

Moment-inertial representation of the Square-cube law in aircraft industry

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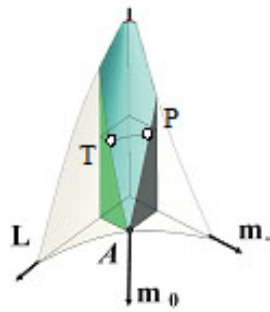
Abstract: *Flight distance depends on the dimension of the aircraft, but the designers stand against an insurmountable barrier caused by the dimension of the aircraft. In the process of analysis, alternative variants of the moment-inertial layout of fuel, engines, and commercial loads and their influence on the aircraft mass change are considered. A comparative analysis of the characteristics of the moment-inertial layouts of the main aircraft of the normal aerodynamic configuration and the aircraft made according to the flying wing scheme obtained as a result of a numerical experiment showed a clear advantage in the moment-inertia characteristics of the aircraft made according to the “Flying Wing” scheme. A number of unconditional advantages in the moment-inertial shape were revealed, such as more rational placement of the target load, fuel tanks and engines, which ensured a gain in aircraft mass up to 7-8%, only due to the rational moment-inertial layout. The moment of inertia of the aircraft depends to a fifth degree on the change in the linear type size of the aircraft.*

Key Words: *linear dimension, lift, moment of inertia, “Square-Cuba” law, “Cargo-distance” diagram*

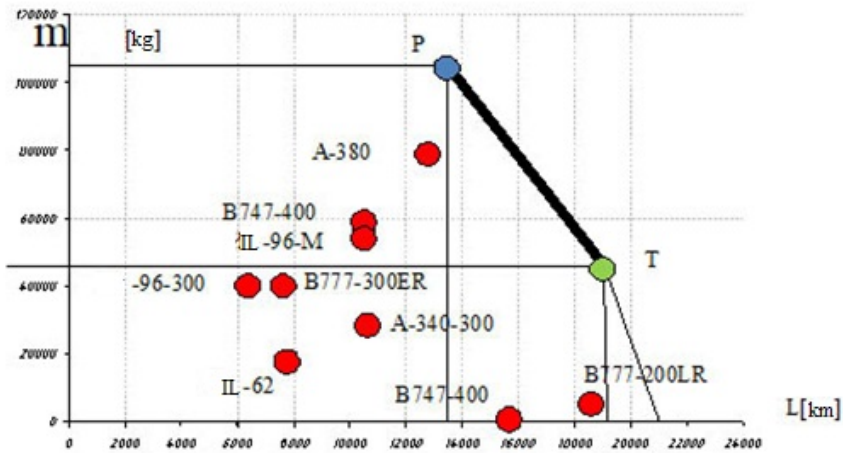
1. INTRODUCTION

At each historical turn of the development of high-tech mechanical engineering, the designer faces an insurmountable barrier caused by the dimensions of the aircraft [1], [2], [3], [4], [5], [6]. The pride of the Russian air fleet of 1913 is a heavy bomber “Ilya Muromets” designed by I. I. Sikorsky today seems a little dwarf on the background of the Tu-160. And there are dozens of such dialectic pairs. Both the Boeing 747 and the Antey An-22 seemed to be the limit of the dimensions of the aircraft, but as time went on, new materials and new technologies appeared and they were replaced by other champions, for example, the A-380 and the Mriya An-225.

- Proportional to area (n^2)
 - Structural strength, muscle strength.
 - Pressure force (in Newton! – not pressure, which in $N/m^2 = 1 Pa \approx 10^{-5} atm$) in a thermal, hydraulic or pneumatic machine.
 - The area of support.
 - Body surface
 - The area and lift force of the wing.
 - Air resistance
 - Surface heat and gas exchange.
 - Pipe capacity.
- In proportion to volume (n^3)
 - Weight
 - Required engine power.



a)



b)

Fig. 2 – “Load-distance” chart of a long-distance aircraft

Adding the dimensions of the aircraft to the third axis, for example, expressed by the take-off mass, we can visually show the extreme dimensions of the aircraft, expressed by modern infrastructural limitations (Figure 2a-b). For example, at point A, the aircraft is degenerated as a transport system [15], [16]. Splicing the points of the estimated flight distance P, which is due to the restriction of the target load on the volume of the passenger compartment, and the point T – maximum flight distance with a target load, due to the volume of fuel tanks. To be exact, the plane begins to carry itself, with zero weight return on the payload [17], [18]. An

interesting example of a flight on the route New York – Singapore “Singapore Airlines” SQ 21 with a length of 15,345 km. Airbus 340-500 aircraft is in the air for 18 hours and 50 minutes. Noteworthy cabin layout. Business class only 98 places (in three-class layout 250 places). That is, the minimum density of the layout for a given volume of the passenger compartment. For many years since June 29, 2004 this flight was the farthest in the world. But it was canceled in October 2012 for purely economic reasons, and in 2018 it was restored again.

