c^2 & -sc \\ -2sc & 2sc & c^2 - s^2 \end{pmatrix} \begin{pmatrix} \epsilon_{x'} \\ \epsilon_{y'} \\ \gamma_{x'y'} \end{pmatrix} \tag{7}$$

The deformation is along the stiffener; therefore, the only nonzero strain is $\epsilon_{x\prime}$:

$$\epsilon_{x\prime} = c^2\,\epsilon_x + s^2\,\epsilon_y - sc\,\gamma_{xy} \tag{8}$$

Idem, we have for stress in the module system of coordinates:

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix} = \begin{pmatrix} c^2 & s^2 & sc \\ s^2 & c^2 & -sc \\ -2sc & 2sc & c^2 - s^2 \end{pmatrix} \begin{pmatrix} \sigma_{x\prime} \\ 0 \\ 0 \end{pmatrix} \tag{9}$$

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix} = \begin{pmatrix} c^2\,\sigma_{x\prime} \\ s^2\,\sigma_{x\prime} \\ -2sc\,\sigma_{x\prime} \end{pmatrix} \tag{10}$$

As the only nonzero stress expression is $\sigma_{x\prime} = E\epsilon_{x\prime} = E(\mathrm{c}^2\,\epsilon_x + s^2\,\epsilon_y - sc\,\gamma_{xy})$, we have:

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix} = E \begin{pmatrix} c^2\,(c^2\,\epsilon_x + s^2\,\epsilon_y - sc\,\gamma_{xy}) \\ s^2\,(c^2\,\epsilon_x + s^2\,\epsilon_y - sc\,\gamma_{xy}) \\ -2sc\,(c^2\,\epsilon_x + s^2\,\epsilon_y - sc\,\gamma_{xy}) \end{pmatrix} \tag{11}$$

To find the stiffener contribution to the distributed force N_x, the balance of forces shall be written as follows:

$$N_x = \frac{F_x}{a} = \frac{1}{a}\int \sigma_x\,dA$$
$$N_x = \frac{1}{a}\int_{\frac{h_p}{2}}^{\frac{h_p}{2}+h_r} \sigma_x\,\frac{t_r}{cos(\phi)}\,dz \tag{12}$$

The force F_x represents the total force on the area of the stiffener. N_x is the distributed force on the side a of the module. The area element dA is equal to the thickness of the stiffener t_r divided by $\cos(\phi)$ and multiplied by dz. We divided by $\cos(\phi)$ because we integrate over the projected transversal area of the stiffener on the x axis. Similarly, the other expressions (13) for all the distributed forces and moments are found:

$$N_x = \frac{E}{a}\frac{t_r}{cos(\phi)}\int_{\frac{h_p}{2}}^{\frac{h_p}{2}+h_r} c^2\left(c^2\,\epsilon_x + s^2\,\epsilon_y - sc\,\gamma_{xy}\right)dz$$
$$N_y = \frac{E}{b}\frac{t_r}{sin(\phi)}\int_{\frac{h_p}{2}}^{\frac{h_p}{2}+h_r} s^2\left(c^2\,\epsilon_x + s^2\,\epsilon_y - sc\,\gamma_{xy}\right)dz$$
$$N_{xy} = \frac{E}{a}\frac{t_r}{cos(\phi)}\int_{\frac{h_p}{2}}^{\frac{h_p}{2}+h_r} -2sc\left(c^2\,\epsilon_x + s^2\,\epsilon_y - sc\,\gamma_{xy}\right)dz$$
$$M_x = \frac{E}{a}\frac{t_r}{cos(\phi)}\int_{\frac{h_p}{2}}^{\frac{h_p}{2}+h_r} c^2\left(c^2\,\epsilon_x + s^2\,\epsilon_y - sc\,\gamma_{xy}\right)zdz \qquad (13)$$
$$M_y = \frac{E}{b}\frac{t_r}{sin(\phi)}\int_{\frac{h_p}{2}}^{\frac{h_p}{2}+h_r} s^2\left(c^2\,\epsilon_x + s^2\,\epsilon_y - sc\,\gamma_{xy}\right)zdz$$
$$M_{xy} = \frac{E}{a}\frac{t_r}{cos(\phi)}\int_{\frac{h_p}{2}}^{\frac{h_p}{2}+h_r} -2sc\left(c^2\,\epsilon_x + s^2\,\epsilon_y - sc\,\gamma_{xy}\right)zdz$$

