Control of uncertain systems by feedback linearization with neural networks augmentation.
Part I. Controller design

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Abstract

The paper highlights the main steps of adaptive output feedback control for non-affine uncertain systems – both in parameters and dynamics – having a known relative degree. Given a reference model, the objective is to design a controller that forces the measured system output to track the reference model output with bounded errors. A single hidden layer neural network is used to counteract feedback linearization error. A dynamic observer of tracking error is added. The treatment of control saturation is also sketched. The mathematical model for the longitudinal dynamics of an experimental helicopter is used as framework.

1. Introduction

One of the most important problems in control theory is that of controlling an uncertain system in order to have its output tracking a given reference signal. A way in treating such a problem is the adaptive control. Research in adaptive output feedback control of uncertain nonlinear systems is of particular importance, taking into account the emerging applications in various fields such as modern fighter and civilian aircrafts, unmanned aerial vehicles (UAV), flexible structures, robotics, flow physics, combustion processes and so on. Modelling for all these applications suffers of uncertainty, both in parameters and dynamics.

To highlight the framework of the paper, let the dynamics of an *observable* nonlinear single-input-single-output (SISO) non affine system be given by the equations

$$\dot{x} = f(x,u), \quad y = g(x) \tag{1}$$

where $x \in D \subset R^n$ is the state vector, $u, y \in R$ (for sake of simplicity) are input signal (control), respectively, output signal (measurement), and f, g *are unknown functions*, sufficiently smooth; moreover, n *need not be necessary prescribed*! For this real or virtual system, for example an airplane or its mathematical model, various problems are stated in control theory. Let consider such a problem: design (more specific, synthesize) a control law $u(y)$, which uses the available measurement y, so that the output y follow asymptotically a prescribed reference signal $y_c(t) \in C^r$, $r < n$. This is the problem of trajectory tracking for an airplane or rocket. In addition, the control law u is subjected to saturating restrictions, $|u| < u_M$.

A premise of solving the problem is the ability of the artificial intelligence techniques – of *neural networks (NNs)*, for example – in compensating the lack of system knowledge, in other works, in compensating the uncertainties in its modeling. For the system (1), the hypothesis of feedback linearization conditions [1] with relative degree r is introduced, which means that by successively differentiation of $g(x)$ by virtue of the system (so called Lie derivatives), the control appears in the order r derivative

$$y^{(r)} = g_r(x, u) \tag{2}$$

Here $g_r := d^r g / dt^r$, such that $\partial g_i / \partial u = 0$ for $0 \le i < r$ and $\partial g_r / \partial u \ne 0$. This hypothesis is also not restrictive, because in any system the output depends finally on input. Feedback linearization is carrying out by a transformation of variable

$$v = \hat{g}_r(y, u), \; u = \hat{g}_r^{-1}(y, v) \tag{3}$$

where v is the *pseudo control* and $\hat{g}_r(y, u)$ represents any available approximation of $g_r(x, u)$ that is invertible with respect to its second argument. Thus, the uncertain system (1) will be represented by a linear dynamics of r integrators

$$\begin{gathered} y^{(r)} = v + \Delta \\ \Delta := g_r\left(x, \hat{g}_r^{-1}(y,v)\right) - \hat{g}_r\left(y, \hat{g}_r^{-1}(y,v)\right) \end{gathered} \tag{4}$$

where Δ is the inversion error, which acts as a disturbance signal on system. Performing $(n-1)$

