Near Optimal Explicit Guidance Law with Impact Angle Constraints for a Hypersonic Re-entry Vehicle

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DOI: 10.13111/2066-8201.2018.10.2.5

Received: 05 February 2018/ Accepted: 16 March 2018/ Published: June 2018

Aerospace Europe CEAS 2017 Conference,
16th-20th October 2017, Palace of the Parliament, Bucharest, Romania
Technical session Guidance, Navigation & Control I

Abstract: *Two guidance laws are discussed in this paper. The first one is the Vector guidance law. This guidance law when equipped with the appropriate gains has the capability to hit a target at the desired impact angle. A parametric method to find the optimal gains of this guidance law which will maximize the impact velocity and keep the miss distance and impact angle errors within bounds is developed in this research. Further, it is seen that the separation in the upper and lower bounds increases with increase in one of the gain values. Also, it is found that, only one of the gain values is independent and that the other dependent gain value is related through a simple straight-line expression. Next guidance law to be discussed is the Diveline guidance law. This law uses multiple divelines to hit a target at the desired impact angle. In the present research, the capability of this Diveline guidance law using a single diveline is analyzed. A method to derive the Diveline guidance law from the Vector guidance law is given in this study. The miss distance and impact angle errors evolving because of reducing the maximum acceleration limit is studied using simulations. Finally, three methods to increase the capture region (i.e. bounds on the set of initial states to achieve zero errors) of a guidance law is discussed.*

Key Words*: Diveline guidance, Explicit Guidance, Impact angle constrained Guidance, Reentry guidance*

NOMENCLATURE

X, Y, Z = Position of the vehicle from the origin
$\ddot{X}, \ddot{Y}, \ddot{Z}$ = Acceleration of the reentry vehicle

C_L = Lift coefficient
C_D = Drag coefficient
C_L^* = Critical lift coefficient
ρ = Atmospheric density
V_m = Vehicle velocity magnitude
$U_{L_x}, U_{L_y}, U_{L_z}$ = Unit lift vector components of the guidance command
β_m = Ballistic coefficient
C_{D_0} = Zero-lift drag coefficient

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1. INTRODUCTION

Explicit guidance laws are used on-board re-entry vehicles to determine the re-entry path. These laws do not require stored trajectories. Vector guidance law is a type of explicit guidance law which uses vector cross product and dot product to generate the guidance commands. The path that has to be flown is decided on-board based on the current vehicle range to the target, current velocity of the vehicle, and velocity and range vector orientation relative to the desired impact angle orientation. Proportional navigation and pursuit guidance are also types of explicit guidance laws used against maneuvering targets. These laws do not shape trajectory nor control impact angle. In this paper, the vector guidance law is analyzed in the context of diveline guidance. The effect of various gain values on the shape of the trajectory, the miss distance and impact angle error is studied. This helps in choosing the value of the gains. Diveline guidance which is a modified vector guidance equation is also analyzed. The diveline guidance law is derived from the vector guidance equation.

There are many guidance laws that deal with impact angle constraints. Guidance laws such as proportional navigation (PN) [1], trajectory shaping guidance [2] (a form of augmented PN), diveline guidance [3], [4] (a form of vector guidance [5]), tangent cubic guidance [6] (a form of characteristic curve guidance [7]), orientation guidance, composite guidance (modification of PN), state-dependent Riccati equation-based guidance law, adaptive PN [8], [9] (adaptive gains for PN), and optimal guidance [10], [11], [12], [13], [14], [15], [16] using linear quadratic regulator are some of the guidance laws that are found in the literature which satisfy the impact angle requirements.

Impact angle constrained guidance for re-entry vehicles requires attention because of the complexity in maneuvering the vehicle to achieve required terminal constraints. Maneuvering at hypersonic speeds requires intricate control system design. Trajectory design is the first step to mitigate the errors in meeting these terminal conditions. Due to increased accuracy requirements and the need for evasion, maneuverable re-entry vehicle technology requires immediate attention. This paper deals with the onboard trajectory design for re-entry vehicles using vector guidance law. The guidance law is three-dimensional cross-product steering law which can be customized for both space shuttle re-entry and re-entry missiles. Desired azimuthal impact angle and elevation impact angle can be accomplished using this guidance law.

