Flutter analysis of the IAR 99 SOIM aircraft

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Abstract: *Flutter analysis is considered for the minimum altitude at which the minimum designed Mach number is achieved, for the maximum altitude at which the maximum designed dynamic pressure is obtained and for the minimum altitude at which transonic effects begin occurring. Moreover, analyzes is performed for any other altitudes considered necessary. Flutter analysis results is graphically presented in equivalent speed with the curves of structural damping coefficient g required for flutter according to the flutter speed. Flutter analysis aims to determine the speeds of IAR 99 SOIM.*

Key Words: *flight loads, boundary conditions, flutter, finite element method*

1. INTRODUCTION

In FEM analysis, static loads are applied to geometric and scalar points in a variety of ways, including:

- Loads directly applied to grid points.
- Pressure loads on surfaces.
- Distributed and concentrated loads on elements.
- Gravity loads.
- Centrifugal loads due to steady rotation.
- Tangential loads due to angular acceleration.
- Loads resulting from thermal expansion.
- Loads resulting from enforced deformations of a structural element.
- Loads resulting from enforced displacements at a grid point.

The import capability allows the user to retrieve an aerodynamic model into the current FEM analysis database.

2. BUILDING SPECIALIZED IDEALIZED MODELS FOR AEROELASTIC CALCULATION OF IAR99 SOIM (HAWK) AIRCRAFT

The subject of this chapter is developing the model of the IAR 99 HAWK Soim (Figure 1) in a green configuration (empty equipped version) for free vibration, static aeroelastic and flutter analysis (Figure 2 and Figure 3). Creating idealized models is based on the following elements:

- SHELL 63- which is a 4 or 3-nodes flat plate element, subjected to bending and in plane forces,
- LINK 8- a 2-nodes element, subject to compression and tension,
- MASS 21- 1-node mass element (includes inertial moments).

For each structural element the corresponding various material properties are used. Typical material properties include Young's elasticity modulus, density, etc. Each property is indicated by a label, eg. ANSYS-EX, EY, EZ for Young's modulus directional components, DENS for density, etc. All material properties can be considered entry data and temperature functions.

Some material properties used in analysis that are not temperature dependant are called linear properties because the typical solutions with these properties only require one iteration.

The linear properties of materials are fed to the program using the MP command, while non-linear properties are introduced using the TB command.