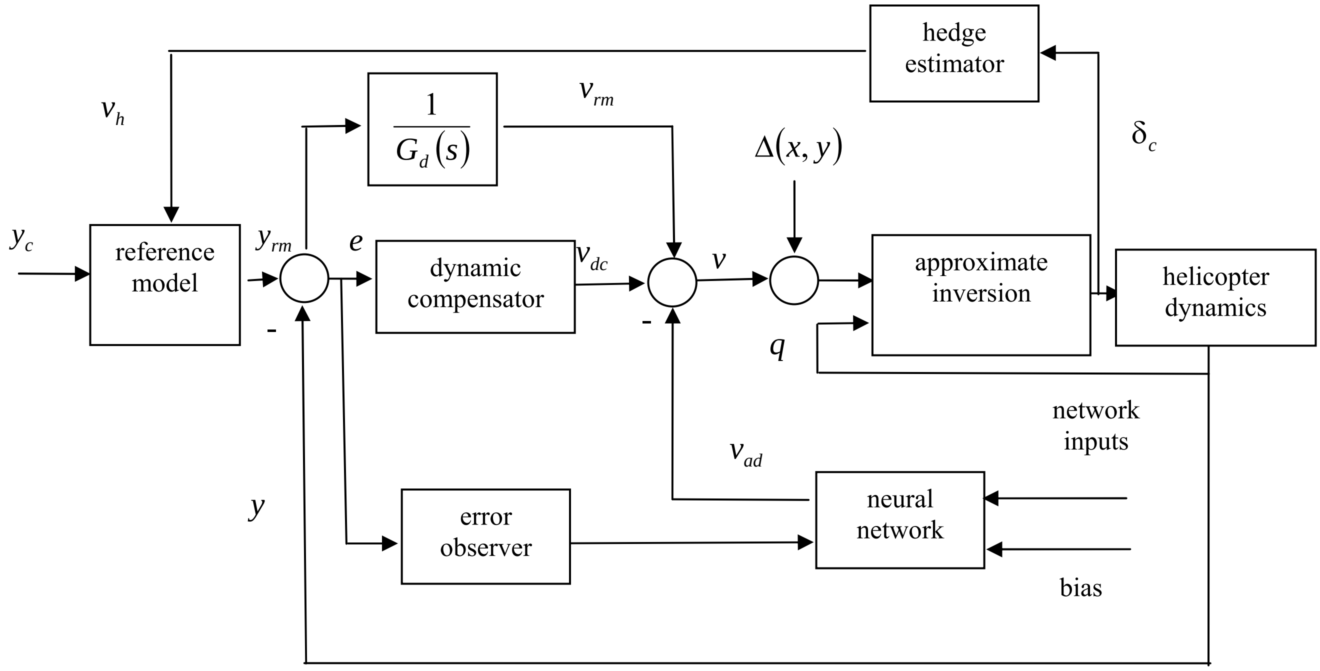

Fig. 1. Implementation block diagram with pseudo control hedging and reference model

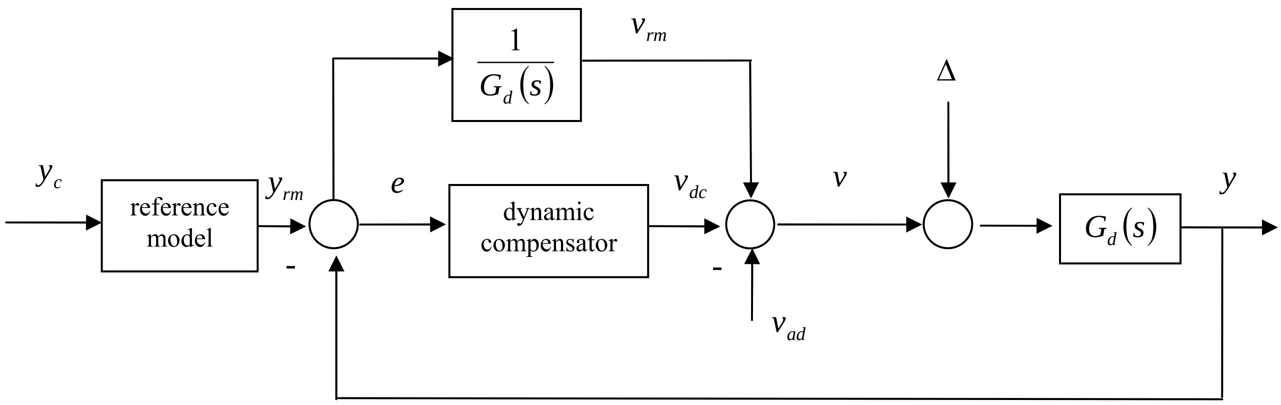

Fig. 2. Control system architecture

times Lie derivatives of the function g yields

$$y = g(x),\ \dot{y} = L_f g(x), \ldots, y^{(n-1)} = L_f^{n-1} g(x) \quad (5)$$

Observability hypothesis in (1) ensures that the right side of system (5) has a full rank and, taking into account (3) and the condition of relative degree r, the following implicit dependence can be stated