2. MATHEMATICAL MODEL OF THE VEHICLE USED

For a non-rotating flat earth, the equations of motion of a lifting reentry vehicle in inertial frame is given by [4]

$$\ddot{X} = \frac{D_x + L_x}{m} \tag{1}$$

$$\ddot{Y} = \frac{D_y + L_y}{m} \tag{2}$$

$$\ddot{Z} = \frac{D_z + L_z}{m} - g \tag{3}$$

The drag components in (1)-(3) are given by

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = -\left(\frac{\rho}{2\beta_m}\right)\left[1+\left(\frac{C_L}{C_L^*}\right)^2\right] V_m \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} \tag{4}$$

And the lift components are given by

$$\begin{bmatrix} L_x \\ L_y \\ L_z \end{bmatrix} = \left(\frac{\rho}{2\beta_m}\right)\left(\frac{C_L}{C_D}\right)\left[1+\left(\frac{C_L}{C_L^*}\right)^2\right] V_m^2 \begin{bmatrix} U_{L_x} \\ U_{L_y} \\ U_{L_z} \end{bmatrix} \tag{5}$$

3. VECTOR GUIDANCE LAW

The present paper discusses a strategy to determine closed loop gains for an explicit vector guidance law which results in near optimal trajectory. The objective of the parameters in any guidance law depends basically on the mission. The objective of the mission being discussed is to hit the target at a particular azimuth angle and elevation angle. The vehicle is fed with the target coordinates and the desired impact angle. There is no onboard scanning for the target because the target is assumed to be stationary. Only the navigation instruments are required to provide the instantaneous position and velocity used by the guidance law to determine lift. Hence the vector guidance law uses the range and velocity information provided by the navigation instruments and gives the lift acceleration required in all the three directions. This is transformed into coordinates along and normal to the velocity vector so that the command can be issued to the vehicle autopilot.

In Fig. 1, an ENU frame is chosen with the origin of the frame at the surface of the earth. The range vector $\vec{R}$ is the line-of-sight vector to the target from the vehicle. When the vehicle velocity vector $\vec{V}$ is aligned with the line of sight (LOS) vector, and is maintained to be so, the vehicle would hit the target. But to hit the target at a particular impact angle, also called the diveline angle ($\vec{D}$), the angle between the velocity vector and the diveline vector should also go to zero. Hence three error angles, $\theta_1, \theta_2, \theta_3$ as shown in Fig. 1, which are the angles between V and $\vec{R}$, $\vec{V}$ and $\vec{D}$, $\vec{R}$ and $\vec{D}$ respectively, have to go to zero so that all the three vectors are collinear at the terminal condition.

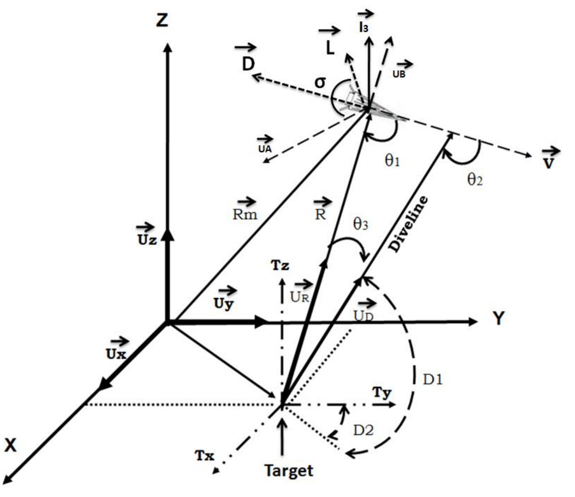

Figure 1: Engagement geometry

The objective is to make $\theta_1 = \theta_2 = \theta_3 = 0$ at $t = t_f$. The cross product of the terms $\vec{V} \times \vec{R}$, $\vec{V} \times \vec{D}$, $\vec{R} \times \vec{D}$, should go to zero and the dot, product of the terms will result in $||V||.||R||, ||R||.||D||, ||V||.||D||$. So the commanded acceleration has to be to zero at the terminal condition. But as long as there is an error angle, the acceleration should not be zero. So the law can be formulated as a simple proportional gain as $G(\vec{V} \times \vec{R}) + H(\vec{V} \times \vec{D}) + I(\vec{R} \times \vec{D})$ where G, H, I are the guidance gains.

If $\vec{V}, \vec{R}, \vec{D}$ are coplanar, then from Fig. 1, we get

$$\theta_1 + \theta_3 + (180^0 - \theta_2) = 180^0 \tag{6}$$

$$\theta_2 = \theta_1 + \theta_3 \tag{7}$$

Hence, if any two angles go to zero, the third angle also goes to zero and so we can omit any one component. If we omit the third component, the resulting equation for lift acceleration can be written as

$$\vec{a}_L = G(\vec{V} \times \vec{R}) + H(\vec{V} \times \vec{D}) \tag{8}$$

Consider a plane containing $\vec{V}$ and $\vec{R}$. The two cross products in the above equation will result in a direction which is out of the plane containing the error angles and will not help in rendering the error angles to be zero. The direction of the lift has to be such that the angular acceleration caused by lift, will mitigate the errors in the plane containing the lift vector. This lift direction should also be orthogonal to the velocity vector. To achieve this, the cross product of the terms with the velocity vector is necessary. This will result in an in-plane vector orthogonal to velocity vector.