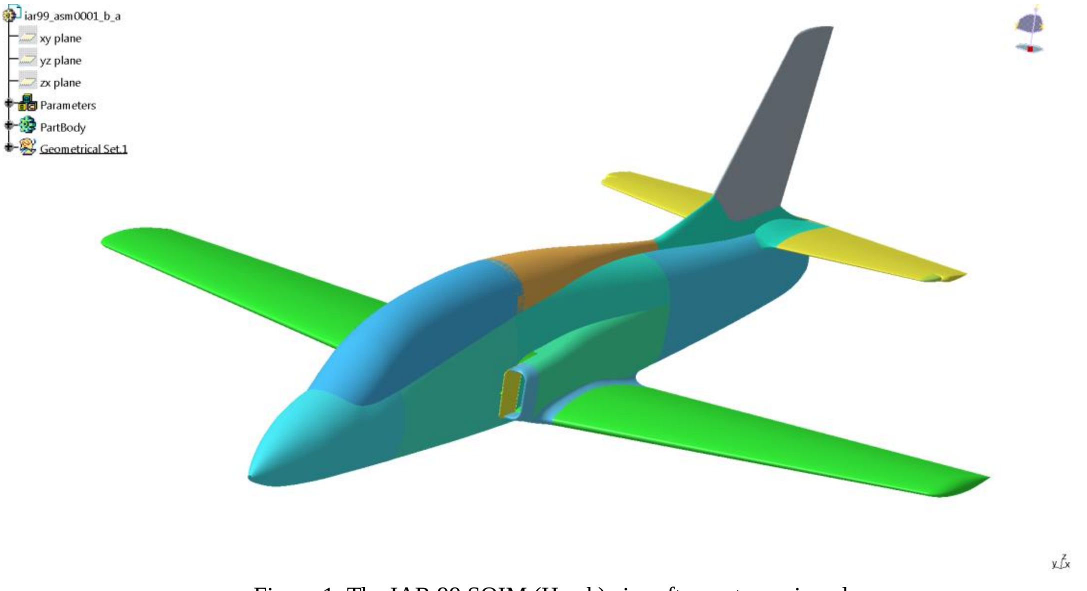


Figure 1. The IAR 99 SOIM (Hawk) aircraft, empty equipped

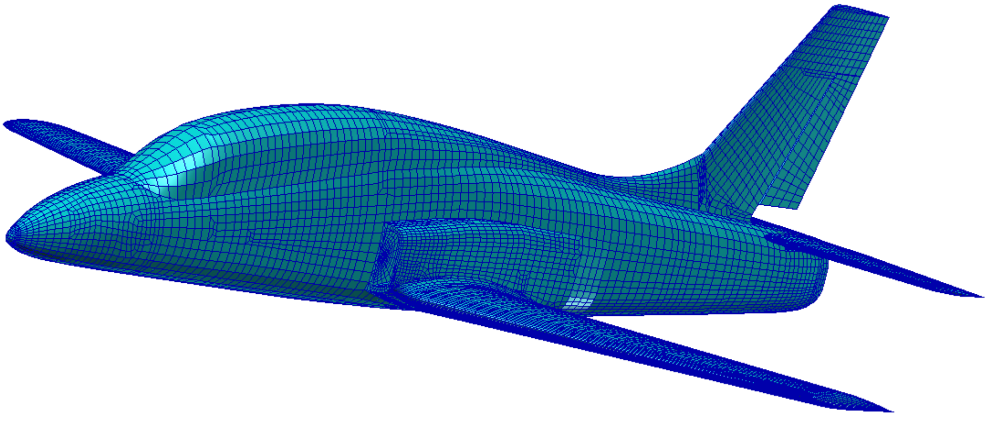


Figure 2. The IAR 99 SOIM (Hawk) aircraft, empty equipped

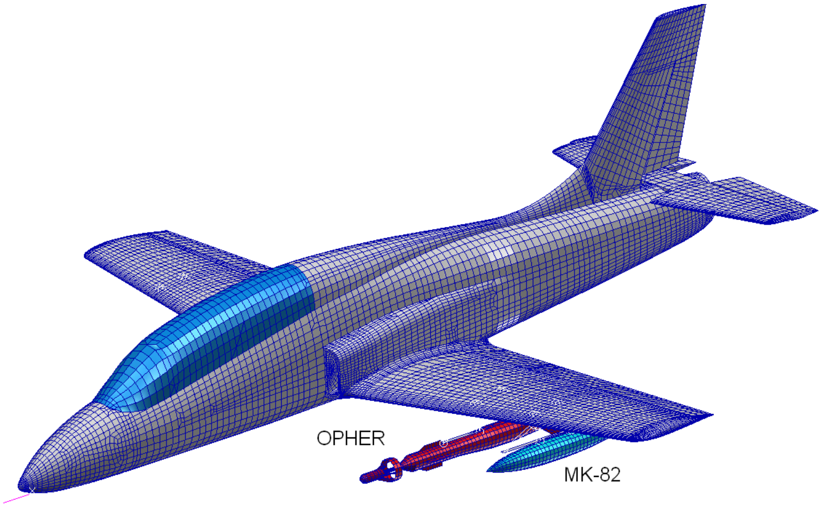


Figure 3. The IAR 99 SOIM (Hawk) aircraft, weapons configurations

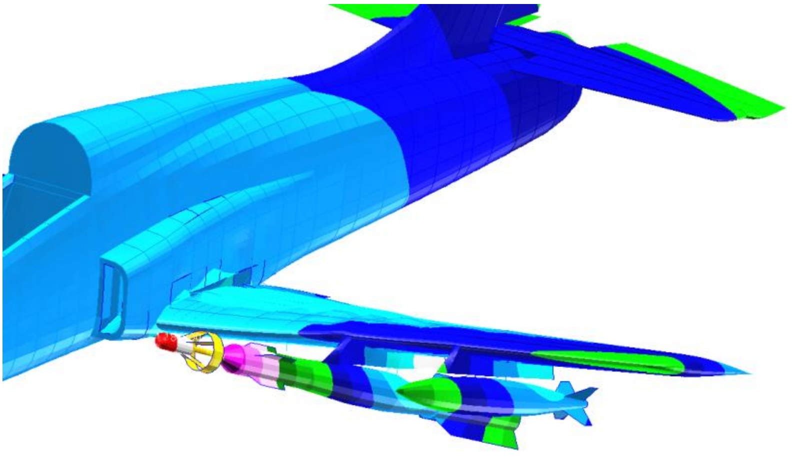


Figure 4. The IAR 99 SOIM (Hawk) aircraft, vibration mode

3. FLUTTER ANALYSIS OF THE IAR 99 AIRCRAFT

3.1 Theoretic summary

Notation

$\mathbf{M}$	mass matrix (positively defined),
$\mathbf{C}$	dampening matrix,
$\mathbf{K}$	rigidity matrix (semipositively defined),
$\ddot{\mathbf{x}}$	acceleration vector,
$\dot{\mathbf{x}}$	speed vector,
$\mathbf{x}$	displacement vector,
$\mathbf{F}_{EXT}$	external force vector,
$\mathbf{F}_{\mathbf{MSC}}$	structural movement dependant forces vector,

ω pulsation [rad/s],

$\mathrm{f} = \dfrac{\omega}{2\pi}$ frequency [Hertz = cycles/s],

$\mathbf{\Phi_K}$ characteristic k – mode vibration form k (k = 1, …, M) corresponding to characteristic frequency $\mathrm{f_K} = 2\pi \cdot \omega_K$,

$\mathbf{\Phi} = [\mathbf{\Phi_1} \cdots \mathbf{\Phi_M}]$ modal form vibration matrix,

$\mathbf{q}$ generalized coordinates,

$\boldsymbol{\mu} = \mathbf{\Phi}^{\mathrm{T}} \cdot \mathbf{M} \cdot \mathbf{\Phi}$ generalized mass matrix,

$\boldsymbol{\beta} = \mathbf{\Phi}^{\mathrm{T}} \cdot \mathbf{C} \cdot \mathbf{\Phi}$ generalized dampening matrix,