$$x = F\left(y, \dot{y}, \ldots, y^{(n-1)}, v, \dot{v}, \ldots, v^{(n-r-1)}\right) \quad (6)$$

A similar expression is obtained for the error

$$\Delta(x, y, v) = G\left(y, \dot{y}, \ldots, y^{(n-1)}, v, \dot{v}, \ldots, v^{(n-r-1)}\right) \quad (7)$$

A theorem of Kolmogorov-Sprecher type (see [2]), ensures the existence of a NN so that Δ may be approximated with good accuracy when the network is operating only on the input-output data (with d a sample time)

$$\begin{gathered} y(t), y(t-d), \ldots, y\left(t-(n_1-1)d\right), v(t), \\ v(t\text{-}d), \ldots, v\left(t-(n_1-r-1)d\right),\ N_1 \geq n,\ d > 0 \end{gathered} \quad (8)$$

2. Controller design

To demonstrate that the developed approach in the paper is adaptive to both parametric uncertainty and unmodeled dynamics (including time delay), we illustrate a step by step controller design using a simplified model for the pitch channel six order dynamics of an R-50 experimental helicopter [3] with a time delay $T_d = 0.03\,\text{sec}$

$$\dot{x}(t) = Ax(t) + B\delta_c(t - T_D) \tag{9}$$

$$x := \begin{bmatrix} u \\ q \\ \theta \\ \beta \\ w \\ \delta \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1/\tau \end{bmatrix}$$

$$A := \begin{bmatrix} X_u & X_q & X_\theta & X_\beta & X_w & X_\delta \\ M_u & M_q & 0 & M_\beta & M_w & M_\delta \\ 0 & 1 & 0 & 0 & 0 & 0 \\ B_u & -1 & 0 & B_\beta & 0 & B_\delta \\ Z_u & Z_q & Z_\theta & Z_\beta & Z_w & Z_\delta \\ 0 & 0 & 0 & 0 & 0 & -1/\tau \end{bmatrix} \tag{9′}$$

$X_u = -0.0553$ $X_q = 1.413$ $X_\theta = -32.1731$
$X_\beta = -19.9033$ $X_w = 0.0039$ $X_\delta = 11.2579$
$M_u = 0.2373$ $M_q = -6.9424$ $M_\beta = 68.2896$
$M_w = 0.002$ $M_\delta = -38.6265$ $B_u = 0.0101$
$B_\beta = -2.1633$ $B_\delta = -4.2184$ $Z_u = 0.0027$
$Z_q = -0.0236$ $Z_\theta - 0.2358$ $Z_\beta = -0.1233$
$Z_w = -0.5727$ $Z_\delta = 0.0698$ $\tau = 0.04\,\text{sec}$

with measured output

$$\begin{bmatrix} \bar{y} \\ y \end{bmatrix} = \begin{bmatrix} q \\ \theta \end{bmatrix} \tag{10}$$

$y := \theta$ - controlled output; $\dot{u}$ – forward velocity; w – vertical velocity; q – pitch rate; θ – pitch angle; β – control rotor longitudinal tilt angle; δ – actuator state; $\delta_c \equiv u$ – longitudinal cyclic input.

Worthy noting, the system (9)-(9′) is only a pretext in view of controller validating by numerical simulations.

Control objectives are regulation and tracking of commanded pitch attitude θ. Main sources of unmodeled dynamics are the control rotor dynamics and time delay. As main assumption on system, *the relative degree was assumed*: one can see that *the controlled output* θ *has relative degree* 3. The controller design will be illustrated step by step with reference to Fig. 1.

