Brinkman–Forchheimer-Darcy flow past an impermeable cylinder embedded in a porous medium

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***Abstract**: For the flow past an impervious cylinder embedded in a fluid saturated porous medium only the linear (Darcy and Darcy - Brinkman) models were used. In this work, the flow past an impermeable cylinder embedded in a fluid saturated porous medium was studied numerically considering a nonlinear model valid (the Brinkman – Forchheimer – Darcy or Brinkman – Hazen – Dupuit – Darcy model). The flow is viscous, laminar, steady and incompressible. The porous medium is isotropic, rigid and homogeneous. The stream function - vorticity equations were solved numerically in cylindrical coordinates system. The influence of the cylinder Reynolds number, Darcy number and Forchheimer term on the velocities field and surface pressure was investigated for two boundary conditions on the surface of the cylinder: slip and no - slip.*

***Key Words:** saturated porous medium, cylinder, stream function, vorticity, no – slip boundary condition, slip boundary condition.*

1. INTRODUCTION

The flow around bluff bodies has been extensively studied due to its academic value and related applications. For the same reasons, many hundreds of papers analyzed the flow in fluid saturated porous media. However, only few articles considered the flow around solid inclusions embedded in a porous medium. Analytical solutions for the flow past an impervious cylinder embedded in a fluid saturated porous medium can be viewed in [1 – 5]. The Darcy – Brinkman (*DB*) model was used in these works. An analytical exact solution for the velocity field was obtained for the case when the velocity far away from the cylinder is uniform. Speilman and Goren [1], Pop and Cheng [2], Chernyakov [3] and Wang [4] consider that the no – slip boundary condition is satisfied on the surface of the cylinder.

An analytic solution for the problem of the incompressible steady viscous flow past an impermeable cylinder / sphere embedded in a porous medium using the *DB* model with Navier boundary condition on the surface of the cylinder / sphere was obtained in [5]. Leont'ev [5] considers that "setting the no-slip condition when using the seepage equations with higher spatial derivatives (Brinkman, Darcy – Lapwood – Brinkman and other models) is generally inadequate".

Thus, for the flow past an impervious cylinder embedded in a fluid saturated porous medium only the *DB* model was used. The aim of this work is the numerical analysis of the flow past an impermeable cylinder embedded in a fluid saturated porous medium using the Brinkman - Forchheimer - Darcy (*BFD*) model. This problem has not been addressed so far. The present computations are focused on the influence of the Darcy number on the velocities field for two boundary conditions on the surface of the cylinder: slip and no - slip.

This paper is organized as follows: Sect. 2 describes the mathematical model of the problem. The numerical experiments and the results obtained are presented in Sect. 3. Finally, some concluding remarks are briefly mentioned in Sect. 4.

2. MODEL EQUATIONS

Consider the laminar, viscous, steady, axisymmetric, incompressible flow of a Newtonian fluid with a superficial velocity U_0 past an impervious circular cylinder embedded in a porous medium with permeability *K*. The porous medium is rigid, isotropic, homogeneous and fluid saturated. The following additional assumptions are considered valid:

- during the flow, the volume and shape of the circular cylinder are constant;
- the surface tension effects are considered negligible;
- the physical properties of the cylinder and ambient porous medium are uniform, isotropic and constant;
- no phase change.

Under these assumptions, the dimensionless *BFD* model equations (the radius of the cylinder, *a*, is considered as the length scale and the free stream velocity, U_0, as the velocity scale) are:

- continuity equation

$$\nabla \cdot V = 0 \tag{1}$$

- momentum equation

$$-\nabla p + \frac{2}{\mathrm{Re}} \nabla^2 V - \frac{\varepsilon}{2\,\mathrm{Re}\,Da} V - \frac{\varepsilon\, C_F}{2\, Da^{1/2}} \| V \| V = 0 \tag{2}$$

where *V* is the Darcy dimensionless velocity vector $V = (V_R, V_\theta)$, *p* is the dimensionless local average pressure, ε is the porosity of the porous medium, C_F the Forchheimer constant and

$$Da = \frac{K}{d^2}, \mathrm{Re} = \frac{U_0\, d\, \rho}{\mu}, \| V \| = \sqrt{V_R^2 + V_\theta^2}\ .$$

In the previous relations *d* is the diameter of the cylinder, $d = 2\,a$, μ is the dynamic viscosity of the fluid and ρ is the density of the fluid.

