Nonlinear automatic control of the satellites by using the quaternion and the angular velocities' vectors

Romulus LUNGU[1], Mihai LUNGU*[,1], Mihai IOAN[2]

*Corresponding author
*[,1]University of Craiova, Faculty of Electrical Engineering,
Carol Blv., no. 6, Craiova, Romania
romulus_lungu@yahoo.com, lma1312@yahoo.com*
[2]"POLITEHNICA" University of Bucharest
Splaiul Independenţei 313, 060042, Bucharest, Romania
ioan.emw@rdscv.ro

DOI: 10.13111/2066-8201.2015.7.2.9

***3rd International Workshop on Numerical Modelling in Aerospace Sciences, NMAS 2015**,*
06-07 May 2015, Bucharest, Romania, (held at INCAS, B-dul Iuliu Maniu 220, sector 6)
Section 4 – System design for small satellites

Abstract: *In this paper, the authors propose a new architecture for the control of the satellites' attitude by using a control law mainly based on a proportional-integrator component with respect to the quaternion vector and to the satellite's angular velocity vector. The control law has two nonlinear components with saturation zone; the actuators' saturation will be considered both from the generated gyroscopic couples' point of view and from the gyroscopic frame angular velocities' point of view. The new obtained nonlinear control system is software implemented and validated through complex numerical simulations; the stabilization and the control dynamic characteristics of the system are obtained and analyzed in detail.*

Key Words: *satellite, quaternion, nonlinear control, attitude*

1. INTRODUCTION

To answer well to multiple tasks, the satellites must have good rotational handling and agility. Such satellites need an automatic system for their attitude's control by performing fast slewing maneuvers.

To perform fast slewing maneuvers, such satellites must use an automatic system for their attitude's control because the physical limitations of the sensors/ actuators, the structural rigidity of the satellites and the mission's type influence the repositioning maneuvers of the satellites [1-3].

Sometimes, the Euler equation describing the satellite's evolution of the orientation on its attitude proves to be too difficult to work with and, therefore, the attitude's dynamics must be put into a double integrator form with respect to the parameters describing the satellite's attitude [4].

The parameterization of the satellite's attitude is mainly described by the cosine rotation matrix which is associated to the orthogonal group $SO(3)$ [5-8]. Because the usage of the cosine rotation matrix leads to difficulties in the numerical implementation process, a solution to the problem of satellite's stabilization is to use the quaternion parameterization [9-11]. A new and interesting method to control the satellite's attitude is without angular velocities [12-

16] and, as a consequence, the main purpose is to control the attitude of satellites without using gyros, but the method proved to be very expensive [11].

Because the Euler representation is always characterized by an inherent geometric singularity, a four-parameter description of the satellite's orientation, known as "quaternions" is more often used [17]; the advantage of using quaterninons is related to the fact that successive rotations result in successive multiplications of the quaternion commutative matrices [17].

The present study involves the design of a new architecture for the control of the satellites' attitude by using a control law mainly based on a proportional-integrator component with respect to the quaternion vector and to the satellite's angular velocity vector; the control law to be designed has two nonlinear components with saturation zone; the actuators' saturation will be considered both from the generated gyroscopic couples' point of view and from the gyroscopic frame angular velocities' point of view.

The new architecture is implemented and validated through complex numerical simulations for the case of a mini-satellite involved in a typical maneuver (complete cycle) around its own axis.

2. DYNAMICS OF THE SATELLITE

The motion of the satellite (S) is achieved on an elliptical trajectory in the plane containing the center of Earth.

The attitude of a satellite (Euler angles – θ, φ and ψ) can be defined by means of the quaternion vector $\boldsymbol{q} = [q_1 \ q_2 \ q_3]^T$.

The significances of these angles are similar to the ones expressing the attitude of an aircraft with respect to the Earth tied frame: φ is associated to the roll angle, θ – associated to the pitch angle and ψ – associated to the direction angle [18].

The absolute motion of the satellite is described by the equation [13]:

$$J\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}^{\times} J\dot{\boldsymbol{\omega}} = \boldsymbol{u}, \tag{1}$$

where $\vec{\boldsymbol{\omega}}$ is the vector of the satellite's angular velocities, J – the inertia moment, $\boldsymbol{u}$ – the control law for the stabilization of the satellite, while $\boldsymbol{\omega}^{\times}$ is the following matrix:

$$\boldsymbol{\omega}^{\times} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}, \tag{2}$$

with $\omega_1, \omega_2, \omega_3 -$ the components of the satellite's angular velocity $\left(\vec{\boldsymbol{\omega}} = \omega_1 \vec{i} + \omega_2 \vec{j} + \omega_3 \vec{k}\right)$.

The stabilizing control law can be chosen as [13]:

$$\boldsymbol{u} = -K_p \boldsymbol{q} - K_d \boldsymbol{\omega} + \boldsymbol{\omega}^{\times} J \boldsymbol{\omega}, \tag{3}$$

such that the satellite's dynamics (1) gets the form:

$$J\dot{\boldsymbol{\omega}} = -K_p \boldsymbol{q} - K_d \boldsymbol{\omega}, \tag{4}$$

with K_p and K_d – gain matrices having positive terms.