Assuming that the system output y is required to track a known bounded input y_c, the pseudo control in (3) is chosen to have the form

$$v = v_{rm} + v_{dc} - v_{ad} \tag{11}$$

see [4]-[6]. Therefore, the pseudo control has three components: v_{rm} – the output of a reference model, v_{dc} – the output of a stabilizing linear dynamic compensator for the linearized dynamics in (4) with $\Delta = 0$ and v_{ad} – the adaptive control signal designed to approximately cancel Δ.

Now, to better assimilate the ideas, let refer to control system architecture as shown in Fig. 2. Our system is known only as having a relative degree 3 with respect to controlled output y, fact transcribed algebraically in the last block of main direct loop

$$y = G_d(s)(v + \Delta)$$
$$G_d(s) := \frac{b_0}{s^r + a_{r-1}s^{r-1} + \ldots + a_0} \tag{12}$$

where the coefficients a_i are available to design. Aiming to correlate the blocks in view of simplifying, the block in upper loop is conceived as

$$v_{rm} = \frac{1}{G_d(s)} y_{rm} \tag{13}$$

and substituting (12) in (11)

$$y = G_d(s)\left(\frac{1}{G_d(s)} y_{rm} + v_{dc} - v_{ad} + \Delta\right) \tag{14}$$

one gets successively

$$y = y_{rm} + \frac{b_0}{s^r + a_{r-1}s^{-1} + \ldots + a_0} v_{dc} \tag{15}$$

$$\sum_{i=0}^{r} a_i e^{(i)} + b_0 v_{dc} = 0,\ e := y_{rm} - y,\ a_r = 1 \tag{16}$$

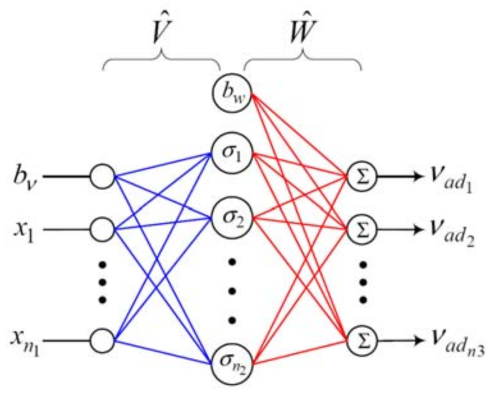

Fig. 3. Single hidden layer neural network

Therefore – **a first step of design** concerns the necessity of introducing a dynamic stabilizing compensator, in principle of dimension at least $r - 1$

$$\begin{aligned}\dot{\eta} &= A_c\eta + b_c e \\ v_{dc} &= c_c\eta + d_c e, e := \begin{bmatrix} e & \dot{e} & \dots & e^{(r-1)} \end{bmatrix}^{\mathrm{T}}\end{aligned} \tag{17}$$

The second step of design is performed by building an observer for the *error dynamics*

$$\begin{aligned}\begin{bmatrix} \dot{e} \\ \dot{\eta} \end{bmatrix} &= \underbrace{\begin{bmatrix} A - b\left(b_0 d_c c + a\right) & -b_0 b c_c \\ b_c c & A_c \end{bmatrix}}_{\bar{A}} \begin{bmatrix} e \\ \eta \end{bmatrix} + \\ &+ \underbrace{\begin{bmatrix} b \\ 0 \end{bmatrix}}_{\bar{b}} b_0 \left(v_{ad} - \Delta\right)\end{aligned} \tag{18}$$

$$A = \underbrace{\begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & & 0 \\ \vdots & \vdots & & & \\ 0 & 0 & & & 1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}}_{r} \Bigg\} r \quad b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} (r) \tag{18'}$$

$$c = \underbrace{\begin{bmatrix} 1 & 0 & 0 & \dots & 0 \end{bmatrix}}_{r}, a = \begin{bmatrix} a_0 & a_1 & \dots & a_{r-1} \end{bmatrix}$$