$$\vec{a}_L = G.\vec{V} \times (\vec{V} \times \vec{R}) + H.\vec{V} \times (\vec{V} \times \vec{D}) \tag{9}$$

The units are inconsistent on the two sides of the equation, and by choosing the proper values of G and H, the terminal constraints can be achieved. The gains should be non-dimensionalised such that the units are consistent. When we chose $G = \frac{\{A\}}{\{|R|^2\}}$ and $H = \frac{\{A-B\}}{\{|R||D|\}}$ in [9], the units are consistent and the, vectors in the inner cross product can be normalized and the time-to-go ($t_{\{go\}}$) parameter appears [17].

$$\vec{a}_L = \vec{V} \times \left[\frac{A}{||R^2||}.(\vec{V} \times \vec{R}) + \frac{A-B}{||R|| ||D||}.(\vec{V} \times \vec{D}) \right] \tag{10}$$

The unit lift vector required by the vehicle model in Eqn. 5 is obtained by $\vec{U}_L = \frac{\vec{L}}{||\vec{L}||}$.

4. PARAMETRIC STUDY OF THE EFFECT OF GAINS ON THE MISS DISTANCE AND IMPACT ANGLE ERRORS

Table 1: Parameter values

Parameter	Description	Value
C_{D_0}	Zero lift drag coefficient	0.1
$\left(\frac{L}{D}\right)_{max}$	Maximum lift to drag ratio	2.5

l	Reference length	0.4 m
m	Mass	140 kg
β	Ballistic coefficient	1.1141e4 kg/m^2
g	Gravitational acceleration	9.7803 m/s^2
G	Diveline gain	0.5
γ_0	Initial flight path angle	30^0
ψ_0	Initial heading angle	45^0
V_0	Velocity	5 km/s
$[X_0, Y_0, Z_0]$	Initial position	[0,0,25] km
$[X_f, Y_f, Z_f]$	Target position	[3e3,3e3,0] km

The value of the gains A and B decide the shape of the trajectory as well as the miss distance and impact angle errors. Also, the guidance law has an inaccessible area of operation where it cannot hit, irrespective of the gain values. A guidance law has a particular region of operation, called capture region. In vector guidance law the capture region is a function of the maximum initial differences between $\theta_1, \theta_2, \theta_3$.

To study the effects of the guidance parameters a re-entry vehicle with the following parameters are chosen [4]. Here $\gamma = sin^{-1}\left(-\frac{||V_z||}{||V||}\right)$ and $\psi = tan^{-1}\left(\frac{||V_x||}{||V_y||}\right)$. $||V_x||, ||V_y||, ||V_z||$ are the velocity magnitudes in X, Y, Z directions respectively. $||V||$ is the resultant velocity magnitude.

It is assumed that the commanded acceleration is equal to achieved acceleration. Exponential atmospheric approximation is used to find the density. The idea is to first find the range of combinations of A and B which will result in minimum miss distance and impact angle error. Then using that particular range of gains the capture region in terms of maximum angle between the three vectors is found. This is a brute force method of finding the capture region of a guidance law.