Putting together the equation (4) and the differential equations of the quaternions [19]:

$$\begin{aligned} \dot{\boldsymbol{q}} &= -\frac{1}{2}\boldsymbol{\omega}^{\times}\boldsymbol{q} + \frac{1}{2}q_4\boldsymbol{\omega}, \\ \dot{q}_4 &= -\frac{1}{2}\boldsymbol{\omega}^{T}\boldsymbol{q}, \end{aligned} \tag{5}$$

one obtains the equations of the closed loop control system:

$$\begin{aligned} J\dot{\boldsymbol{\omega}} &= -K_p\boldsymbol{q} - K_d\boldsymbol{\omega}, \\ \dot{\boldsymbol{q}} &= -\frac{1}{2}\boldsymbol{\omega}^{\times}\boldsymbol{q} + \frac{1}{2}q_4\boldsymbol{\omega}, \\ \dot{q}_4 &= -\frac{1}{2}\boldsymbol{\omega}^{T}\boldsymbol{q}. \end{aligned} \tag{6}$$

If one chooses the matrices K_p and K_d as:

$$\begin{aligned} K_p &= k_p J = \mathrm{diag}\left[k_{p1}\ \ k_{p2}\ \ k_{p3}\right] J, \\ K_d &= k_d J = \mathrm{diag}\left[k_{d1}\ \ k_{d2}\ \ k_{d3}\right] J, \end{aligned} \tag{7}$$

the closed loop control system is asymptotically stable [20] and the equations (6) become:

$$\begin{aligned} \dot{\boldsymbol{\omega}} &= -k_p\boldsymbol{q} - k_d\boldsymbol{\omega}, \\ \dot{\boldsymbol{q}} &= -\frac{1}{2}\boldsymbol{\omega}^{\times}\boldsymbol{q} + \frac{1}{2}q_4\boldsymbol{\omega}, \\ \dot{q}_4 &= -\frac{1}{2}\boldsymbol{\omega}^{T}\boldsymbol{q}. \end{aligned} \tag{8}$$

These equations describe the automatic control system of the satellite's attitude around its own axis; thus, the motion's control will be achieved by using the quaternion and the angular velocities' vectors.

3. DESIGN OF THE NEW ARCHITECTURE FOR THE CONTROL OF THE SATELLITES' ATTITUDE

Denoting with $\boldsymbol{x} = [\boldsymbol{q}\ ,\boldsymbol{\omega}]^T$ – the state vector of the system (8), then a saturation function depending on the vector $\boldsymbol{x}$(6x1) has the form [21]:

$$\mathrm{sat}\left(\boldsymbol{x}\right) = \left[\mathrm{sat}_1\left(x_1\right)\ \mathrm{sat}_2\left(x_2\right)\ \ldots\ \mathrm{sat}_6\left(x_6\right)\right]^T, \tag{9}$$

where

$$\mathrm{sat}_i\left(x_i\right) = \begin{cases} x_i^- , & x_i < x_i^- , \\ x_i , & x_i \in \left[x_i^- , x_i^+\right] \\ x_i^+ , & x_i > x_i^+ . \end{cases} \tag{10}$$

A command law for the control of the satellite around its own axis, having two nonlinear functions (saturation type) is described by the equation:

$$\boldsymbol{u} = -K_p \text{sat}(P\boldsymbol{q}) - K_d \boldsymbol{\omega} + \boldsymbol{\omega}^{\times} J \boldsymbol{\omega}, \tag{11}$$

with

$$K_p = k_p J = \text{diag}\left[k_{p1} \ \ k_{p2} \ \ k_{p3}\right] J, K_d = k_d J, P = \text{diag}\left[p_1 \ \ p_2 \ \ p_3\right], \tag{12}$$

where $k_{pi}, p_i, i = \overline{1,3}$ and k_d are positive constants (control law's parameters) to be determined; this is equivalent with the control law (4). Using this, the closed loop control system is described by the equations:

$$\begin{aligned} J\dot{\boldsymbol{\omega}} &= -K_p sat(P\boldsymbol{q}) - K_d \boldsymbol{\omega}, \\ \dot{\boldsymbol{q}} &= -\frac{1}{2}\boldsymbol{\omega}^{\times}\boldsymbol{q} + \frac{1}{2} q_4 \boldsymbol{\omega}, \\ \dot{q}_4 &= -\frac{1}{2}\boldsymbol{\omega}^T \boldsymbol{q}. \end{aligned} \tag{13}$$

We consider the motion of a mini-satellite which performs a typical maneuver (a complete cycle) around its own axis (with constrained angular speed); the three phases of the motion are [21]:

1) the accelerated angular motion;
2) the uniform angular motion;
3) the braked motion.