This observer for the tracking error dynamics may be designed of minimal dimension $r-1$, but herein a full order observer of dimension $2r-1$ is preferred

$$\begin{aligned}\dot{\hat{E}} &= \bar{A}\hat{E} + K\left(z - \hat{z}\right) \\ \hat{Z} &= \bar{C}\hat{E}, \bar{C} := \begin{bmatrix} c & 0 \\ 0 & I \end{bmatrix}, E := \begin{bmatrix} e \\ \eta \end{bmatrix}\end{aligned} \tag{19}$$

K is a gain matrix, and should be chosen such that $\bar{A} - K\bar{C}$ is asymptotically stable. Equation (19) provides estimates only for the states that are feedback linearizable, and not for the states that are associated with the internal dynamics [1], [4].

Summarizing until this point, we have to run on computer the system with the input $v_{ad} - \Delta$

$$\begin{aligned}\dot{E} &= \bar{A}E + \bar{b}b_0\left(v_{ad} - \Delta\right) \\ \dot{\hat{E}} &= \left(\bar{A} - K\bar{C}\right)\hat{E} + K\bar{C}E, \hat{Z} = \bar{C}\hat{E} = \begin{bmatrix} \hat{e} \\ \hat{\eta} \end{bmatrix} \\ v_{dc} &= C_c\hat{\eta} + d_c\begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix}\hat{e}\end{aligned} \tag{20}$$

and giving the output v_{dc}.

The third step of design concerns the getting of adaptive control v_{ad}. As mentioned in the last phrase of Introduction, the dynamic inversion error described in (2)-(4) will be counteracted using the property of universal approximator of a NN.

Given $x \in R^{n_1}$, a three layer-layer NN (with a single hidden layer) has an output given by

$$\begin{aligned}\nu_{ad_k} &= b_w\theta_{w,k} + \sum_{j=1}^{n_2} w_{j,k}\sigma_j, k = 1,\dots,n_3 \\ \sigma_j &= \sigma\left(b_v\theta_{v,j} + \sum_{i=1}^{n_1} v_{i,j}x_i\right), j = 1,\dots,n_2\end{aligned} \tag{21}$$

$\sigma(\cdot)$ is so called activation function, v_{jk} are the first-to-second layer interconnection weights, w_{ij} are the second to third layer interconnection weights, θ_{vj} and θ_{wj} are bias terms (see Fig. 3). In fact, a linearly parameterized NN

$$v = W^{\mathrm{T}}\sigma(x) \tag{22}$$

is a universal approximator, if vector function $\sigma(\cdot)$ can be selected as a basis over the domain of approximation, and accordingly, a general function $y(x) \in C^k, x \in D \subset R^n$ can be written as

$$y(x) = W^{\mathrm{T}}\sigma(x) + \varepsilon(x) \tag{23}$$

where $\varepsilon(x)$ is the functional reconstruction error. Various publications show that the NN type function (22) is dense for different activation functions $\sigma(\cdot)$ [7]. The essential results are expressed as theorem, hereby:

Given $\varepsilon^* > 0$*, there exists a set of bounded ideal weights W, such that* Δ (7)*, associated with the system* (1)-(4)*, can be approximated over a compact domain* $D \subset \Omega \times R$ *by a linearly parameterized NN*

$$\Delta = W^{\mathrm{T}}\sigma(\mu) + \varepsilon(\mu), \|\varepsilon\| < \varepsilon^* \tag{24}$$

using the input vector derived from (8)

$$\mu(t) = \begin{bmatrix} 1 & \bar{v}_d^{\mathrm{T}}(t) & \bar{y}_d^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}} \tag{25}$$

provided there exists a suitable basis of activation functions $\sigma(\cdot)$ *on the compact domain D.*