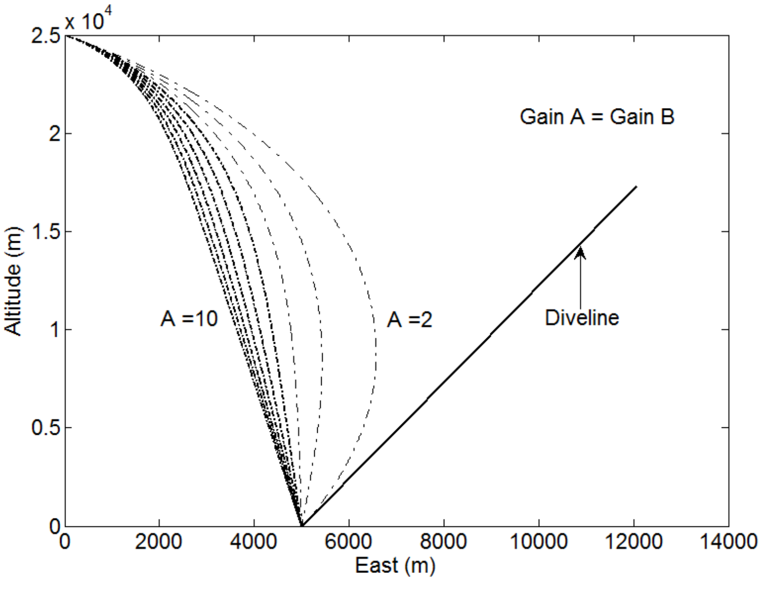

Figure 2: Trajectories for various values of Gain A

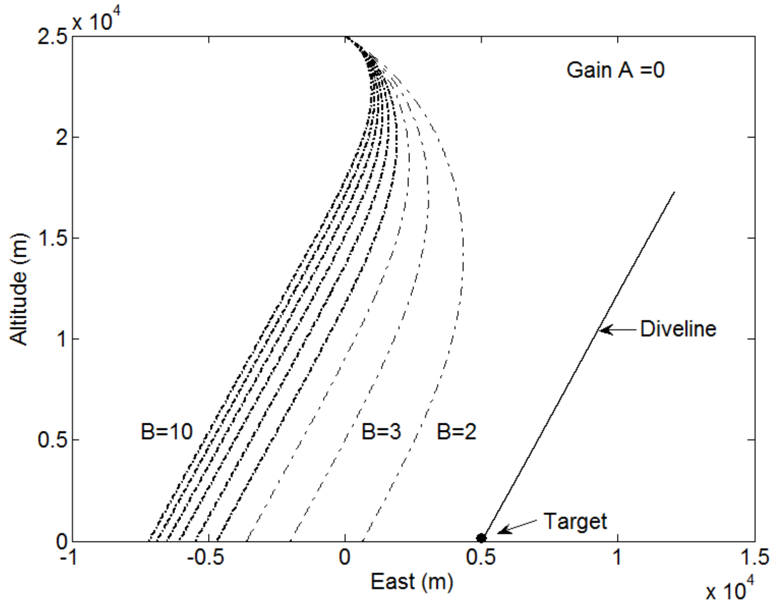

Figure 3: Trajectories for various values of Gain B

To see the effect of individual terms in the vector guidance law, the gains were selected such that only the range goes to zero.

When the gain B is equal to A, the term containing the final diveline becomes zero. Fig. 1 shows trajectories for various values of gain A and the consequent impact angle errors. In Fig. 2, the gains were chosen such that the final impact angle is achieved. So gain A was set to zero and B was varied.

This resulted in miss distances, but the terminal impact angle was satisfied. Now to see the overall effect of varying both the gains, simulations were run, resulting in 400 trajectories.

From these trajectories, the combinations of gains A and B, for which the miss distance and impact angle errors are minima was selected.