$\boldsymbol{\gamma} = \mathbf{\Phi}^{\mathrm{T}} \cdot \mathbf{K} \cdot \mathbf{\Phi}$ generalized rigidity matrix.

$\mathbf{f} = \mathbf{\Phi}^{\mathrm{T}} \cdot \mathbf{F}$ generalized force vector,

$\mathbf{I}$ identity matrix,

$\mathbf{\Omega^2}$ diagonal matrix of characteristic squared pulsations

Equations of Motion

General Equations

General equations of motion for a structure are:

$$\mathbf{M} \cdot \ddot{\mathbf{x}} + \mathbf{C} \cdot \dot{\mathbf{x}} + \mathbf{K} \cdot \mathbf{x} = \mathbf{F}$$

$$\mathbf{F} = \mathbf{F}_{\mathrm{EXT}} + \mathbf{F_{MSC}}$$

where the matrixes are N x N and the vectors N x 1.

Free Vibration Equations

Are obtained if the dampening matrix **C** = **0** and the force vector **F** = **0** namely,

$$\mathbf{M} \cdot \ddot{\mathbf{x}} + \mathbf{K} \cdot \mathbf{x} = \mathbf{0}$$

The equation above leads to the eigenvalue problem:

$$\omega^2 \cdot \mathbf{x} = \mathbf{M^{-1}} \cdot \mathbf{K} \cdot \mathbf{x}$$

Solving the above eigenvalue problem the first M vibration frequencies corresponding to the first M modal forms of structural vibrations can be computed, for M<<N.

Modal Form of Equations of Motion (Figure 4)

In the generalized motion equations the x displacement vector (size N X 1) is approximated with the first M modal vibration forms multiplied by the generalized coordinate vector q (sized M X 1) , that is:

$$\mathbf{x} = \mathbf{\Phi} \cdot \mathbf{q}$$

This means to approximate the N-dimension space by a M<<N dimension space (the Ritz approximation).