Thus, the output of the adaptive clement in Fig. 1 will be designed as [3]

$$v_{ad} - \Delta \cong W^T\sigma\left(V^T\mu\right) - W^{*T}\sigma\left(V^{*T}\mu\right) \tag{26}$$

with the following *weight adaptation laws*

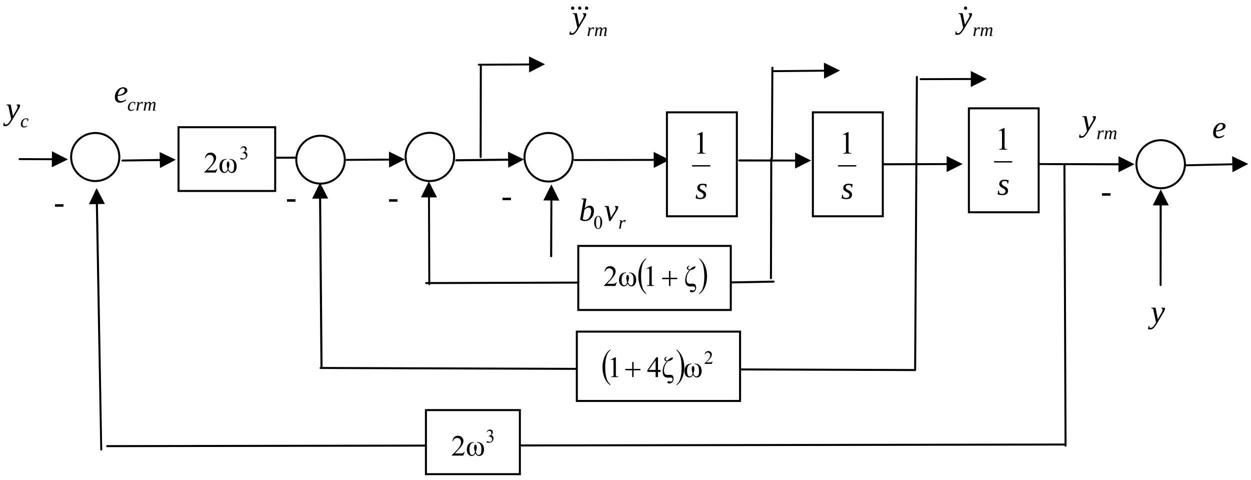

Fig 4. Third order reference model with pseudo control hedging signal

$$\dot{V} = -\Gamma_v\left[\mu\hat{E}^T P\bar{b} W^T \sigma' + k(V - V_0)\right]$$

$$\dot{W} = -\Gamma_w\left[\left(\sigma - \sigma' V^T \mu\right)\hat{E}^T P\bar{b} + k(W - W_0)\right] \quad (27)$$

$$\bar{A}^T P + P\bar{A} = -Q$$

$\left(V^*, W^*\right), \left(V_0, W_0\right)$ are initial guess of NN weights, $Q > 0$ is a suitable matrix, $k > 0$ is a constant adaptation gain, and Γ_V, $\Gamma_w > 0$. The other notations stand for

$$\sigma(z) = \frac{1}{1 + e^{-az}} \quad (28)$$

$$\sigma := \sigma\left(V^T \mu\right) \quad \sigma'(z) := \mathrm{diag}\left(\frac{d\sigma_i}{\partial z_i}\right)$$

where σ is a sigmoidal function and a is an activation potential.

Remark that in (4) Δ depends on v_{ad} through v and v_{ad} has to be designed to cancel Δ. To guarantee existence and uniqueness of a solution for v_{ad}, the *contraction hypothesis* over the entire domain of interest

$$\left|\partial\Delta / \partial v_{ad}\right| < 1 \quad (29)$$

is introduced concerning map $v_{ad} \to \Delta$

The fourth step of design. Usually, the reference signals are filtered. For example, as shown in Fig. 4, in the case of R-50 experimental helicopter dynamics, the commanded pitch attitude will be processed through a linear 3rd order reference model

$$y_{rm} = \frac{2\omega^3}{(s + 2\omega)\left(s^2 + 2y\omega s + \omega^2\right)} y_c \quad (30)$$