Also, the Darcy and cylinder Reynolds numbers are symbolized by *Da* and *Re*. The dimensionless stream function is defined by,

$$V_R = -\frac{1}{r}\frac{\partial \psi}{\partial \theta}, \quad V_\theta = \frac{\partial \psi}{\partial r} \tag{3}$$

and the dimensionless vorticity by (for axisymmetric flow, the vorticity vector has only one non-zero component),

$$\zeta = \frac{\partial V_\theta}{\partial r} - \frac{1}{r}\frac{\partial V_R}{\partial \theta} + \frac{V_\theta}{r} \tag{4}$$

Eliminating V_R and V_θ from (3) and (4), we obtain the stream function equation in dimensionless cylindrical coordinate system (r, θ) as,

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} = \zeta \tag{5}$$

Using the relation

$$\nabla^2 V = \nabla \nabla \cdot V - \nabla \times \left(\nabla \times V\right),$$

the momentum equation (2) can be rewritten as,

$$-\nabla p - \frac{2}{\mathrm{Re}} \nabla \times \zeta - \frac{\varepsilon}{2\,\mathrm{Re}\,Da} V - \frac{\varepsilon C_F}{2\,Da^{1/2}} \| V \| V = 0 \tag{6}$$

Applying the *curl* operator to equation (6) and taking into consideration that

$$\nabla \times \nabla p = 0, \nabla \times \left(f V\right) = f \ \nabla \times V + \nabla f \times V,$$

for any scalar function f, it results, in dimensionless cylindrical coordinate system (r, θ),

$$\frac{\partial^2 \zeta}{\partial r^2} + \frac{1}{r}\frac{\partial \zeta}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \zeta}{\partial \theta^2} - \frac{\varepsilon}{4\,Da}\zeta - \frac{\varepsilon C_F \,\mathrm{Re}}{4\,Da^{1/2}}\left(\| V \| \zeta + \nabla \| V \| \times V\right) = 0 \tag{7}$$

The boundary conditions are:

- axis of symmetry, $\theta = 0, \pi$,

$$\psi = \zeta = 0, \tag{8a}$$

- surface of the cylinder, $r = 1$,

i) no – slip

$$\psi = \frac{\partial \psi}{\partial r} = 0, \tag{8b}$$

ii) slip

$$\psi = 0, \zeta = \frac{1 + 2\beta}{\beta}\frac{\partial \psi}{\partial r}, \tag{8c}$$

- free stream, $r \to \infty$,

$$\frac{\partial}{\partial r}\left(\psi - r \sin \theta\right) = 0, \ \zeta = 0, \tag{8d}$$

where β is the dimensionless slip coefficient, [5] ($\beta = 0$ means no – slip while $\beta \to \infty$ means perfect slip).

A value of β between these two limits corresponds to partial slip on the surface of the cylinder. The present mathematical model is composed by equations (5), (7) and boundary conditions (8).

The mathematical model equations were solved numerically. The radial coordinate r was replaced by x using the transformation $r = \exp(x)$.

The central finite – difference scheme was used to discretize the equations of the mathematical model. The algorithm employed to solve the discrete equations is the classical multigrid (MG) - Full Approximation Storage (FAS) algorithm, [6], suitable for both linear and nonlinear problems.

The algorithm is well described in [7]. For this reason, it is not necessary to repeat its presentation in this work.

However, the following need to be mentioned: (1) the spatial discretization steps on the finest mesh are: $\Delta\theta = \pi / 256$, $\Delta x = 1 / 256$; (2) the numerical algorithm converged for all parameters values employed in the numerical experiments made.

3. RESULTS

The dimensionless parameters of the *BFD* model are: C_F, Da, Re, β and ε. The value considered in this work for ε is, $\varepsilon = 0.9$. This selection does not restrict the area of interest of the present results. The dimensionless slip coefficient β takes value in the range, $0 \div \infty$.

The value considered for the Forchheimer constant is, $C_F = 0.55$. Initially, the Forchheimer constant C_F was considered a universal constant with a value approximately equal to 0.55. Solving numerically the flow equations at pore scale, Coulaud et al. [8] have shown that for $\varepsilon \leq 0.61$, C_F depends on the geometry of the medium. Additional information concerning this problem can be viewed in [9].

Non – Darcy effects (Forchheimer term) occur when the microscopic Re number exceeds a critical value (≈ 1). The microscopic Re number is defined using as characteristic length a pore scale characteristic length (the hydraulic radius, for example). Usually, the diameter of the cylinder is larger than the pore scale characteristic length. Under these circumstances, the values considered in this work for the cylinder Re number are, $Re \geq 10$. The Darcy number, Da, takes values in the range, $Da \leq 10$.