In the first phase, S makes a rotation around its awn axis with the angular velocity $\boldsymbol{\omega} \approx t$, $\dot{\boldsymbol{\omega}} \approx \boldsymbol{u} = \boldsymbol{M}$ ($\boldsymbol{M}$ – the equivalent gyroscopic moment) with the components of the vector $\boldsymbol{M}$ being constant ($M_i = \text{const.}$).

In the second phase, S is rotated also around its awn axis with the angular velocity $\boldsymbol{\omega} = \boldsymbol{\omega}^* \cong \text{const.}, \dot{\boldsymbol{\omega}} \approx \boldsymbol{u} \cong 0$ (response to a null input), while, in the third phase, S is rotated around its awn axis with the angular velocity $\boldsymbol{\omega} \approx t, \dot{\boldsymbol{\omega}} \approx \boldsymbol{u} = -\boldsymbol{M}$.

A gyro system consisting of four control moment gyros, is used to control the satellite; both the speed gyros' saturation and the saturation of the actuators will be taken into account. We also consider that the moments generated by the actuators (gyros) $-M_i$ satisfy the inequality:

$$|M_i| \le \overline{M}_i, i = \overline{1,m}, \tag{14}$$

where m is the number of the actuators and $\overline{M}_i -$ the maximum value of the moment M_i; generally, $m > 3$ or $m - 3 \ge 1$ to be assured that there is at least one actuator that works under the failure of three of them.

The control vector is $\boldsymbol{u} = [u_1 \ \ u_2 \ \ u_3]^T$, with $u_1, u_2, u_3 -$ the control moments acting around the fixed axes of the satellite linked frame.

Also, we denote with $\boldsymbol{M} = [M_1 \ \ M_2 \ \cdots \ M_m]^T$ – the vector of the moments generated by the actuators with saturation and with $\boldsymbol{M_g} = \left[M_{g1} \ \ M_{g2} \ \cdots \ M_{gm}\right]^T -$ the vector of the moments generated by the actuators without saturation (see fig. 1). In these circumstances, the vector $\boldsymbol{u}$ can be expressed as follows:

$$\begin{aligned}\boldsymbol{u} = B\boldsymbol{M} &= [b_1\ b_2\ \dots\ b_m][M_1\ M_2\ \dots\ M_m] = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ b_{31} & b_{32} & \cdots & b_{3m} \end{bmatrix}\begin{bmatrix} M_1 \\ M_2 \\ \vdots \\ M_m \end{bmatrix} = \\ &= \begin{bmatrix} b_{11}M_1 + b_{12}M_2 + \cdots + b_{1m}M_m \\ b_{21}M_1 + b_{22}M_2 + \cdots + b_{2m}M_m \\ b_{31}M_1 + b_{32}M_2 + \cdots + b_{3m}M_m \end{bmatrix} = \underbrace{\begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix}}_{b_1} M_1 + \underbrace{\begin{bmatrix} b_{12} \\ b_{22} \\ b_{32} \end{bmatrix}}_{b_2} M_2 + \cdots + \underbrace{\begin{bmatrix} b_{1m} \\ b_{2m} \\ b_{3m} \end{bmatrix}}_{b_m} M_m = \\ &= b_1 M_1 + b_2 M_2 + \cdots + b_m M_m\,;\end{aligned} \tag{15}$$

B is the transformation matrix (the distribution matrix of the vector $\boldsymbol{M}$ around the command axes of the satellite), while $b_{ij}\,, i = \overline{1,3},\ j = \overline{1,m},$ are the weights of the induced moments by the moment of the actuator j (M_j) around the axes i of the satellite; $b_i^T b_i = 1.$ At least three vectors b_i from the m vectors must be linearly independent in order to achieve the control of the satellite after its three axes.

If $B^+ = \left(B^T B\right)^{-1} B^T$ is the pseudo-inverse of the transformation matrix B, then:

$$\boldsymbol{M}_g = B^+ \boldsymbol{u}_c\,. \tag{16}$$

In the ideal case when $\boldsymbol{M} = \boldsymbol{M}_g\,,$ the matrix B is invertible and, therefore, $B^+ = B^{-1};$ in that case, $\boldsymbol{u} = \boldsymbol{u}_c$, i.e. S is controlled by the command vector $\boldsymbol{u}_c$. If the saturation appears to at least one of the actuators, then $\boldsymbol{M} \neq \boldsymbol{M}_g\,, \boldsymbol{u} \neq \boldsymbol{u}_c\,.$