The calculations are the same for the thin plate. The final stiffness matrix is obtained through summing up all the stiffness matrix of the stiffeners and the plate. Having these final relations, we can extract the terms for the final stiffness matrix as follows:

- The stiffness matrix terms are calculated for the curved plate.
- We choose a stiffener and calculate the formulas from above.
- We extract the stiffness matrix for the component.
- Repeat last two steps for all stiffeners.
- Final stiffness matrix is obtained by summing all component stiffness matrices.

5. COMPUTING THE STRESS IN THE AXIAL SIMETRIC CASE

The calculation is simple. Having the stiffness matrix, we can write the relationship between the distributed forces and moments and strains and curvatures (14):

$$\begin{pmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & 0 & 0 & 0 & 0 \\ A_{21} & A_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \overline{\epsilon_x} \\ \overline{\epsilon_y} \\ \overline{\gamma_{xy}} \\ k_x \\ k_y \\ k_{xy} \end{pmatrix} \qquad (14)$$

The unknowns are strains and curvatures. The knowns are F_x, the axial force and R, the cylinder radius. As N_x is F_x divided by the circumference of the cylinder, and the rest of the forces and moments are zero, we have:

$$\begin{pmatrix} \overline{\epsilon_x} \\ \overline{\epsilon_y} \\ \overline{\gamma_{xy}} \\ k_x \\ k_y \\ k_{xy} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & 0 & 0 & 0 & 0 \\ A_{21} & A_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} \frac{F_x}{2\pi R} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{15}$$

If the stiffness matrix is separated into its component matrices, calculated in chapter 4, the stresses on each component of the module can be computed. The strains, which were previously computed (15), are the same for all components. For this, the stress can be computed using the usual formulas.

6. COMPUTING THE BUCKLING FACTOR

We shall first write the equilibrium equations in general form:

$$\begin{aligned} &\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \\ &\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \\ &\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + N_x w_{xx} + N_y w_{yy} + 2N_{xy} w_{xy} - \frac{N_y}{R} = p \end{aligned} \tag{16}$$

The pressure p reprezents the pressure distributed along the circumference of the cylinder.

Cancelling all the derivatives in y and the elongation v from (16), we have:

$$\begin{aligned} N_x &= A_{11}\left(u_x + \frac{1}{2}w_x^2\right) + A_{12}\left(\frac{w}{R}\right) \\ N_y &= A_{21}\left(u_x + \frac{1}{2}w_x^2\right) + A_{22}\left(\frac{w}{R}\right) \\ N_{xy} &= 0 \\ M_x &= A_{44}(-w_{xx}) \\ M_y &= 0 \\ M_{xy} &= 0 \end{aligned} \tag{17}$$

And the system of differential equations (16) becomes:

$$\begin{aligned} &\frac{\partial N_x}{\partial x} = 0 \\ &\frac{\partial^2 M_x}{\partial x^2} + N_x w_{xx} - \frac{N_y}{R} = p \end{aligned} \tag{18}$$

From the first equation in (18), we have that $N_x = N$ constant. If we decompose the second equation we have:

$$A_{44}\frac{d^4 w}{dx^4} - N\frac{d^2 w}{dx^2} + \frac{N_y}{R} + p = 0 \tag{19}$$

We equate N_y as a function of w/R from the expressions for N_x şi N_y in (17), so that a fourth order differential equation can be obtained.

$$N_y = \frac{1}{A_{11}}\left(A_{21}N + (A_{11}A_{22} - A_{12}A_{21})\frac{w}{R}\right) \tag{20}$$

Therefore, the final differential equation is:

$$A_{44}\frac{d^4w}{dx^4} - N\frac{d^2w}{dx^2} + \frac{A_{11}A_{22} - A_{12}A_{21}}{A_{11}}\frac{w}{R^2} + \frac{A_{21}}{A_{11}}N + p = 0 \tag{21}$$