The numerical experiments focused on the following problem: the influence of the Darcy number and Forchheimer term on the flow characteristics. The quantity used to analyze these effects is the velocity on the surface of the cylinder ($V_{\theta,s}$ ($\theta = \pi / 2$), *de facto*). The solutions of the *BFD* model have all the characteristics of the *DB* model (symmetric stream function and vorticity, symmetric / antisymmetric velocities, and so on) (the axis of symmetry / antisymmetry is $\theta = \pi / 2$).

Under these conditions, the influence of the Forchheimer term on the flow field for different Da values will be investigated comparing the present numerical results with the analytical solution of Leont'ev [5].

Figures 1 and 2 show the influence of the Da number and dimensionless slip coefficient on the surface velocity for $Re = 10$. The symbols from figures 1 and 2 refer to the analytical solution of Leont'ev [5].

The present numerical results are depicted as lines. For graphical reasons, the values of $V_{\theta,s}$ for $\beta = 0$ and $\beta > 10$ are not plotted in figures 1 and 2.

Concerning these aspects the following must be mentioned: (1) regardless the value of the Da number, $V_{\theta,s} = 0$ for $\beta = 0$; (2) the maximum relative difference between $V_{\theta,s}$ ($\beta = 10$) and $V_{\theta,s}$ ($\beta = \infty$) is around 3%.

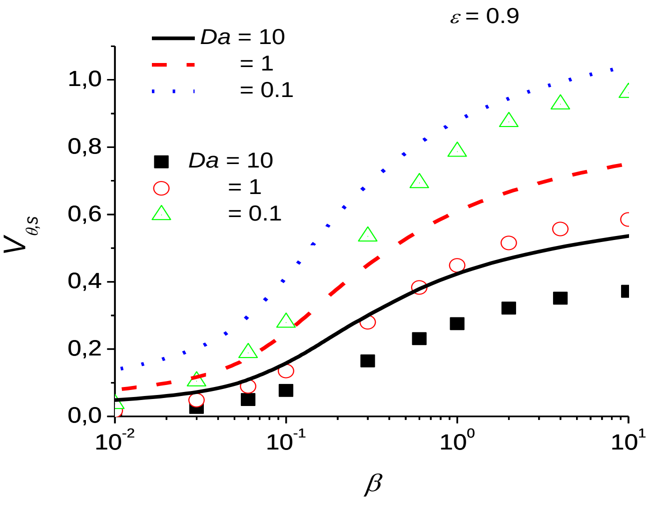

Figure 1. The influence of the dimensionless slip coefficient β on the surface velocity for $Da \geq 0.1$ and $Re = 10$

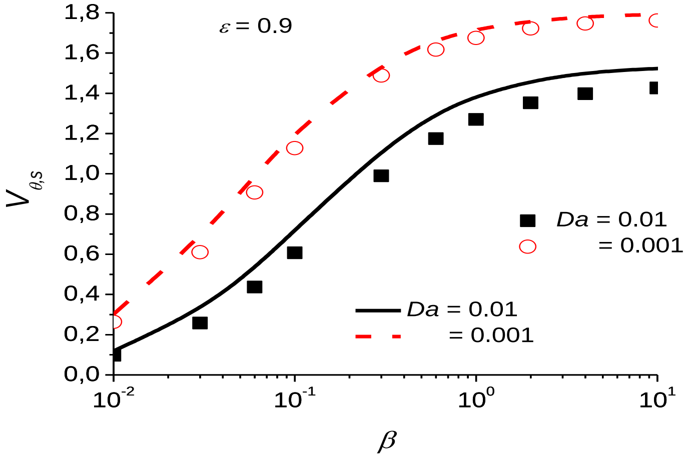

Figure 2. The influence of the dimensionless slip coefficient β on the surface velocity for $Da < 0.1$ and $Re = 10$

The numerical data presented in figures 1 and 2 show that:

- for a given, fixed value of *Da*, the increase in β increases the values of $V_{\theta,s}$;
- for a given, fixed value of β, $\beta > 0$, the decrease in *Da* increases the values of $V_{\theta,s}$;
- for a given, fixed value of β, $\beta \geq 0$, the decrease in *Da* decreases the differences between the results obtained with the *DB* and *BDF* models;
- the influence of the Forchheimer term increases with the increase in *Da*.

The numerical simulations made and not presented in figures 1 and 2 have shown the followings: (1) for very small values of *Da*, i.e. $Da \leq 10^{-5}$, $V_{\theta,s}$ ($\beta \to \infty$) tends to the Darcy (potential) flow value; (2) the numerical results obtained for $Re = 20$ and 30 are similar; obviously, for $Re > 10$, the differences between the values provided by *DB* and *BFD* models increase compared to those plotted in figures 1 and 2.