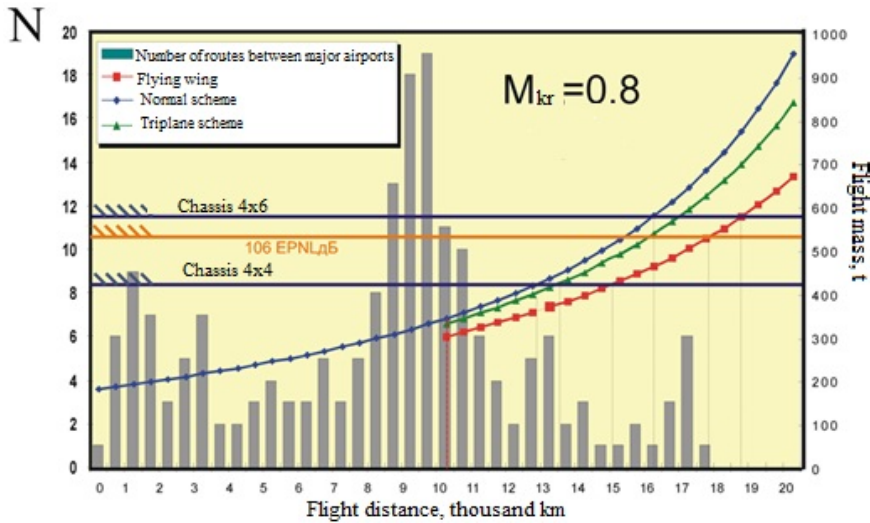


Fig. 3 – The dependence of the take-off mass of the aircraft from the flight distance

Figure 3 shows the histogram of the distribution of flights N between the largest airports in the world that can operate long-distance aircraft (B747, A-380, B777, A-340). The nature of the distribution of large airports is characterized by a demographic factor. The European and North American regions are so vividly expressed, but the countries of the “tiger belt” are of particular interest. These are countries from Japan to Australia.

The horizontal boundary lines (Figure 3) show the limits on the dimension of the take-off weight of the aircraft in the chassis layout options for the four carts on four wheels scheme and for the four carts on six wheels. An independent limitation is shown for noise on the ground, based on the dimension of the take-off mass calculated from the covered surface area of the aircraft [19], [20]. On the right is the take-off weight scale and the dependence of the take-off weight of the aircraft in tons on the flight distance in thousands of kilometers.

3. RESULTS AND DISCUSSIONS

Histogram analysis shows three distinct flight zones. The first to 3000 km corresponds to inland flights. The second in the distance from 8000 to 11000 is the transatlantic distance. Historically, the B747 was designed for this distance. The next frontier at 13,000 kilometres for the task of mastering the Asia-Pacific traffic from Europe and America. A bright representative of the aircraft of this generation is the A-380. On the histogram there is a subject area in the area of 17 thousand kilometres. These are the tasks of future generations. These are the so-called “kangaroo flights” [21]. These are flights from Australia to Europe and America, for example, London – Sydney (16,994 km), or Melbourne (16,903km), or Oakland (18338 km).