The following motion equations result:

$$\mathbf{M} \cdot \mathbf{\Phi} \cdot \ddot{\mathbf{q}} + \mathbf{C} \cdot \mathbf{\Phi} \cdot \dot{\mathbf{q}} + \mathbf{K} \cdot \mathbf{\Phi} \cdot \mathbf{q} = \mathbf{F}$$

and they are projected in M-sized space by left-multiplying with the transposed of the modal form matrix, that is:

$$\mathbf{\Phi}^{\mathrm{T}} \cdot \mathbf{M} \cdot \mathbf{\Phi} \cdot \ddot{\mathbf{q}} + \mathbf{\Phi}^{\mathrm{T}} \cdot \mathbf{C} \cdot \mathbf{\Phi} \cdot \dot{\mathbf{q}} + \mathbf{\Phi}^{\mathrm{T}} \cdot \mathbf{K} \cdot \mathbf{\Phi} \cdot \mathbf{q} = \mathbf{\Phi}^{\mathrm{T}} \cdot \mathbf{F}$$

Or, by introducing the generalized mass, dampening and rigidity matrices and the generalized force vector the above equation becomes:

$$\boldsymbol{\mu} \cdot \ddot{\mathbf{q}} + \boldsymbol{\beta} \cdot \dot{\mathbf{q}} + \boldsymbol{\gamma} \cdot \mathbf{q} = \mathbf{f}$$

As modal form vectors are the eigenvalue vectors of a matrix, it means they are defined up to an arbitrary multiplicative constant. If the scaling of the modal forms (of the eigenvalue vectors) is done such as the generalized mass matrix is equal to the unit matrix **I**, then the generalized rigidity matrix is equal to the diagonal matrix of the squared characteristic pulsations, that is:

$$\boldsymbol{\mu} = \begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \cdots & \vdots \\ 0 & \cdots & 1 & \cdots & 0 \\ \vdots & \cdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 1 \end{bmatrix} = \mathbf{I} \quad \Leftrightarrow \quad \boldsymbol{\gamma} = \begin{bmatrix} \omega_1{}^2 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \cdots & \vdots \\ 0 & \cdots & \omega_K{}^2 & \cdots & 0 \\ \vdots & \cdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & \omega_M{}^2 \end{bmatrix} = \mathbf{\Omega^2}$$

In this case the equations of motion in modal form are written as:

$$\ddot{\mathbf{q}} + \boldsymbol{\beta} \cdot \dot{\mathbf{q}} + \mathbf{\Omega^2} \cdot \mathbf{q} = \mathbf{f}$$

3.2 Matching the structural and aerodynamic models

3.2.1 Theoretical overview

The used method is called **Thin-Plate Spline (TSP)**.

The main equations describing the method are given below. Displacement w(x,y,z) in an arbitrary point P(x,y,z) caused by the forces F_J applied in the points given by (x_J, y_J, z_J) can be written as:

$$w(x,y,z) = \mathrm{a_o} + a_1 \cdot x + a_2 \cdot y + a_3 \cdot z + \sum_{I=1}^{N} \mathrm{S_I} \cdot F_I \tag{1}$$

where:

$$\mathrm{S_I} = \mathrm{r_I}^2 \cdot \ln \mathrm{r_I}^2 \quad \text{cu} \quad \mathrm{r_I}^2 = (\mathrm{x_I} - \mathrm{x})^2 + (\mathrm{y_I} - \mathrm{y})^2 + (\mathrm{z_I} - \mathrm{z})^2 \tag{2}$$

The conditions imposed on the forces F_I are that the sum and their moments with respect to the three axes are zero, that gives:

$$\begin{cases} \sum_{I=1}^{N} F_I = 0 \\ \sum_{I=1}^{N} \mathrm{x_I} \cdot F_I = 0 \\ \sum_{I=1}^{N} \mathrm{y_I} \cdot F_I = 0 \\ \sum_{I=1}^{N} \mathrm{z_I} \cdot F_I = 0 \end{cases} \Leftrightarrow \begin{cases} \mathbf{1}^{\mathrm{T}} \cdot \mathbf{F} = 0 \\ \mathbf{x}^{\mathrm{T}} \cdot \mathbf{F} = 0 \\ \mathbf{y}^{\mathrm{T}} \cdot \mathbf{F} = 0 \\ \mathbf{z}^{\mathrm{T}} \cdot \mathbf{F} = 0 \end{cases} \Leftrightarrow \begin{bmatrix} \mathbf{1}^{\mathrm{T}} \\ \mathbf{x}^{\mathrm{T}} \\ \mathbf{y}^{\mathrm{T}} \\ \mathbf{z}^{\mathrm{T}} \end{bmatrix} \cdot \mathbf{F} = \mathbf{0} \Leftrightarrow [\mathbf{1} \ \mathbf{x} \ \mathbf{y} \ \mathbf{z}]^{\mathrm{T}} \cdot \mathbf{F} = \mathbf{0} \tag{3}$$