Due to the fact that solving this equation (21) is extremely long, we will only present a brief summary:

- The characteristic polynomial that has four solutions is written.
- The generalised solution is written with four arbitrary constants.
- The following boundary conditions are applied: at each end $w = 0$ and $w_{xx} = 0$.
- A 4x4 linear system is obtained.
- The determinant is set equal to zero and a relation between all four constants is obtained.
- That relation contains the variable N:

$$N = 2\sqrt{\left(\frac{A_{11}A_{22} - A_{12}A_{21}}{A_{11}}\right)\frac{A_{44}}{R^2}} \tag{22}$$

It is known that N is the distributed force on the circumference of the cylinder:

$$N = \frac{F_x}{2\pi R} \tag{23}$$

Through a simple substitution in (22), we arrive at the final formula for the first buckling force of a cylinder under axial symmetrical load:

$$P_{cr} = 4\pi\sqrt{\left(\frac{A_{11}A_{22} - A_{12}A_{21}}{A_{11}}\right)A_{44}} \tag{24}$$

7. NUMERICAL RESULTS

The code used to implement this method is:

```
numer:true$
ratprint:false$

l1:[]$
l2:[]$
l3:[]$
for i:1 thru 5 step .2 do(
    L:2.534,                        /*cylinder height*/
    R:.575,                         /*cylinder radius*/
    n:.3,                           /*cylinder material Poisson ratio*/
    E:7.1*10^10,                    /*cylinder material Young's modulus*/
    tsh:.0015,                      /*cylinder shell thickness*/
    tr:.001*i,                      /*cylinder stiffener thickness*/
    hr:.002,                        /*cylinder stiffener height*/
    A1:E*tsh/(1-n^2),
```

```
    A2:E*tsh^3/12/(1-n^2),
    a:2*%pi*R/36,               /*module base*/
    b:a*2.5,                    /*module height*/

    F:320000,                   /*axial force*/
    Nx:F/(2*%pi*R),             /*distributed axial force*/
    Nf:0,
    Nxf:0,
    Mx:0,
    Mf:0,
    Mxf:0,

    Mp:matrix(                  /*plate stiffness matrix*/
        [A1,n*A1,0,0,0,0],
        [n*A1,A1,0,0,0,0],
        [0,0,A1*(1-n)/2,0,0,0],
        [0,0,0,A2,A2*n,0],
        [0,0,0,n*A2,A2,0],
        [0,0,0,0,0,A2*(1-n)/2]
    ),
    ex:ex0+z*kx,
    ey:ey0+z*ky,
    gxy:gxy0+z*kxy,
    s(phi):=E*matrix(           /*stiffner stiffness matrix*/
        [hr/cos(phi)*cos(phi)^2*(sin(phi)^2*ex+cos(phi)^2*ey+sin(phi)*cos(phi)*gxy)/a],
        [hr/sin(phi)*sin(phi)^2*(sin(phi)^2*ex+cos(phi)^2*ey+sin(phi)*cos(phi)*gxy)/b],
        [hr/cos(phi)*(-2)*sin(phi)*cos(phi)*(sin(phi)^2*ex+cos(phi)^2*ey+sin(phi)*cos(phi)*gxy)/a]
    ),
    a1:integrate(s(phi),z,tsh/2,tsh/2+tr),
    a2:integrate(s(phi)*z,z,tsh/2,tsh/2+tr),
    l:[a1[1],a1[2],a1[3],a2[1],a2[2],a2[3]],
    l:flatten(l),
    exp:coefmatrix(l,[ex0,ey0,gxy0,kx,ky,kxy]),
    al:b/a/3,
    Mt:Mp+ev(exp,phi=al)+ev(exp,phi=-al)+ev(exp,phi=atan(b/a))+ev(exp,phi=-atan(b/a))+2*ev(exp,phi=%pi/2)+2/3*a*ev(exp,phi=0),

    vt:[Nx,Nf,Nxf,Mx,Mf,Mxf],       /*force vector*/
    e:invert(Mt).transpose(vt),     /*strain vector*/

    vr:Mr.e,                        /*stiffner forces vector*/

    vp:Mp.e,                        /*plate forces vector*/
    sigma_x:vp[1]/t*1e-6,           /*sigma_x*/

    Pcr:(2*(Mt[4,4]*(Mt[1,1]*Mt[2,2]-Mt[1,2]*Mt[2,1])/Mt[1,1])^.5)*2*%pi,    /*buckling force*/
    mass:L/b*(hr*tr*(3^.5/2*(1+(b/a)^2)^.5*a+b+2*(a^2+b^2)^.5)+a*b*tsh)*36*2800.0,
    push(Pcr/F,l1),
    push(mass,l3),
    push(i,l2),
    display(i)
```

```
)$
wxplot2d([discrete,l2,l1], grid2d,
      [yx_ratio, 1], [axes, solid], [ylabel,"K_b"],[xlabel,"Stiffner height [mm]"], [title, "Buckling factor"]);
wxplot2d([discrete,l2,l3], grid2d,
      [yx_ratio, 1], [axes, solid], [ylabel,"mass[kg]"],[xlabel,"Stiffner height[mm]"], [title, "Mass[kg]"]);
```