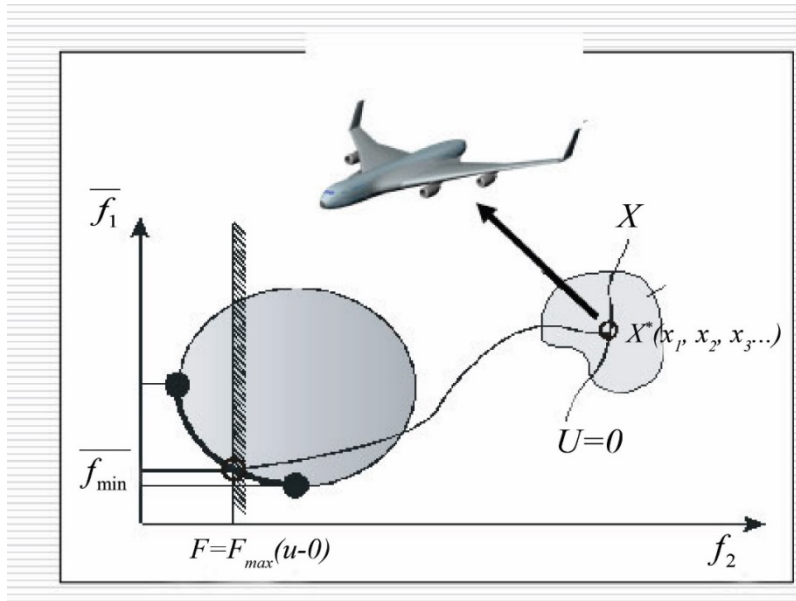


Fig. 4 – Graphic model for finding the rational image of the aerodynamic balancing scheme of a long-distance aircraft in the plane of infrastructure terminal restrictions

In Fig. 4 shows a graphical interpretation of the mathematical model for searching for a rational appearance of the aerodynamic balancing scheme of a long-distance aircraft in the plane of infrastructure terminal restrictions. If we take the ordinate axis for the target load, and the abscissa axis for the flight distance, then in the obtained minimax task, the load-distance diagram rests on the limitations on the maximum flight distance. It is necessary to find vector X design parameters (x_i), to set the parameters X and U constraints. If the distance constraints are considered “hard”, then it suffices to check the points at the intersection of the load-distance diagram and the distance limitations on the project’s feasibility using the aircraft’s equation of state:

$$1 = \frac{\partial m_a}{\partial m_0} + \frac{\partial m_e}{\partial m_0} + \frac{\partial m_f}{\partial m_0} + \frac{\partial m_{eq\&c}}{\partial m_0} + \frac{\partial m_{sl}}{\partial m_0} + \frac{\partial m_{tl}}{\partial m_0}, \tag{1}$$

where m_a – the mass of the aircraft; m_e – the mass of the engines; $m_{eq\&c}$ – the mass of equipment and control; m_f is the mass of the fuel; m_{tl} – the mass of a given target load; m_{sl} is the mass of service load and equipment.

Hence, within the framework of the study, the relative mass of an empty equipped aircraft, which clearly reflects the volume-weight efficiency, and, in conjunction with the relative mass of fuel, and the possibility of creating an airplane, was considered as an independent criterion.

The relative weight of an empty equipped aircraft is a criterion that reflects the perfection of the design and layout and makes it possible to exclude from consideration aerodynamic and flight characteristics, as well as the requirements for a combat operation. Mathematically, the expression for calculating the relative mass of an empty equipped aircraft can be written as:

$$\bar{m}_{e.eq} = \bar{m}_e + \bar{m}_a + \bar{m}_{eq} + \bar{m}_{sl} \tag{2}$$

Each of the addends is the sum of the relative masses of the aggregates, the magnitudes of which change during the composition process, and the aggregates whose masses are constant. The mass of the aircraft and, in particular, the mass of the fuselage is the main

component, reflecting the perfection of the layout, because the fuselage is formed in the process of composition and its relative mass is much greater than the relative masses of the aggregates, which are also formed in the process of composition and included in other terms.

If, as a limitation (Figure 3), to take a landing gear with four trolleys on four wheels, using airplanes according to the flying-wing scheme allows us to increase the flight distance to 15,000 km, in contrast to 13,000 km for a normal aerodynamic balancing scheme. At the same time, the use of a triple-support chassis with four trolleys with six wheels allows the flying wing to fly to 19,000 km with a take-off weight of about 600 tons, that is, to realize the maximum possible flight distances that can theoretically occur on Earth at the equatorial zone. For example, when flying from Buenos Aires to Shanghai, the distance is 19,602 km, Auckland – Madrid is 19,628 km, and between Taipei and Asuncion, by the arc of large circle is 19,918 km.

The dialectic of the development of design ideas at all stages, when engineers stopped on the dimension of the aircraft, allowed to overcome the line, for example, of sound barrier or thermal barrier. In the situation with the flight distance, we are faced with the action of the Law of “square-cube”. There cannot be infinitely increasing dimension aircraft. In accordance with it, the mass of the structure and the entire aircraft (m), depending on its volume (V), increases in proportion to the cube of the increase in linear dimensions (L^3) while maintaining the geometric similarity, while the lifting force (Y), depending on the wing area (S) grows in proportion to the square of the linear dimensions (L^2). The increase in aircraft mass, the rapid growth of lift, must inevitably limit the extreme increase in its linear dimensions [1]:

$$1L - 2Y(L^2) - 3m(L^3) \quad (3)$$

The economic aspects of the design force us to go to perfection in the structural design of the units and systems of the aircraft, i.e. reduce the relative mass of an empty equipped aircraft. Using Forsyth [3] as a method for determining promising development trends, it can be stated that the use of profile laminating, composite and Nano materials, hybrid and integral structures, additive technologies, etc. will contribute to weight reduction.