The fifth step of design provides an approximate inversion law. Let exemplify with system (9′) having relative degree 3 with respect to output $y = \theta$

$$y = \theta, \dot{y} = q$$

$$\ddot{y} = M_u u + M_q q + M_\beta \beta + M_w w + M_\delta \delta$$

$$\dddot{y} = M_u \dot{u} + M_q \dot{q} + M_\beta \dot{\beta} + M_w \dot{w} - \frac{M_\delta}{\tau}\delta + \frac{M_\delta}{\tau}\delta_c \quad (31)$$

where we are considering only q and θ leaving other states as unmodeled dynamics. Thus, choosing

$$G_d(s) = \frac{b_0}{s^3 + b_0 s^2} \quad (32)$$

and taking into account (12), we have

$$b_0 v \cong \dddot{y} + b_0 \ddot{y} = b_0 M_q q + \frac{M_\delta}{\tau}\delta_c + M_q^2 q \quad (33)$$

therefore

$$u := \delta_c = \frac{\tau}{\hat{M}_q}\left[b_0 v - \hat{M}_q\left(\hat{M}_q + b_0\right) q\right] \quad (34)$$

where $\widehat{M}_q, \widehat{M}_\delta$ are introduced to account for parametric uncertainty in $\widehat{M}_q, \widehat{M}_\delta$, respectively.

The sixth step of design, and the last, concerns the hedging of pseudo control to prevent the adaptation law from shortcomings such as actuator position and

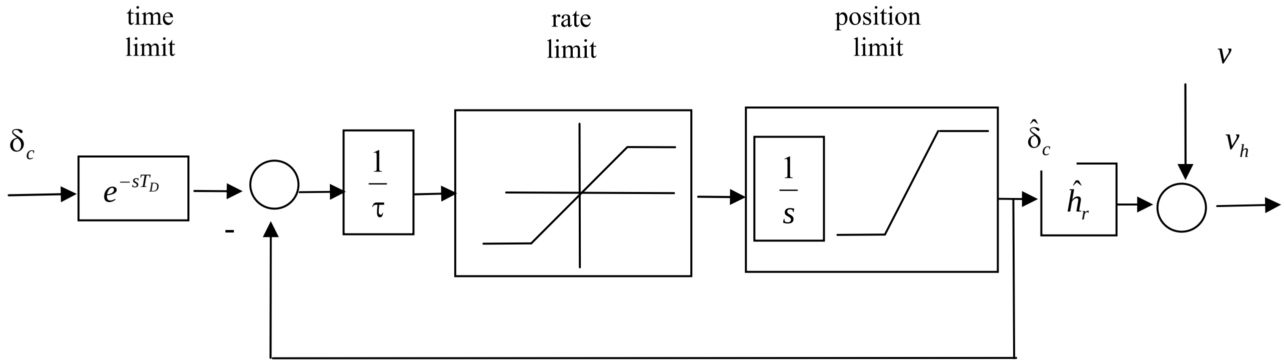

Fig. 5. Computation of the pseudo control hedging signal

rate limits. When saturations are ignored, the phenomenon referred to as *reset windup* can produce the worst undesirable transients. An antiwindup compensator is proposed in [8]. In the present paper, the empiric procedure proposed in the works of E. N. Johnson of coworkers [9] is adopted. The idea is simple: an estimate of actuator position is firstly obtained, and then this estimate is used to compute the difference between commanded pseudo control v and the estimated achievable pseudo control (see Fig. 5)

$$v_h = h(\xi,\delta) - \hat{h}(\xi,\hat{\delta}) = \hat{M}_\delta \big/ b_0\tau \left(\delta_c - \hat{\delta}_c\right) \qquad (35)$$

3. Conclusion

The paper presents as a state-of-art the adaptive output feedback control of uncertain systems in which both the dynamics and the dimension of the regulated plant may be unknown, but knowledge of relative degree is required. More specifically, given smooth references, the problem is to design controllers that force the system measurements to track them with bounded errors. The involved solution includes a linear observer for the output tracking error, a neural network to cancel the modeling error and a pseudo control hedging signal to counteracting actuators limits.

A Part II of the paper will validate the controller by numerical simulations of mathematical model (9)-(10).

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