The model used for the validation of the numerical results is:

- Radius = 0.5 m
- Material = Aluminium
- E = 70 GPa
- ν = 0.33
- F = 100000 N

Fig. 6 presents the cylinder with module height to base ratio of 1, and Fig. 7 shows the cylinder with module height to base ratio of 2. The entire cylinder is made of the same material. The numerical results are given in Table 1. Numerical Results.

Table 1. Numerical Results

Figure number	Height-base ratio	Stiffener height [mm]	Analytical buckling factor	Numerical buckling factor	Relative error [%]
Fig. 8	1	0	24.2	23.9	1.25
Fig. 10	1	1	24.5	24.4	0.40
Fig. 12	1	2	25.4	25.4	0.00
Fig. 14	1	3	27.0	27.2	0.73
Fig. 16	1	4	29.2	30.1	2.99
Fig. 18	1	5	32.2	34.1	5.57
Fig. 20	1	6	35.9	38.6	6.99
Fig. 22	1	7	40.1	44.0	8.86
Fig. 24	1	8	44.8	51.0	12.15
Fig. 9	2	0	24.2	23.9	1.25
Fig. 11	2	1	25.5	24.2	5.37
Fig. 13	2	2	27.6	24.8	11.29
Fig. 15	2	3	30.4	26.0	16.92
Fig. 17	2	4	34.0	27.8	22.30
Fig. 19	2	5	38.3	30.6	25.16
Fig. 21	2	6	43.0	34.2	25.73
Fig. 23	2	7	48.3	38.5	25.45
Fig. 25	2	8	54.1	43.5	24.36

The following figures show the modes of the buckling process.