But none of these areas does radically change the situation. For example, a bracket made using additive technologies reduces its weight by 30%, but this is only 300 grams [4]. Yes, in terms of the entire aircraft, for thousands of structural details, this is a significant gain, but it also has limits. Somewhere additive technologies are not applicable due to the dimensions of the aggregates, etc. Often, when solving technical problems, it becomes necessary to move from a single-criterion problem to a poly-criteria one. Poly-criterial tasks create objects with a developed hierarchical structure. This feature is very characteristic for the aircraft both from design point and from the design process itself.

In addition to the requirement of decomposability of criteria, the principle of minimizing the number of particular criteria plays a very important role in multi-criteria tasks. The most common method of convolving particular criteria into a global one is using the weighting method [1]. Its flaw is in the difficulty of correctly setting the values of the weights.

In the majority of works devoted to the optimization of aggregates, “artificial” evaluation criteria are used. In this case, it is assumed that the parameters determining the shape and connection with other aggregates are unchanged. In the works of Galin L. Ya. It is shown that the construction of a mathematical model of an aircraft, as a complex two-level system, allows us to consider the problem of finding its optimal parameters as a mathematical programming problem with the feature that the objective function (top-level function) is presented in additive-separable form. This feature allows us to apply the nonlinear decomposition method

and transform the task of finding $\min m_0 = \min \sum_i m_i$ in the task of finding $\min m_0 = \min \sum \min m_i$. In the notation of mathematical programming, this can be written as:

$$\min m_0 = \sum_i \min_y [m_i(y) | y_i = f_i(y, x_i), g(y) = 0] \quad (4)$$

where $g(y) = 0$ means requirements of flight performance, presented by the system of restrictions, y means output characteristics that interconnect independent parameters x .

We introduce the function W_i , which we define as follows:

$$W_i(x_i) = \min_y [m_i | y_i = f_i(y, x_i), g(y) = 0] \quad (5)$$

This expression shows that the value of W_i is completely determined by the independent parameters of the i -th unit, with a fixed state of the entire aircraft as a whole. Then the common problem can be represented as a sum of formally independent functions:

$$\min m_0 = \min_x \sum_i W_i(x_i) \quad (6)$$

However, the direct construction of the assessment criterion for W_i is difficult, since the functions f and g are generally non-linear, and f_i depends not only on x_i but also from y . A. A. Badyagin has developed an approximate method of gradients for changing the characteristics of an aircraft depending on the variation of parameters. The method is based on the linearization of small (finite) increments. The simplest problem is solved in cases where the value of the take-off mass of the aircraft is taken as the evaluation [4].

The gradient method is based on the following assumptions:

- private (local) changes, if they are made simultaneously, are not directly related to each other, are independent and are determined by the designer;
- making any changes to the project, the designer seeks to maintain a given target load and basic flight data according to the statement of work or TTT;
- the changes are small, i.e. do not exceed 10 ... 15% of the initial value (the greater the magnitude of the changes, the greater the error in the method).

Based on the first assumption, the total differential of the evaluation criterion is:

$$dW = \frac{\partial W}{\partial i_1} di_1 + \frac{\partial W}{\partial i_2} di_2 + \dots + \frac{\partial W}{\partial i_n} di_n; \quad (7)$$

where i is a parameter.

Assuming that the differentials and finite increments are equivalent, we can write:

$$\Delta W = \frac{\partial W}{\partial i_1} \Delta i_1 + \frac{\partial W}{\partial i_2} \Delta i_2 + \dots + \frac{\partial W}{\partial i_n} \Delta i_n, \quad (8)$$

We use gradient method to solve the problem of choosing task solutions that provide for "hard" infrastructure constraints. For example, consider the i -th level of the hierarchical structure of the aircraft. As a criterion, we take the take-off mass of the aircraft, and as variable independent parameters – additional mass of any part of the aircraft, due to the satisfaction of the i -th infrastructural constraint m_{infr} , and the increment of some other parameters i_n affecting the take-off weight of the aircraft.