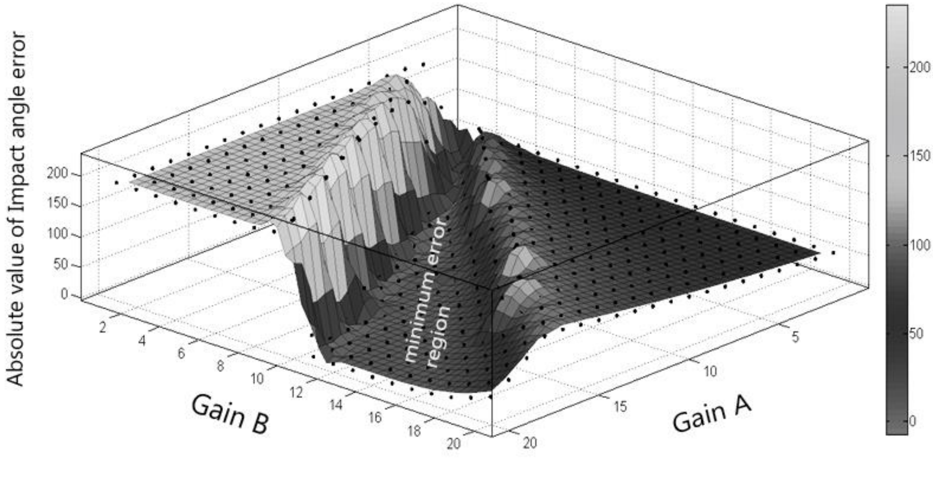

Figure 4: Cubic spline interpolation of impact angle errors of Vector guidance

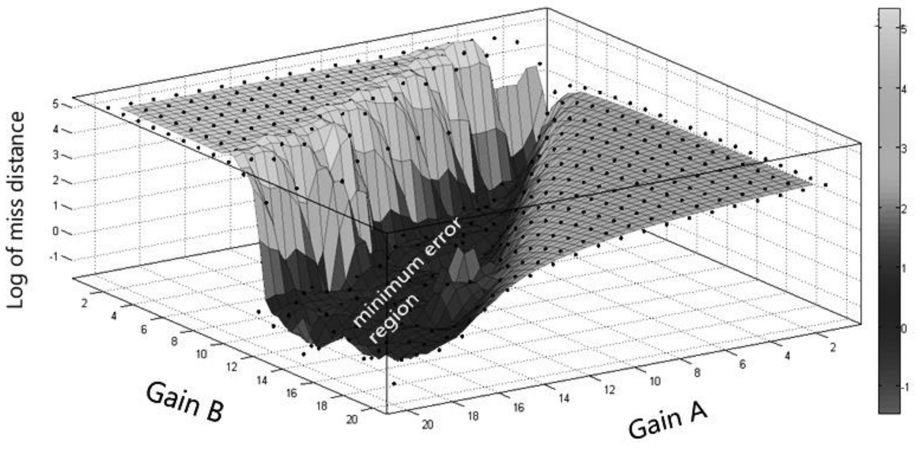

Figure 5 : Cubic spline interpolation of miss distance of Vector guidance

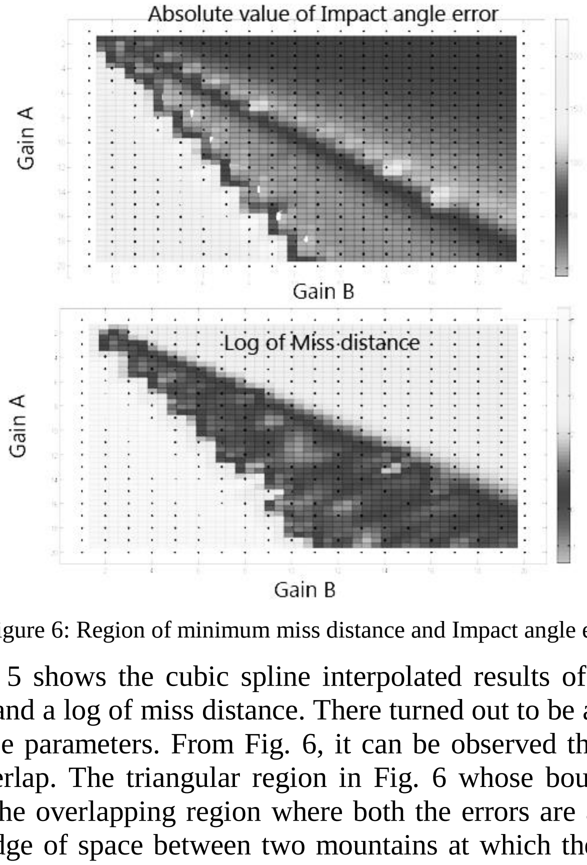

Figure 6: Region of minimum miss distance and Impact angle error

Fig. 4 and Fig. 5 shows the cubic spline interpolated results of the absolute value of impact angle errors and a log of miss distance. There turned out to be a pattern in which both the gains affect these parameters. From Fig. 6, it can be observed that there is a region of overlap and not overlap. The triangular region in Fig. 6 whose boundary is given by the triangular mark, is the overlapping region where both the errors are at their minimal. Both the errors form a ridge of space between two mountains at which the errors are minimum. From these figures, the combinations of A and B for which minimum errors would occur can be found. The analytical investigations are not possible because of the dependency of the error on the vehicle model. Hence every vehicle model has to be flown in-order to find the combinations of gains A and B.

Now the value of gains been found, the vehicle cannot hit the target beyond a particular range. The method of using the constant gain values of A and B limits the capture region.

Hence the gains have to be adapted according to the scenarios which will be discussed later. In the minimal error combinations of A and B, there exist one or more values of B for single value of A for which minimal error occurs and the range of B increases with A. An approximate polynomial equation relating A and B can be fitted as $B = 1.6A - 0.8137$ which will result in a value of B which is the midpoint, i.e. there are particular set of values above and below this value for which the minimal error occurs.

As said earlier the shape of the trajectory depends on the gains. Given this gain limits, we can change the shape by changing the gains within these gain bounds. The figure shows a 6th order polynomial fit which passes through the middle of the upper and lower bounds of the gain B. The analysis done so far ensures that the value of the gains A and B decides the terminal accuracy. The gains can be used as weights to a trade-off between allowable miss distance and impact angle error depending on the scenario. In the case of evasive maneuvering, the bounds on these gains help in determining the possible trajectory shapes with minimum terminal errors. The results show that there are well defined lower and upper bounds on the gains that result in a near optimal terminal guidance solution.

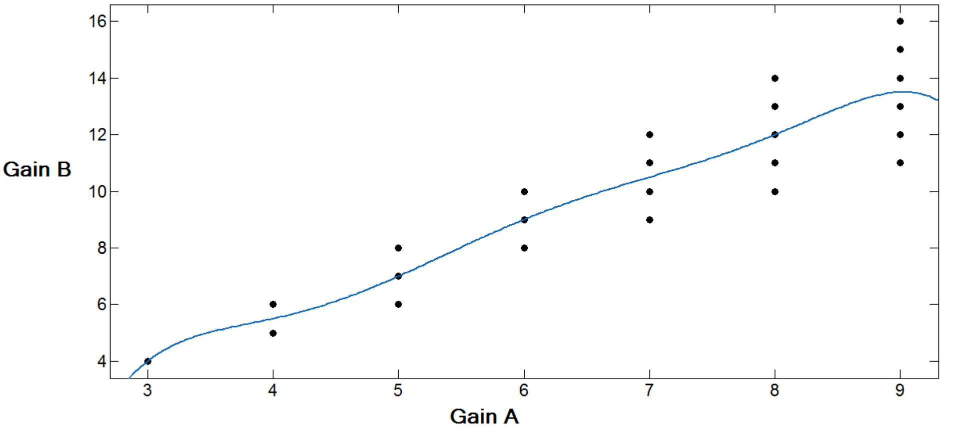

Figure 7: Polynomial fit for choosing gains A and B

5. DIVELINE GUIDANCE LAW

The diveline guidance may also be called as a three-dimensional waypoint guidance since, instead of points, directions are used. In a three-dimensional space, a diveline is represented by five parameters.