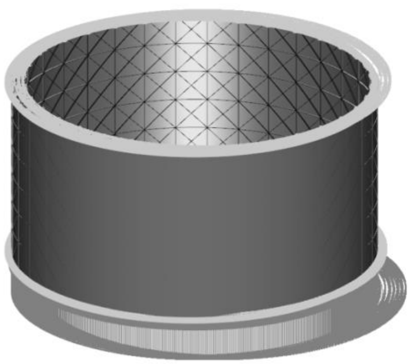

Fig. 6. Isogrid cylinder b/a=1

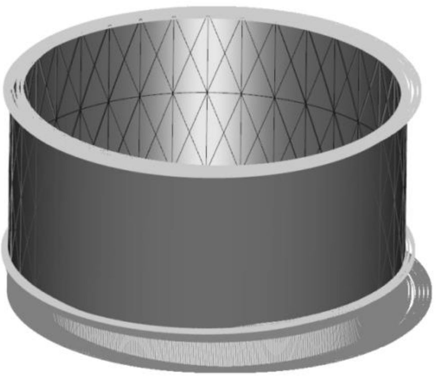

Fig. 7. Isogrid cylinder b/a=2

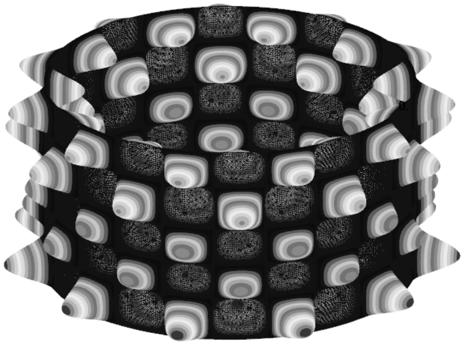

Fig. 8. b/a=1; hp=0

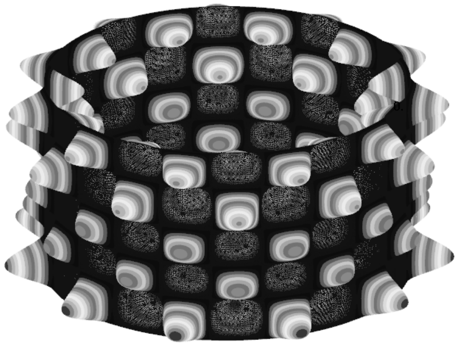

Fig. 9. b/a=2; hp=0

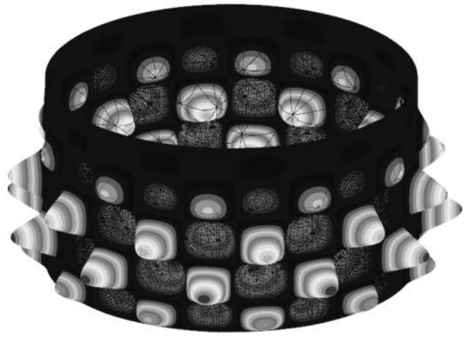

Fig. 10. b/a=1; hp=1

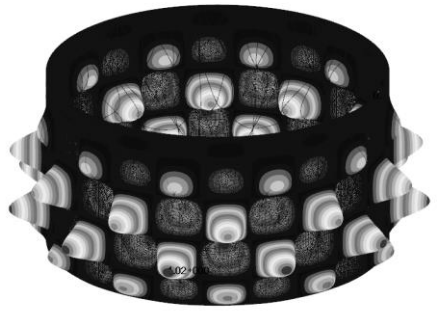

Fig. 11. b/a=2; hp=1

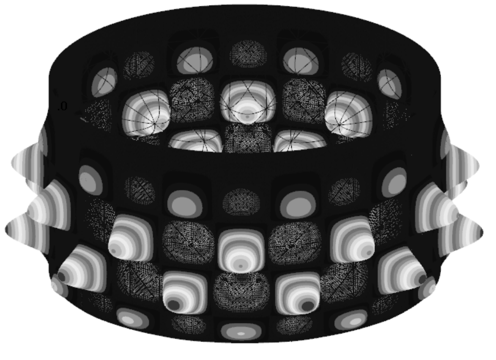

Fig. 12. b/a=1; hp=2

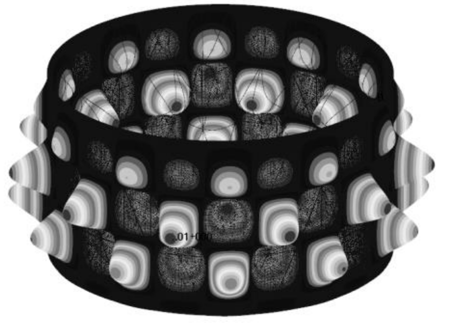

Fig. 13. b/a=2; hp=2

Fig. 14. b/a=1; hp=3

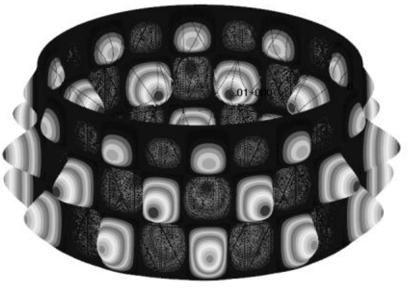

Fig. 15. b/a=2; hp=3

Fig. 16. b/a=1; hp=4

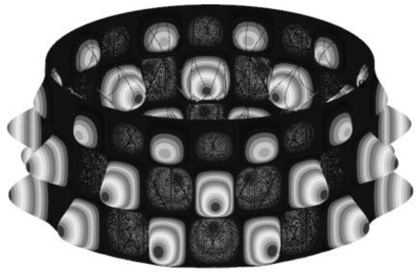

Fig. 17. b/a=2; hp=4

Fig. 18. b/a=1; hp=5

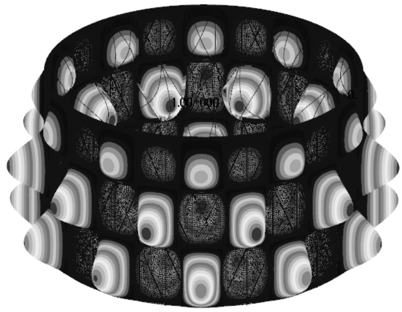

Fig. 19. b/a=2; hp=5

Fig. 20. b/a=1; hp=6

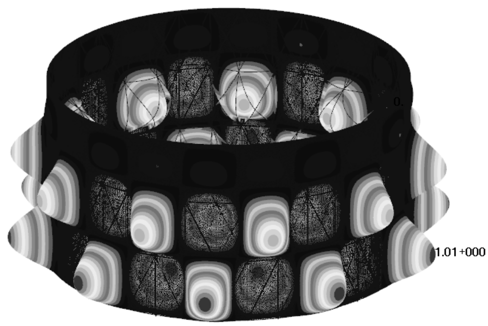

Fig. 21. b/a=2; hp=6

Fig. 22. b/a=1; hp=7

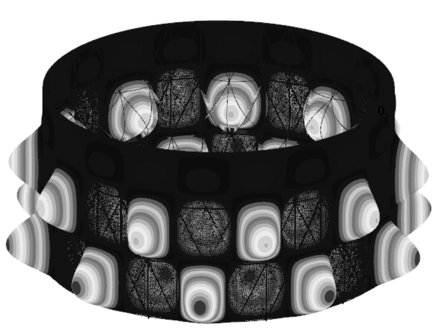

Fig. 23. b/a=2; hp=7

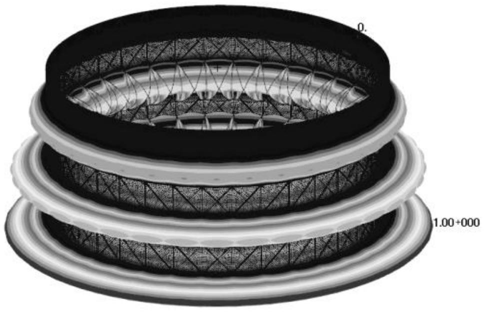

Fig. 24. b/a=1; hp=8