$$\Leftrightarrow \mathbf{R}^{\mathrm{T}} \cdot \mathbf{F} = \mathbf{0}$$

matrix **R** being:

$$\mathbf{R} = \begin{bmatrix} \mathbf{1} & \mathbf{x} & \mathbf{y} & \mathbf{z} \end{bmatrix} \tag{4}$$

Imposing the conditions that in the J=1,…, N structural points the displacement is equal to that resulting from structural calculations, we obtain:

$$w(x_J, y_J, z_J) = w_J = a_o + a_1 \cdot x_J + a_2 \cdot y_J + a_3 \cdot z_J + \sum_{I=1}^{N} S_{IJ} \cdot F_I \quad \text{unde} \quad J = 1,...,N \tag{5}$$

Or using matrices:

$$\mathbf{W} = \mathbf{R} \cdot \mathbf{a} + \mathbf{S} \cdot \mathbf{F} \tag{6}$$

where:

$$\mathbf{a} = \begin{bmatrix} a_o \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} \text{ si } S_{IJ} = r_{IJ}^2 \cdot \ln r_{IJ}^2 \quad \text{cu} \quad r_{IJ}^2 = r_I^2 = (x_I - x_J)^2 + (y_I - y_J)^2 + (z_I - z_J)^2 \tag{7}$$

From (3) and (6) we get:

$$\begin{cases} \mathbf{0} \cdot \mathbf{a} + \mathbf{R}^T \cdot \mathbf{F} = \mathbf{0} \\ \mathbf{R} \cdot \mathbf{a} + \mathbf{S} \cdot \mathbf{F} = \mathbf{W} \end{cases} \Leftrightarrow \begin{bmatrix} \mathbf{0} & \mathbf{R}^T \\ \mathbf{R} & \mathbf{S} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{a} \\ \mathbf{F} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{W} \end{bmatrix} \Leftrightarrow \mathbf{C} \cdot \begin{bmatrix} \mathbf{a} \\ \mathbf{F} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{W} \end{bmatrix} \Leftrightarrow \begin{bmatrix} \mathbf{a} \\ \mathbf{F} \end{bmatrix} = \mathbf{C}^{-1} \cdot \begin{bmatrix} \mathbf{0} \\ \mathbf{W} \end{bmatrix} \tag{8}$$

The displacements $w(x_k,y_k,z_k)$ in an aerodynamic point $P(x_k,y_k,z_k)$ corresponding to structural displacements **W** are given by equation (1), yielding:

$$w(x_K, y_K, z_K) = a_o + a_1 \cdot x_K + a_2 \cdot y_K + a_3 \cdot z_K + \sum_{I=1}^{N} S_{IK} \cdot F_I \tag{9}$$

where:

$$S_{IK} = r_{IK}^2 \cdot \ln r_{IK}^2 \quad \text{cu} \quad r_{IK}^2 = (x_I - x_K)^2 + (y_I - y_K)^2 + (z_I - z_K)^2 \tag{10}$$

Equation (9) can successively be written as:

$$w_K = \begin{bmatrix} 1 & x_K & y_K & z_K \end{bmatrix} \cdot \mathbf{a} + \begin{bmatrix} S_{1K} & \cdots & S_{IK} & \cdots & S_{NK} \end{bmatrix} \cdot \mathbf{F} \tag{11}$$

$$w_K = \left[1 \;\; x_K \;\; y_K \;\; z_K \,\vdots\, S_{1K} \;\cdots\; S_{IK} \;\cdots\; S_{NK} \right] \cdot \begin{bmatrix} \mathbf{a} \\ \mathbf{F} \end{bmatrix} \tag{12}$$

$$w_K = \left[1 \;\; x_K \;\; y_K \;\; z_K \,\vdots\, S_{1K} \;\cdots\; S_{IK} \;\cdots\; S_{NK} \right] \cdot \left(\mathbf{C}^{-1} \cdot \begin{bmatrix} \mathbf{0} \\ \mathbf{W} \end{bmatrix} \right) \tag{13}$$