Let us denote with $\dot{\theta}_{\max}$ – the constrained maximum rate of S during its rotation around its own axis (rotation described by the equations (13)), with the saturation of the gyros and $\boldsymbol{q}(0) \neq 0\,, \boldsymbol{\omega}(0) = 0;$ also, let us suppose that t^* is the time moment at which the conditions $\left|p_i q_i\left(t^*\right)\right| = 1$ and $q_i(0) q_i(t) > 0\,, i = \overline{1,3},\ \forall\, t \in \left[0\,, t^*\right]$ are fulfilled. In [21] it is proved that if one chooses:

$$k_{pi} = k_d \frac{\left|q_i(0)\right|}{\left\|\boldsymbol{q}(0)\right\|}\dot{\theta}_{\max}\,, K_p P = kJ\,, \tag{17}$$

where $k = \operatorname{diag}\left[k_{p1}p_1\ \ k_{p2}p_2\ \ k_{p3}p_3\right],$ with $k_{pi}\,, p_i$ – positive constants, then, for $t \in \left[0\,, t^*\right],$ the following properties are true: 1) the motion of the satellite is achieved around an own axis $\boldsymbol{q}(0)$; 2) the constrained maximum rate of S is finite, i.e. $\left\|\boldsymbol{\omega}(t)\right\| \le \dot{\theta}_{\max}\,;$ 3) the attitude error $\left\|\boldsymbol{q}(t)\right\|$ is monotonically decreasing; 4) at the time moment t^*, one has: $\left|p_i q_i\left(t^*\right)\right| = 1\,, i = \overline{1,3}.$

We choose the control law $\boldsymbol{u}_c$, with nonlinear element having saturation, which ensures the achievement of a typical maneuver by the satellite;

$$\boldsymbol{u}_c = -K_p \operatorname{sat}(P\boldsymbol{q}) - K_d \boldsymbol{\omega}\,, \tag{18}$$

with K_p, K_d and P of forms (12); the relationships between the control law's parameters (k_{pi} and k_d) are (17).

Putting together the equations (15), (16), (18), the equation describing the dynamics of the satellite, the equations of the quaternions – (5) and the correlation formulas between the components of the quarternion vector $\boldsymbol{q}$ and the satellite attitude angles [19]:

$$\begin{aligned} \theta &= \operatorname{atan}\frac{2(q_1q_3 + q_2q_4)}{-q_1^2 - q_2^2 + q_3^2 + q_4^2}, \\ \varphi &= \operatorname{asin}\left[2\left(q_1q_4 - q_2q_3\right)\right], \\ \psi &= \operatorname{atan}\frac{2(q_1q_2 + q_3q_4)}{-q_1^2 + q_2^2 - q_3^2 + q_4^2}, \end{aligned} \tag{19}$$

one obtains the automatic system for the control of S around its awn axis – fig. 1.

By using the criteria in [21], the nonlinearity of the actuators' saturation under normalized form is described by the next two equations:

$$\sigma(\boldsymbol{q}, \omega) = \max_i \left|\frac{M_{gi}}{\overline{M}_i}\right|, i = \overline{1, m}; \tag{20}$$

$$\boldsymbol{M} = \operatorname*{sat}_{\sigma}\left(\boldsymbol{M}_g\right) = \begin{cases} \boldsymbol{M}_g, & \sigma(\boldsymbol{q}, \omega) \le 1, \\ \overline{\boldsymbol{M}}_\sigma = \dfrac{\boldsymbol{M}_g}{\sigma(\boldsymbol{q}, \omega)}, & \sigma(\boldsymbol{q}, \omega) > 1. \end{cases} \tag{21}$$

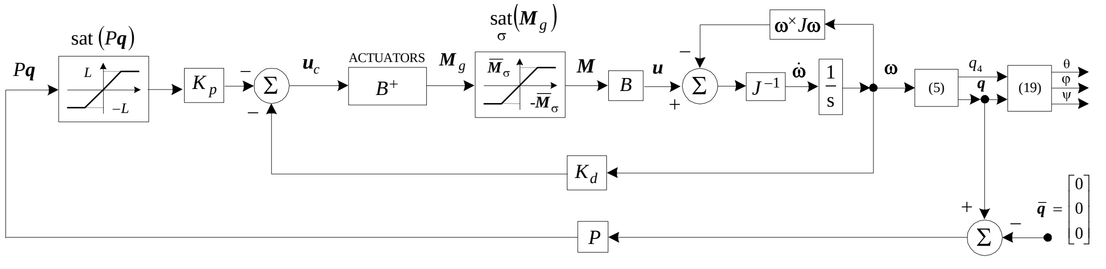


Fig. 1 – The automatic system for the control of the satellite around its awn axis

4. NUMERICAL SIMULATION AND CONCLUSIONS

In this section, the satellite's attitude control system (fig. 1) is the software implemented and validated in Matlab/Simulink environment, for the case of a mini-satellite.

The attitude of the satellite (the angles θ, φ and ψ) will be controlled by using a control law mainly based on a proportional-integrator component with respect to the quaternion vector $\hat{\boldsymbol{q}} = [q_1\ q_2\ q_3\ q_4]^T$ and to the satellite's angular velocity vector (ω).

According to (5), the modification of the vector $\boldsymbol{\omega}$ leads to other expressions of the quaternion vector $\boldsymbol{q}$, this resulting in the change of the satellite attitude (see eq. (19)).