$$\Delta m_0 = \frac{\partial m_0}{\partial m_{infr}} \Delta m_{infr} + \frac{\partial m_0}{\partial i_2} \Delta i_2 + \dots + \frac{\partial m_0}{\partial i_n} \Delta i_n, \quad (9)$$

Approximately for small (finite) increments, we can assume that the increments are equal to the relative masses of the corresponding units and systems [3], [4]:

$$\frac{\partial m_{infr}}{\partial m_0} \approx \bar{m}_{infr}. \quad (10)$$

Thus, in the work, additional relative masses of any part of the aircraft will be used as elements of the characteristic matrix, due to the satisfaction of the i -th infrastructure constraint m_{inf} . Wide opportunities for reducing the energy cost of the engine to control the aircraft provides a way to reduce the moments of inertia of the aircraft. In general terms, the expressions for calculating the moments of inertia of the aircraft are:

$$\begin{cases} I_x = \int_m (y^2 + z^2) dm = \int_V (y^2 + z^2) \rho dV = \iiint_{xyz} (y^2 + z^2) \rho dx dy dz; \\ I_y = \int_m (x^2 + z^2) dm = \int_V (x^2 + z^2) \rho dV = \iiint_{xyz} (x^2 + z^2) \rho dx dy dz; \\ I_z = \int_m (y^2 + x^2) dm = \int_V (y^2 + x^2) \rho dV = \iiint_{xyz} (y^2 + x^2) \rho dx dy dz; \end{cases} \quad (11)$$

where: m , ρ and V are the mass, density and volume of the aircraft, x , y , z are the coordinates of the mass centers of decomposed elements of the aircraft in the associated coordinate system of the aircraft, having a volume dV and a mass dm .

Analysis of expressions shows that the moment of inertia depends only on the shape of the body and the location of the masses relative to the axis, and the relative criterion of the rationality of the layout of aggregates in the layout field can be the radius of inertia:

$$r_{xyz} = \sqrt{\frac{I_{xyz}}{\sum_i m_i}} \quad (12)$$

At the same time, the analysis is available in the works [22], [23], [24], [25], [26], [27], [28], [29] equations to determine the moments of inertia of the aircraft show that they either completely or partially do not take into account the features of the layout of the aircraft. For the convenience of taking into account the effect of the layout of the aircraft aggregates on the magnitude of the moment of inertia, we write expressions (5) through the Steiner theorem in the following form:


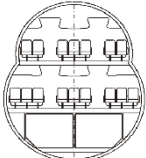


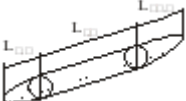

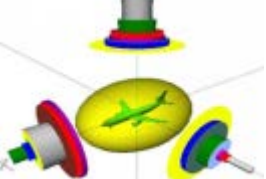
$$I_{1-1} = I_{0-0} + m^* R_{1-0}^2, \quad (13)$$

The expression characterizes the moment of inertia of the body about the axis 1-1 through the moment of inertia of the body relative to its own axis of inertia 0-0, mass and radius vector between axes 1-1 and 0-0. The representation of the expression for determining the moment of inertia allows explicitly, through the distance expressed by the radius-vector, to take into account changes in the moments of inertia of the aircraft depending on the layout of the units and systems of the aircraft. Considering that the mass according to the Square-Cube Law depends to a third degree on the linear dimension change, then the mass multiplied by the square of the inertia radius causes the fifth degree of dependence of the inertia moment on the linear dimension [30], [31]. Taking the target load corresponding to the same transport operation, we can assume that this criterion gives information about the fuel supply necessary for the performance of the flight task.

$$L - 2Y(L^2) - 3m(L^3) - 5J(L^5), \tag{14}$$

This circumstance introduces changes in the order and procedure for assembling the units and systems of the aircraft, which is conventionally divided into several stages. The process can be summarized as follows (Table 1) [3], [4]:

Table 1. – Stages of formation of the moment-inertial appearance of the aircraft

Layout Stage	Influence on moment-inertia characteristics	
Aerodynamic		J_x J_z J_y
Requirements and restrictions	$u \hat{I} U$	J_x J_z J_y
Finding composable sections based on the number of economy class passengers		J_z
Longitudinal deck layout		J_z
Cargo and passenger fuselage layout		J_z
Geometrical parameters of the fuselage		J_z
The layout of the engines and fuel tanks		J_x
Integral assessment of moment-inertia characteristics $J_y = f(J_x + J_z)$		J_x J_z J_y
Verification at TOR level	Airplane = $f(\text{TOR})$	J_{xyz}

Within each stage that constitutes a closed cycle, described by formal models of linking equations of aggregates, the coordinates of the binding of these aggregates are determined. The equations are connected by the layout procedures, the combination of which allows us to solve the system of equations linking the appearance of the aircraft [32], [33].