The derivative of displacements along x in aerodynamic points is:

$$\left.\frac{\partial w}{\partial x}\right|_{x=x_K} = \left[0\ 1\ 0\ 0 \,\vdots\, S_{,x_{1K}} \ \cdots\ S_{,x_{IK}} \ \cdots\ S_{,x_{NK}}\right]\cdot\left(\mathbf{C}^{-1}\cdot\begin{bmatrix}\mathbf{0}\\ \mathbf{W}\end{bmatrix}\right) \tag{14}$$

where:

$$S_{,x} = \frac{\partial S}{\partial x} = \frac{\partial}{\partial x}\left(r_I^{\,2}\cdot\ln r_I^{\,2}\right) = \frac{\partial r_I^{\,2}}{\partial x}\cdot\ln r_I^{\,2} + r_I^{\,2}\cdot\frac{\partial r_I^{\,2}}{\partial x}\cdot\frac{1}{r_I^{\,2}} = \left(1+\ln r_I^{\,2}\right)\cdot\frac{\partial r_I^{\,2}}{\partial x} \tag{15}$$

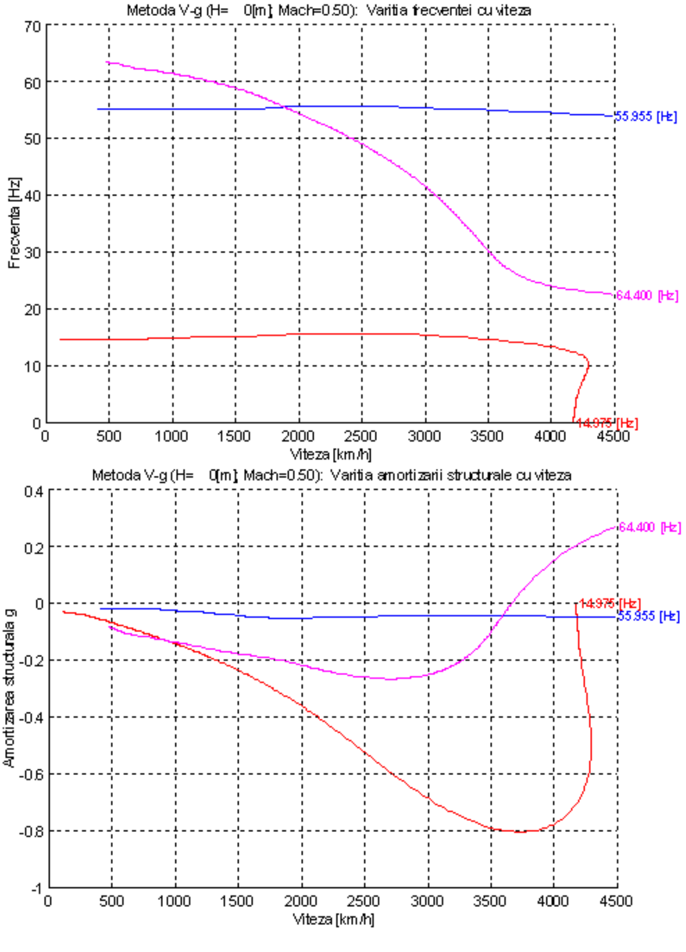
with

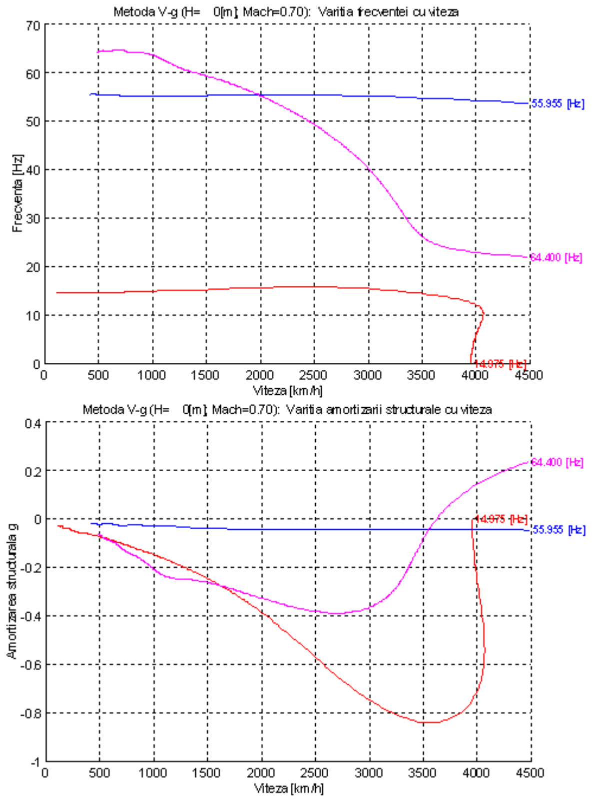
$$\frac{\partial r_I^{\,2}}{\partial x} = \frac{\partial}{\partial x}\left[(x_I - x)^2 + (y_I - y)^2 + (z_I - z)^2\right] = -2\cdot(x_I - x) \tag{16}$$

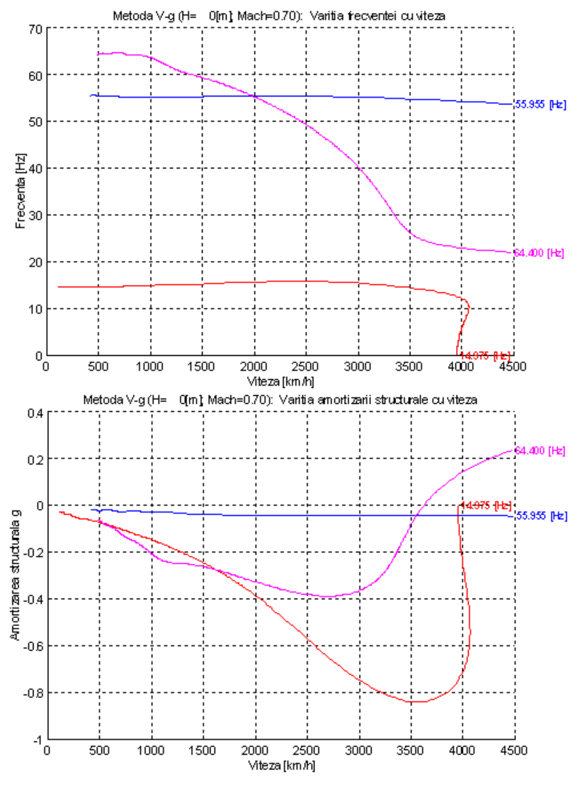
And thus:

$$S_{,x_{IK}} = -2\cdot\left(x_I - x_K\right)\cdot\left(1+\ln r_{IK}^{\,2}\right) \tag{17}$$

Flutter Results for the V-g Method