As we already stated above, the motion of the satellite is described by three phases: the accelerated angular motion, the uniform angular motion and the braked motion [21]. The control system is also described by four actuators ($m = 4$) which are placed such that the matrix B has the form [21]:

$$B = \begin{bmatrix} \cos\alpha & -\cos\alpha & \cos\alpha & -\cos\alpha \\ \sin\alpha & 0 & -\sin\alpha & 0 \\ 0 & -\sin\alpha & 0 & \sin\alpha \end{bmatrix}, \alpha = 45\,\text{deg}. \tag{22}$$

also, the following values are used:
$\overline{M}_i = 0.25\,\text{Nm}, \dot{\theta}_{\max} = 0.3\,\text{deg/s}$, $J = \text{diag}\,[20\ 20\ 15]\,\text{kg}\cdot\text{m}^2$,
$\hat{\boldsymbol{q}}(0) = [0.45\ 0.5\ -0.5\ 0.5454]^T, \boldsymbol{\omega}(0) = 0_{3\times1}, \boldsymbol{q}(0) = [q_1(0)\ q_2(0)\ q_3(0)]^T$; $\boldsymbol{\omega}$ and $\boldsymbol{\omega}^\times$ have the forms $\boldsymbol{\omega} = [\omega_1\ \omega_2\ \omega_3]^T$ and (2), respectively.

The nonlinearity $\underset{\sigma}{\text{sat}}(\boldsymbol{M}_g)$ is described by the equation (21), with $\sigma(\boldsymbol{q}, \boldsymbol{\omega})$ of form (20); $M_{gi}, i = \overline{1,4}$, are the components of the vector $\boldsymbol{M}_g$ and their values are introduced every iteration in the equation (20) for the calculation of $\sigma(\boldsymbol{q}, \boldsymbol{\omega}) = \sigma(\boldsymbol{q}(t), \boldsymbol{\omega}(t))$. $\boldsymbol{M}_g$ is calculated by means of equation (16).

We chose $\xi = 0.707$ and $\omega_n = 0.1\,\text{rad/s}$; using these values, one obtains: $K_d = k_d J = 2\xi\omega_n J, K_p P = kJ = 2\omega_n^2 J$; it resulted $k_d = 2\xi\omega_n = 0.1414, k = 2\omega_n^2 = 0.02$ and

$$K_d = \text{diag}\,[2.828\ \ 2.828\ \ 2.121], K_p P = \text{diag}\,[0.4\ \ 0.4\ \ 0.3]. \tag{23}$$

The matrix K_p is determined with $K_p = k_p J$, where $k_p = \text{diag}\,[k_{p1}\ \ k_{p2}\ \ k_{p3}]$. The coefficients $k_{pi}, i = \overline{1,3}$, are calculated using (17); it yields $K_p = 10^{-2}\text{diag}\,[46\ \ 51\ \ 38]$. Matrix P is determined by means of equation (23) with K_p presented above; it results:

$$P = kK_p^{-1}J = \text{diag}\,[87.14\ \ 78.43\ \ 78.43]. \tag{24}$$

The nonlinearity $\text{sat}_i(P\boldsymbol{q}) = \text{sat}(\tilde{P}_i)$ is described by the equation:

$$\text{sat}_i(P\boldsymbol{q}) = \text{sat}(\tilde{P}_i) = \begin{cases} -L_i, & \tilde{P}_i < L_i, \\ \tilde{P}_i, & \tilde{P}_i \in [-L_i, L_i], \\ L_i, & \tilde{P}_i > L_i, \end{cases} \tag{25}$$

with $L = [L_1\ \ L_2\ \ L_3]^T$. $L_i, i = \overline{1,3}$, are calculated from the condition associated to the steady regime $(\boldsymbol{u}_c = 0)$, i.e.

$$K_p L = K_d \|\boldsymbol{\omega}_{\max}\| = K_d \dot{\theta}_{\max} \tag{26}$$

or

$$k_p L = k_d \dot{\theta}_{\max}, \tag{27}$$

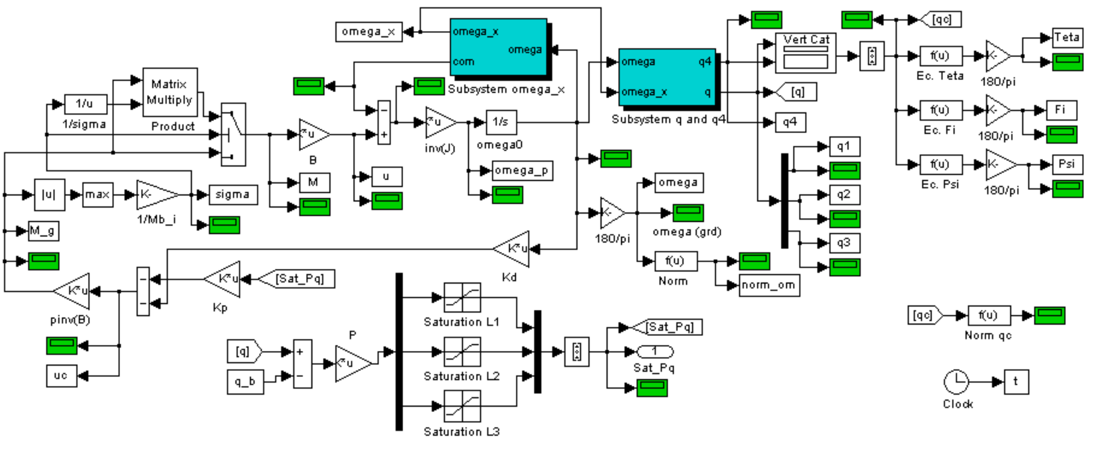
equivalent with the following one:

$$L_i = \frac{k_d}{k_{pi}}\dot{\theta}_{\max} \overset{(17)}{=} \frac{\|\boldsymbol{q}(0)\|}{|q_i(0)|}, i = \overline{1,3}. \tag{28}$$

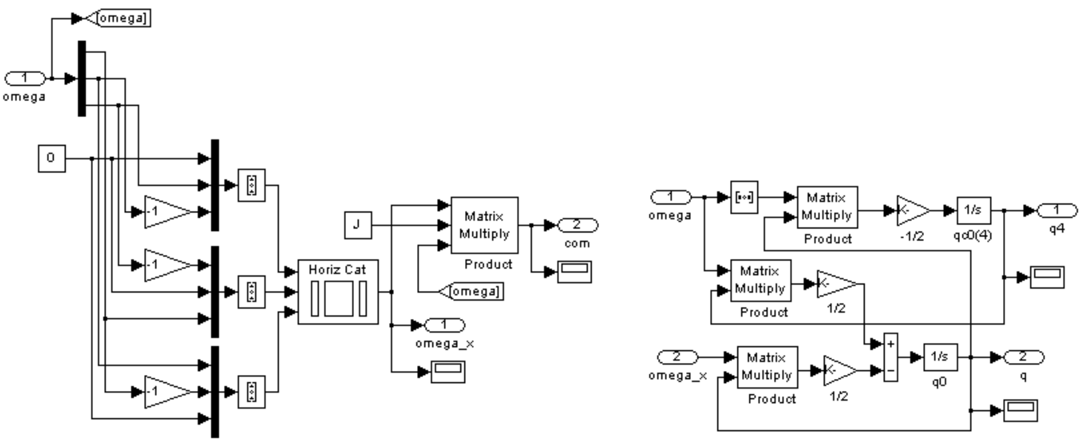
The Matlab/Simulink model used in the validation process of the control system in fig. 1 is the one presented in fig. 2.a; this model includes two sub-systems:

1) "Subsystem omega_x" – fig. 2.b (mainly based on equation (2));

2) "Subsystem q and q4" – fig. 2.c (used for the calculation of the quaternion $\hat{\boldsymbol{q}}$ by using information from the vector of angular velocities $(\boldsymbol{\omega})$ – equation (5)).



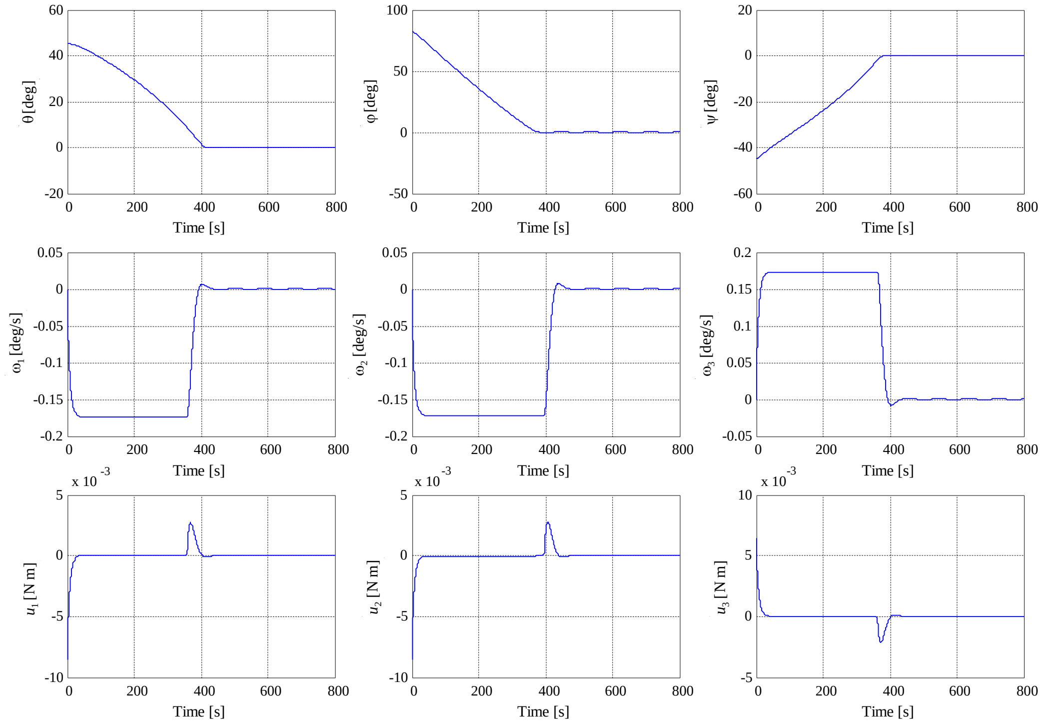
a.