- the point from which it originates (X, Y, Z), and
- the azimuth angle (D1) and the elevation angle(D2) of the line as shown in Fig.1.

A single line with its origin at the target is the simplest case. But this does not provide flexibility regarding trajectory shaping. In the case of off-line trajectory shaping, many divelines are used with the final diveline emanating from the target in the desired direction. Optimal selection of divelines is required for each application if multiple divelines are considered. The vehicle's velocity vector is steered toward the divelines by a cross-product guidance law. A simple two-dimensional cross-product guidance law drives the error angle between the vehicle's velocity vector and the range vector to the target to zero as fast as possible. But in the case of diveline guidance, the three-dimensional cross-product steering law, henceforth called diveline guidance law, sets a direction of the lift vector such that the velocity vector of the vehicle is commanded along the diveline. It then reduces the angle between the range vector to the origin of the diveline and the diveline direction. To accomplish this, the vehicle may take a considerable amount of time depending on the L/D ratio of the vehicle used.

A sensible selection of the divelines is required for the vehicle to achieve all intended waypoints. The successful accomplishment of the waypoint depends on the current vehicle velocity, attitude, range vector to the waypoint, and the angles D1, D2 of the approaching diveline. The time required for the vehicle to arrive at the waypoints is important in the case of missile re-entry, of lesser importance in the case of manned reentry.

In manned re-entry applications, multiple trajectories are generated offline and are stored in the onboard computer and the optimal of the trajectories is used as the primary trajectory. But as the vehicle re-enters the atmosphere, depending on the measured dispersion from the nominal trajectory, a decision is taken whether the guidance law can make the vehicle steer to the primary trajectory. If not then another trajectory is chosen from the multiple trajectories stored in onboard computer. The diveline guidance algorithm can also be derived from the vector guidance equations. From the vector guidance laws let us take $G = \vec{R}.\vec{D}$ and $H = -\left||R|\right|^2$. Substitute the values in eqn. (9)

$$\vec{a}_L = \left(\vec{V} \times \left(\vec{V} \times \vec{R}\right)\right) \vec{R}.\vec{D} - \left(\vec{V} \times \left(\vec{V} \times \vec{D}\right)\right) \left||R|\right|^2 \tag{11}$$

This can be written as

$$\vec{a}_L = \vec{V} \times \left[\left(\vec{V} \times \vec{R}\right)\left(\vec{R}.\vec{D}\right) - \left(\vec{V} \times \vec{D}\right)\left(\vec{R}.\vec{R}\right)\right] \tag{12}$$

Simplifying this equation we get

$$\vec{a}_L = \vec{V} \times [\vec{V} \times [\vec{R} \times (\vec{R} \times \vec{D})]] \tag{13}$$

$$\vec{a}_L = \vec{V} \times [\vec{V} \times \vec{Q}] \tag{14}$$

where $\vec{Q} = \vec{R} \times (\vec{R} \times \vec{D})$. This can be normalized and written as

$$\vec{U}_L = \vec{U}_V \times [\vec{U}_V \times \vec{Q}] \tag{15}$$

And the $\vec{Q}$ becomes $\vec{U}_R \times (\vec{U}_R \times \vec{U}_D)$. When the weights are given to the components of $\vec{Q}$ we get

$$\vec{Q} = F.\vec{U}_R\left(\vec{U}_R.\vec{U}_D\right) - G.\vec{U}_D(\vec{U}_R.\vec{U}_R) \tag{16}$$

Let $\vec{W}$ be a vector achieved by substituting F=1 in $\vec{Q}$

$$\vec{W} = \vec{U}_R\left(\vec{U}_R.\vec{U}_D\right) - G\vec{U}_D \tag{17}$$

$$= \vec{U}_R cos\theta_3 - G\vec{U}_D \tag{18}$$

The $\vec{Q}$ in (17) can be replaced by $\vec{W}$ can be written as

$$U_L = \vec{U}_V \times (\vec{U}_V \times \vec{W}) \tag{19}$$

This is the general form of Diveline guidance law [4]. A successful diveline guidance maneuver will result in

$$\vec{U}_V = \vec{U}_D \tag{20}$$

Substituting in (21) we get

$$\vec{U}_L = \vec{U}_D(\vec{U}_D.\vec{W}) - \vec{W} \tag{21}$$

Substitute (19) in (23) we get

$$\vec{U}_L = \vec{U}_D[\vec{U}_D\{\vec{U}_R(\vec{U}_R.\vec{U}_D) - G(\vec{U}_D.\vec{U}_D)\}] - \vec{U}_R(\vec{U}_R.\vec{U}_D) + G.\vec{U}_D \tag{22}$$