Metoda V-g (H= 0[m]; Mach=0.70): Varitia frecventei cu viteza
Frecventa [Hz]
Viteza [km/h]
55.955 [Hz]
14.075 [Hz]
Metoda V-g (H= 0[m]; Mach=0.70): Varitia amortizarii structurale cu viteza
Amortizarea structurala g
Viteza [km/h]
14.075 [Hz]
55.955 [Hz]


Metoda V-g (H= 0[m]; Mach=0.70): Varitia frecventei cu viteza
Frecventa [Hz]
Viteza [km/h]
55.955 [Hz]
14.075 [Hz]
Metoda V-g (H= 0[m]; Mach=0.70): Varitia amortizarii structurale cu viteza
Amortizarea structurala g
Viteza [km/h]
14.075 [Hz]
55.955 [Hz]

4. CONCLUSIONS

The idealized model of the IAR 99 SOIM (HAWK) aircraft, in an empty equipped configuration, with a mass of 3300 kg, is to be used in the following analyses:

- free vibration analysis for the entire aircraft and comparison of theoretical results with the experimental ones.
- flutter analysis of the entire aircraft, based on theoretical vibration modes and comparisons with the experimental flutter analysis based on measured vibration modes.
- static aeroelastic analysis and pressure distribution comparison on the elastic and rigid aircraft.

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