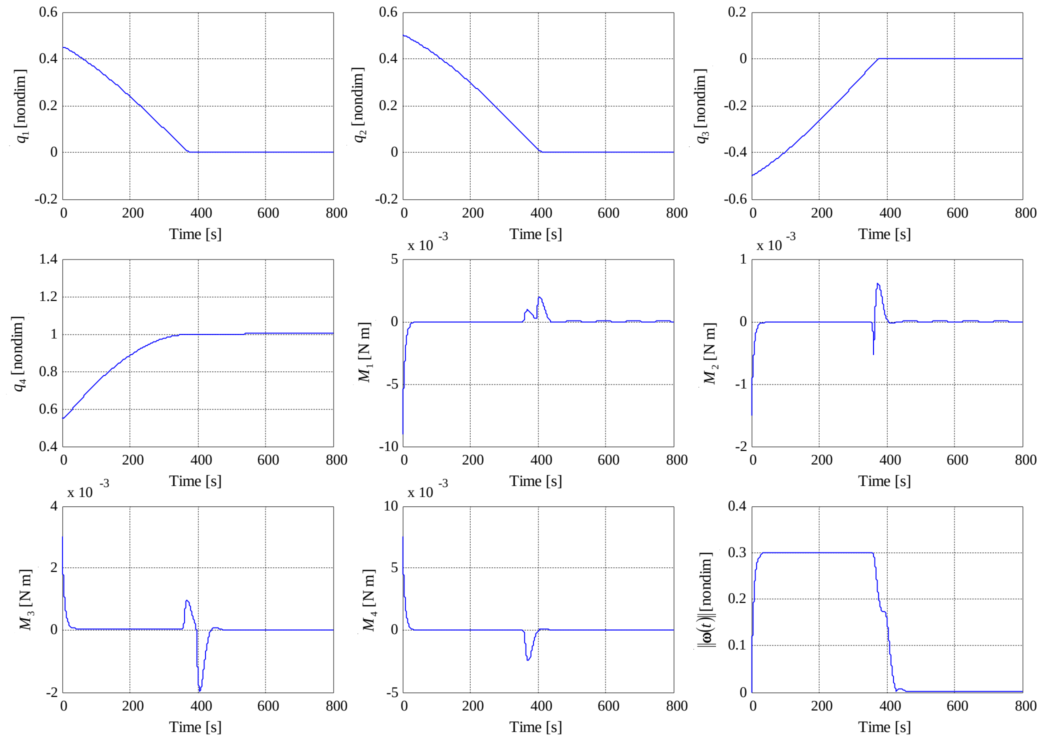
b. c.

Fig. 2 – Matlab/Simulink models for the satellite's attitude control system

In fig. 3.a we present the time histories of the satellite's attitude angles (θ, φ, ψ), angular velocities $(\omega_1, \omega_2, \omega_3)$ and of the three components of the control law $\boldsymbol{u}$; in fig. 3.b we present the time histories associated to the components of the quaternion $\hat{\boldsymbol{q}}(q_1, q_2, q_3, q_4)$, the components of the vector $\boldsymbol{M}(M_1, M_2, M_3, M_4)$ and the norm $\|\omega(t)\|$.



a.



b.

Fig. 3 – Time histories of the main variables associated to the satellite's attitude control system

As one can see in fig. 1.a, the control of the satellite's attitude is achieved by controlling the quaternion vector $\boldsymbol{q}$ and the satellite's angular velocity vector $(\boldsymbol{\omega})$. Actually, the first component of the designed control law is of proportional type and ensures the convergence of the quaternion vector $\boldsymbol{q}$ to the desired quaternion $\bar{\boldsymbol{q}} = [0\ 0\ 0]^T$, while the second component of the control law cancels the deviation of the angular velocity vector from the one associated to the reference frame.

The closed loop control system has good convergence, global asymptotically stability and $\boldsymbol{q}(t) \to 0, \boldsymbol{\omega}(t) \to 0$; on the other hand, as one can notice from fig. 3, the cancellation of the vectors $\boldsymbol{q}$ and $\boldsymbol{\omega}$ leads to the cancellation of other variables: the deviation of the satellite's attitude angles with respect to their desired values, the components of the vector $\boldsymbol{M}(M_1, M_2, M_3, M_4)$ and the three components of the control law $\boldsymbol{u}$.

The closed loop control system has been proved to be characterized by convergence and global asymptotic stability.