As part of the formation of the moment-inertial appearance of the aircraft, the stages of the ardistancement of the target load, fuel tanks and the engine are inextricably linked, since

they make the greatest contribution to the formation of inertia moments, both in terms of quantitative value, contributing in some configurations to 40% of the total moment of inertia about a given axis, and in terms of quality – these units have a certain freedom of movement of their own center of mass, and can, by changing the layout parameters, conceptually change the moment-inertial shape of the aircraft.

In the process of analysis, alternative variants of the moment-inertial layout of fuel, engines, and commercial loads and their influence on the aircraft mass change are considered.

Taken together, the results of this analysis provided a basis for studying the mutual influence of the relative mass of the wing, the fuselage, and the parameters of the moment-inertial layout in the form of the inertia radii of passenger and the luggage compartments. This, in turn, made it possible to form the dependence of the flight distance on the parameters of the moment-inertial layout of the aircraft. The table in Fig. 5 shows the characteristics of the aircraft corresponding to these three characteristic flight distance zones (Figure 3).

It is noteworthy that all three aircraft are designed for operation in the same aviation infrastructure, flying at the same speed, with the same type of the four-propulsion engine system and as fuel, using kerosene. The analysis shows that when compared with the implemented projects, the data obtained as optima on the graphs of the regions of allowable values of moment-inertial characteristics, can improve flight performance to 7-8% by reducing and stabilizing the moment-inertia appearance during the flight.

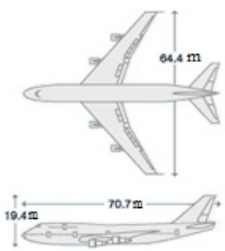
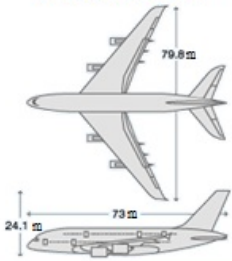
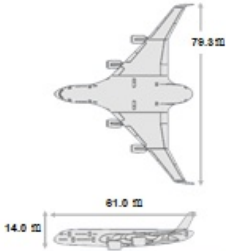
Boeing 747-400	Airbus A380-800	DMC-5
		
Length 70.66 m Wingspan 64.3 m Height 19.32 m Fuselage diameter 6.58 m Number of engines 4 Cruising speed 907 km/h Range of flight 11 200 km Passenger sits 412 Takeoff weight 362 875 kg	Length 73 m Wingspan 79.8 m Height 24.1 m Fuselage diameter 6.58 m Number of engines 4 Cruising speed 902 km/h Range of flight 14 205 km Passenger sits 555 Takeoff weight 540 000kg	Length 61 m Wingspan 79.3 m Height 14 m Fuselage diameter 7 m Number of engines 4 Cruising speed 903 km/h Range of flight 17 000 km Passenger sits 616 Takeoff weight 495 000 kg

Fig. 5 – Long-distance passenger aircraft

The revealed dependence of the flight distance variation on the take-off mass at the optimal values of the moments of inertia reflects the change in the degree of influence of the moment-inertia parameters on the appearance and flight-technical characteristics of the aircraft with an increase in the size of the aircraft. These studies confirm the relevance of work aimed at optimizing the moment-inertia appearance for long-distance high-capacity aircraft.

4. CONCLUSIONS

1. A comparative analysis of the characteristics of the moment-inertial layouts of the main aircraft of normal aerodynamic configuration and the aircraft made according to the flying wing scheme obtained as a result of a numerical experiment showed a clear advantage in the moment-inertia characteristics of the aircraft made according to the flying wing scheme.
2. A number of unconditional advantages in the moment-inertial shape were revealed, such as more rational placement of the target load, fuel tanks and engines, which ensured a gain in aircraft mass up to 7-8%, only due to the rational moment-inertial layout.
3. For promising long-distance aircraft of large passenger capacity, the results of design studies at the modern level of scientific and technical development have confirmed the relevance of using the proposed methods for forming the moment-inertia appearance.
4. The moment of inertia of the aircraft depends to a fifth degree on the alteration in the linear type size of the aircraft.

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