When the diveline is accomplished successfully, the distance between the vehicle and the diveline i.e. $\vec{U}_R = 0$. Hence substituting in the previous equation we get

$$\vec{U}_L = \vec{U}_D[-G] + G[\vec{U}_D] \tag{23}$$

$$\vec{U}_L = 0 \tag{24}$$

Thus theoretically the diveline guidance command goes to zero at $t = t_f$. This behaviour is seen notably in optimal guidance laws [3].

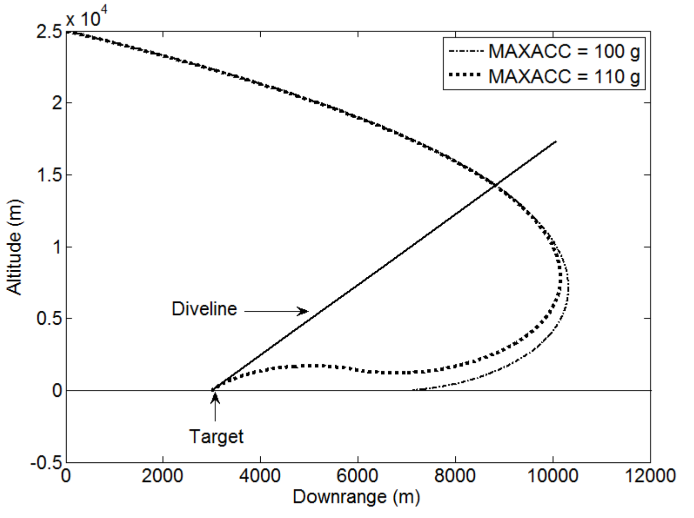

Figure 8: Trajectories for maximum acceleration limit of 100 g and 110 g

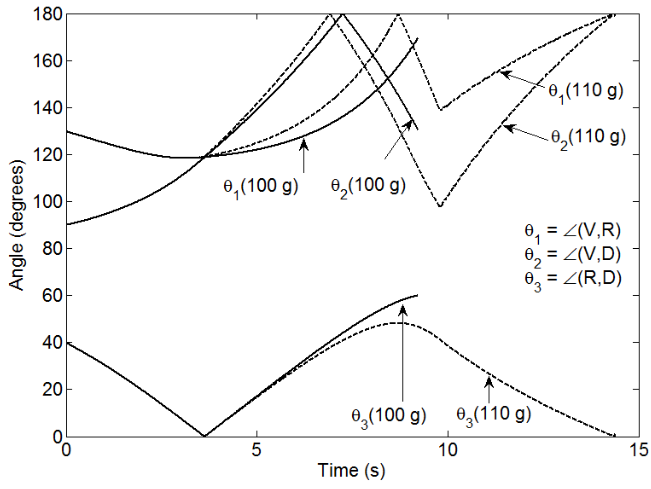

Figure 9: Angle between the vectors $\vec{V}, \vec{R}, \vec{D}$

In Fig. 9, the guidance law tries to nullify θ_3 followed by θ_2 and θ_1. This sequence results in a miss if the maximum permissible limit is set to 100 g as seen in Fig. 8. When the limit is increased to 110 g there is zero miss. In reality, increasing the maximum acceleration limit is infeasible. This problem in terminal accuracy can be mitigated by using moving aim-point technique. This technique can also be used for trajectory shaping.

5.1 Multiple aim-point techniques

The terminal accuracy can be improved by using multiple aim-points. These aim-points are placed at different altitudes in a plane which is orthogonal to the XY plane and contains the final diveline. Initially, the guidance law attempts to steer the reentry vehicle toward these apparent aim-points and finally reaches the target. Every guidance law has a region beyond which it cannot hit. The capture region of any guidance law can be improved by this technique. The moving aim-point technique may be said as a trajectory following guidance with fewer points compared to the nominal trajectory tracking guidance which requires the nominal states at coarse intervals. Aim-points are usually selected at the boundaries of the capture region. So the boundaries of the capture region are extended in a cascade fashion.

Suppose the angle constraint in the intermediate aim-points is violated, then the subsequent aim-points are also missed. This can be avoided by using a switching algorithm which switches the current aim-point to the subsequent when the vehicle goes out of the capture region [17]. Every aim-point has a capture region. i.e. only at particular initial velocity, flight path angle can the aim-point be achieved. To efficiently use the multiple aim-point techniques, the capture region of the guidance law being used needs to be known. The number of aim-points required and the location of the aim-points are preloaded in the on-board computer. The previous problem of increasing the maximum permissible side acceleration can be solved using this technique without increasing the upper bound.