ACKNOWLEDGEMENTS

This work was supported by the project "Computational Methods in Scientific Investigation of Space", project number 72/29.11.2013, of the Romanian National Authority for Scientific Research, Program for Research - Space Technology and Advanced Research – STAR.

REFERENCES

[1] N. Jovanovic, *Aalto-2 satellite attitude control system*. Thesis of Master – Science in Technology. Aalto University, School of Electical engineering, 2014.

[2] J. Bouwmeester and J. Guo, Survey of worldwide pico- and nano- satelitte missions, distributions and subsystem technology. *ActaAstronautica*, vol. **67**, pp. 854-862, 2010.

[3] V. F. Lavet, *Study of passive and active attitude control systems for the OUFTI nanosatellites*. Thesis of Master – Engineering Physics. University of Liege, Faculty of Applied Sciences, 2010.

[4] K. Kreutz, Manipulator control by exact linearization, *IEEE Transactions on Automatic Control*, vol. **34**, no. 7, pp. 763-767, 1989.

[5] R. Bayadi and R. N. Banavar, Almost global attitude stabilization of a rigid body for both internal and external actuation schemes, *European Journal of Control*, vol. **20**, pp. 45-54, 2014.

[6] T. Lee, Robust adaptive attitude tracking on so(3) with an application to a quadrotor UAV, *IEEE Transactions on Control Systems Technology*, vol. **21**, no. 5, pp. 1924-1930, 2013.

[7] N. Chaturvedi, A. Sanyal and N. McClamroch, Rigid-body attitude control, *IEEE Control Systems Magazine*, vol. **31**, no. 3, pp. 30-51, 2011.

[8] R. Mahony, T. Hamel and P. J.-M., Nonlinear complementary filters on the special orthogonal group, *IEEE Transactions on Automatic Control*, vol. **53** , Issue: 5, pp. 1203-1218, 2008.

[9] Z. Zhu, Y. Xia, M. Fu, Adaptive sliding mode control for attitude stabilization with actuator saturation, *IEEE Transactions on Industrial Electronics*, vol. **58**, pp. 4898-4907, 2011.

[10] C. G. Mayhew, R. G. Sanfelice and A. R. Teel, Robust global asymptotic attitude stabilization of a rigid body by quaternion-based hybrid feedback, *Joint 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference*, 2009.

[11] L. Benziane, A. Benallegue, Y. Chitour and A. Tayebi, Inertial Vector Based Attitude Stabilization of Rigid Body Without Angular Velocity Measurements, *Mathematics – Optimization and control*, 2015.

[12] D. Thakur, *Adaptation, gyro-free stabilization, and smooth angular velocity observers for attitude tracking control applications*, Ph.D. dissertation, The University of Texas at Austin, August 2014.

[13] L. Benziane, A. Benallegue and A. Tayebi, Attitude stabilization without angular velocity measurements, in *IEEE International Conference on Robotics & Automation (ICRA)*, pp. 3116–3121, 2014.

[14] A. Tayebi, A. Roberts and A. Benallegue, Inertial vector measurements based velocity-free attitude stabilization, *IEEE Transactions on Automatic Control*, vol. **58**, no. 11, pp. 2893-2898, 2013.

[15] B. Xiao, Q. Hu and P. Shi, Attitude stabilization of spacecrafts under actuator saturation and partial loss of control effectiveness, *IEEE Transactions On Control Systems Technology*, vol. **21**, pp. 2251-2263, 2013.

[16] R. Schlanbusch, E. I. GrÃžtli, A. Loria and P. J. Nicklasson, Hybrid attitude tracking of rigid bodies without angular velocity measurement, *Systems & Control Letters*, vol. **61**, pp. 595–601, 2012.

[17] S. M. Joshi, A. G. Kelkar and J. T. Wen, Robust Attitude Stabilization of Spacecraft using Quaternion Feedback. *IEEE Transactions on Automatic Control*, vol. **40**, no. 10, pp. 1800-1803 , 1995.

[18] C. Heiberg, D., Bailey and B. Wie, Precision Spacecraft Pointing using Single-Gimbal Control Moment Gyroscopes with Disturbances. *Journal of Guidance, Control, and Dynamics*, vol. **23**, no. 1, pp. 77-85, 2000.

[19] J. T. Wen and K. K. Delgado, The Attitude Control Problem. *IEEE Transactions on Automatic Control*, vol. **36**, no. 10, pp. 1372-1379, 1991.

[20] B. Wie, H. Weiss and A. Arapostathis, Quaternion Feedback Regulator for Spacecraft Eigenaxis Rotations. *Journal of Guidance, Control and Dynamics*, vol. **18**, no. 6, pp. 375-380, 1989.

[21] B. Wie and J. Lu, Feedback Control Logic for Spacecraft Eigenaxis Rotations Under Slew Rate and Control Constraints. *Journal of Guidance, Control and Dynamics*, vol. **18**, no. 6, pp. 1372-1379, 1995.