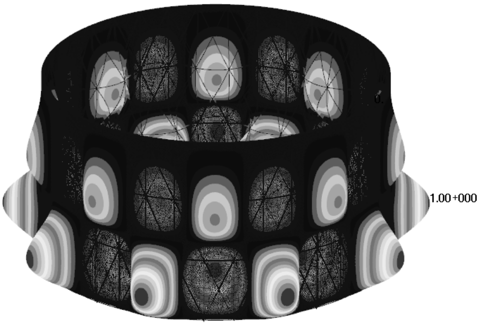

Fig. 25. b/a=2; hp=8

As can be seen the relative error is quite small for stiffener heights of 8 mm to a shell thickness of 3, up to 25%.

8. CONCLUSIONS

The numerical error of this analytical method is quite small, considering the fact that it is intended just for a preliminary feasibility study of such structures. It should be mentioned that the real buckling factor can be far lower in reality than even the FEM buckling factor we used to validate our results. This fact just reinforces the fact that this calculation should not take a considerable amount of time, just to obtain results that are basically not trustworthy.

The original contributions of the author are:

- A methodology for computing the buckling factor for isogrid reinforced cylinders under symmetric axial load.
- A faster way of realising a feasibility study of using isogrid structures.
- A unified way of computing buckling factor for different types of stiffener configurations.

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