Placing an intermediate aim-point at an altitude of 20 km helps the vehicle to achieve the target without increasing the upper bound on the maximum permissible acceleration limit. The location of the aim-point was found by trial and error. Since the analytical framework for the capture region of diveline guidance law is not yet developed, it is difficult to locate the aim-points manually.

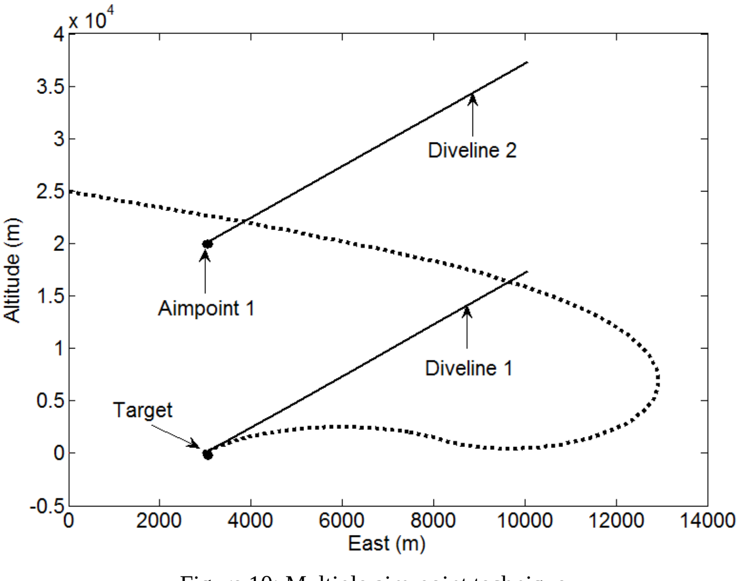

Figure 10: Multiple aim-point technique

Some cases might require more than one aim-point and locating them is a tough task until we know the capture region. Thus the multiple aim-point technique helps in achieving the terminal accuracy.
Simulation results show that they can be used for trajectory shaping as well. The simulation result is shown in Fig. 10 has a maximum acceleration limit of 100 g and can achieve the impact angle constraints whereas in Fig. 8, the value of the limit is increased to 110 g which is beyond the allowable force limit of the reentry vehicle. Thus the moving aim-point technique has the potential to reduce structural costs.

5.2 Adaptive gain updation

Adaptive update of the gains is a widely used technique and is successfully implemented in proportional navigation. The base guidance law with no adaptive update of its gains generally gives zero miss distance but the terminal impact angle constraints are not satisfied. Hence the guidance parameters are updated based on some closed loop non-linear adaptation laws [8].
Using these gains, the guidance law is able to steer the vehicle successfully to the target. From Fig. 6, the boundaries for the values of gains A and B within which the gains can be adapted are found.

5.3 Null Angle method

We propose a novel method to achieve the terminal conditions. The vector guidance law brings the angle between $\vec{R}$ and $\vec{D}$ (θ_3) to zero first, then exerts a pull up maneuver to bring the angle between $\vec{V}$ and $\vec{D}$ (θ_2) to zero. Finally the angle between $\vec{R}$ and $\vec{V}$ (θ_1) is brought to zero. This can be seen in Fig. 9 where the angles are given for both 100g and 110g.
The problem with the current method is that the angle goes to zero first and they skip back and finally at the end point they all go to zero at the same time. This might result in undesired commands. This skip occurs because the guidance law first tries to nullify θ_3. But this is achieved by changing the velocity direction.
Hence θ_1, θ_2 also changes. Once θ_3 reaches zero, the guidance law tries to nullify θ_2. But the inertia and the maximum acceleration limits of the vehicle doesn't allow the vehicle to turn instantaneously.
This makes θ_3 to vary from its zero position. The same happens for θ_1 also. This skip can be avoided by adapting the gains such that there is no skip and all the three angles go to zero simultaneously. This method will result in increased time spending of the vehicle in the diveline which increases the impact velocity.

6. CONCLUSIONS

The present study describes a strategy to arrive at near optimal closed loop guidance gains for meeting the terminal constraints of impact angle and miss distance, for the Vector guidance and Diveline guidance technique. A parametric study was set up to obtain the applicable gain ranges, from which the upper and lower bounds on gains are determined that provide least impact angle error and the least miss distance. The determination of these gain bounds is the first step towards determining the capturability region analytically, for the vector guidance law considered in this study. The study also establishes a formal basis for using multiple divelines in reducing the `g' loads on the re-entry vehicle, in addition to achieving the terminal accuracy.

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