The velocity profiles obtained, practically V_θ (r, $\theta = \pi/2$) and V_R (r, $\theta = 0(\pi)$), have shown that the influence of the Darcy and Forchheimer terms on the flow field is similar to that presented in [7] and [10]. As in the case of the flow past an impermeable sphere, the velocities vary from the surface value to the asymptotic values of the unperturbed flow with the increase in *r*. The variation of the tangential velocity is not monotonic for all situations; in some cases, especially for small values of the *D*a number, there is a velocity overshoot near the surface of the cylinder (the tangential velocity has characteristic *ears*). The influence of the *Da* number on the velocity profiles is significant only inside a viscous film that develops on the surface of the cylinder; the decrease in *Da* decreases the thickness of the viscous film. The Forchheimer term increases the gradient of the tangential velocity near the surface of the cylinder while the increase in β decreases the gradient of the tangential velocity near the surface of the cylinder. For these reasons, we consider unnecessary the graphical presentation of these results in this work.

The influence of the *Da* number on the cylinder surface pressure coefficient, $C_{P,s}$, is presented in figures 3 (β = 0, no-slip boundary condition) and 4 (β = 6, slip boundary condition). The methodology used to calculate the pressure coefficient is presented in [10]. Figures 3 and 4 show that: (1) the decrease in *Da* decreases the values of the surface pressure coefficient; (2) the influence of β on the surface pressure coefficient is not very strong.

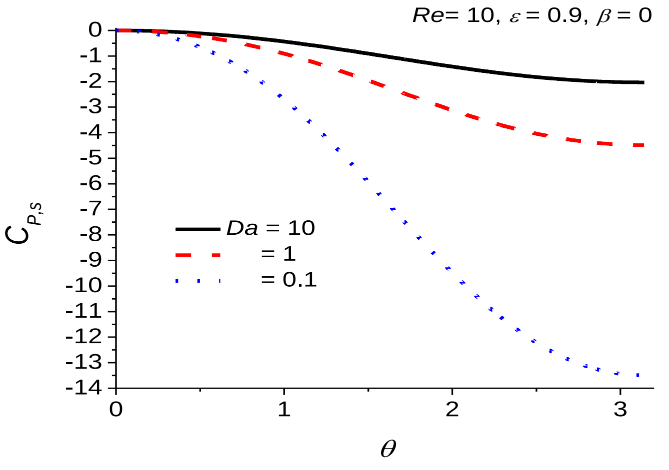

Figure 3. The influence of the *Da* number on the surface pressure coefficient for β = 0 and *Re* = 10

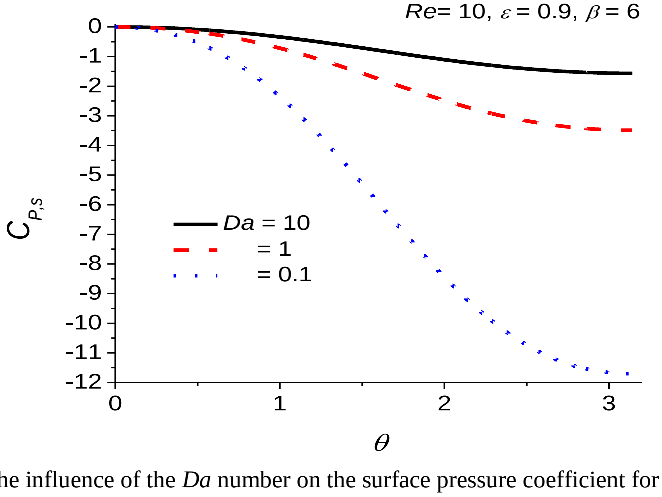

Figure 4. The influence of the *Da* number on the surface pressure coefficient for β = 6 and *Re* = 10

4. CONCLUSIONS

The objective pursued in this work is the numerical solution of the flow past an impermeable cylinder embedded in a fluid saturated porous medium using the Brinkman – Forchheimer - Darcy model. The influence of the *Da* number and Forchheimer constant on the flow characteristics was investigated for cylinder *Re* number in the range, $Re \geq 10$, and two types of boundary conditions on the surface of the cylinder: slip and no - slip.

The numerical results presented in the previous section show that the effect of the Forchheimer term on the velocity profiles strongly depends on *Da* values for $Re \geq 10$. It becomes negligible for very small values of *Da*, i.e. $Da < 0.001$. Only in these situations, the analytical solution of Leont'ev [10] can be used. However, the final decision concerning the velocity profile that can be used for the flow past an impermeable cylinder embedded in a fluid saturated porous medium can be taken only after the following important problem is solved: for which values of *Re* and *Da*, are the macroscopic inertial terms from the generalized momentum balance equation negligible ?

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