The torsion of the multicell sections

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Abstract: *The paper is focused on the stress analysis of thin-walled multicell sections subjected to pure torsion. The shear flow and stiffness characteristics of the cross section for torsion are given. Example: aircraft wing section. The theory for thin-walled closed sections used in this paper was developed by Bredt [3]. The shear flows obtained are used in the design of skins and interior webs, ribs and fasteners at skin splices, skin web junctions and the joints where the ribs meet the skin or webs.*

Key Words: multicell section, torsion, stress analysis, shear flow, shear stress, software

1. INTRODUCTION

The problems involving torsion are common with aircraft structures. The material covered wing and fuselage of the airplane are basically thin walled tubular structures subjected to large torsion moments under many flight and landing conditions; therefore the necessary knowledge about the torsion stresses and distortions of components is particularly necessary with aircraft structural design (See Figure 1).

The objective is to calculate the stiffness and shear flows for a wing with one or more cells in closed cross sections for the conceptual design. This is the solution for the majority of the wings and tail planes.

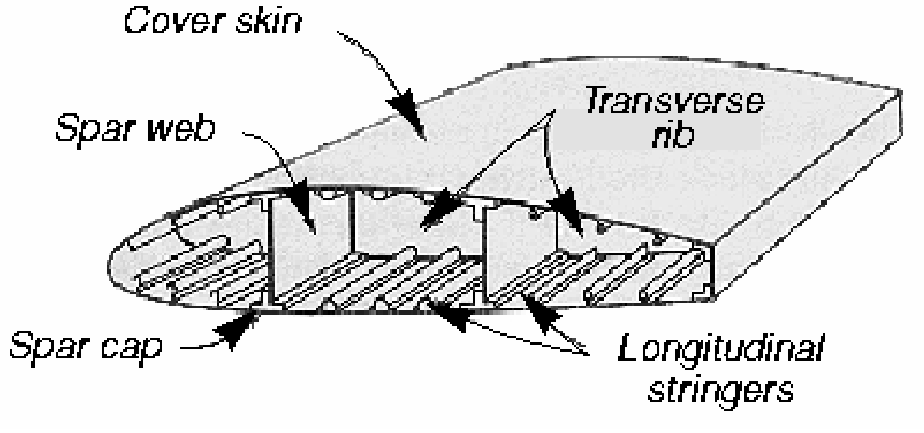

Figure 1 Classical wing structure

The thinner wing boxes used on fighters and supersonic aircraft utilize multi-cell boxes which provide a much more efficient structure (See Fig.2)

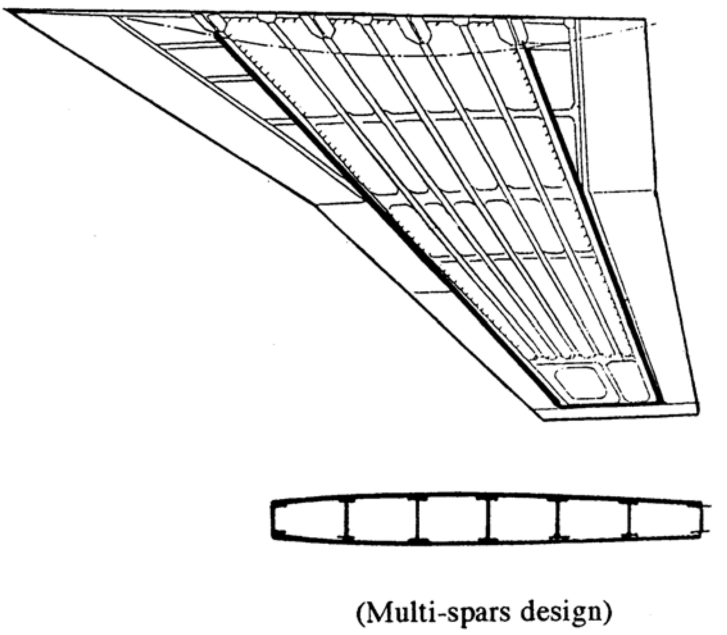

Figure 2 Fighter wing (multi-cell box)

In thin-walled beams the wall thickness is assumed to be much smaller than a representative dimension of the cross sections (f.i. chord) $t/c \ll 1$.

Consider a thin-walled shell structure of arbitrary constant cross section as shown in Fig.3. The area bounded by the outer wall is subdivided into an arbitrary number of cells, which are separated by thin webs. It is assumed that cross-sectional changes during twisting are prevented by transverse stiffening members (ribs or frames) which are considered to be rigid within their planes (so that the cross section is maintained unchanged during loading) but perfectly flexible with regard to deformations normal to their planes [1].

The pure torsion case is considered. In the analysis we assume that no axial constraint effects are present and that the shape of the wing (or tail plane) section remains unchanged by the load application.

In the absence of any axial constraint there is no development of direct stress in the wing section so that only shear stress are present which resulting in the shear flows **q**. It follows that the presence of booms does not affect the analysis in the pure torsion case [1].

The theory for the thin-walled closed sections was developed by Bredt [1], [7].

2. TORSION OF THE MULTICELL SECTIONS

Let's consider a wing subjected to torsion. The torque M_t will be divided over the several boxes. The wing section shown in Fig.3 comprises **n** cells and carries a torque M_t which generates individual but unknown torques in each of the **n** cells. The case of pure torsion is studied. Each cell develops a constant shear flow q_1, q_2,..q_i,..q_n. The shear flow in any interior wall is equal to the difference of the shear flows in the adjoining exterior walls. A structure with **n** cells will have n unknown shear flows [1].

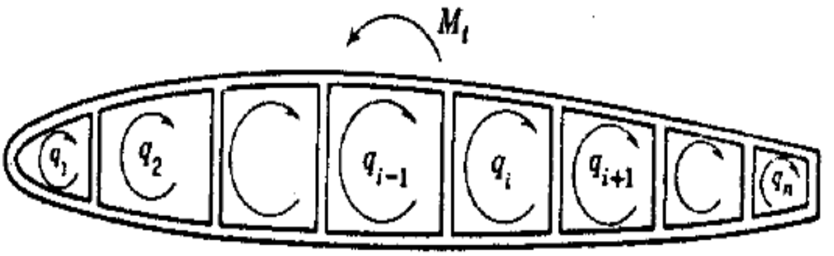

Figure 3. Multicell wing section subjected to torsion

The shear flows must be in equilibrium with the applied torque.

For equilibrium, the applied torque must equal the sum of the torques in the cells:

$$M_t = \sum_{i=1}^{n} 2A_i\, q_i \tag{1}$$

where A_i is the area enclosed by the midline of the $\mathbf{i}_{th}$ cell (Eq. 1 is called the Bredt-Batho formula).

Equation (1) is sufficient for the solution of the special case of a single cell section which is statically determinate. For an **n**-cell section additional equations are required because the structure is statically indeterminate. These are obtained by considering the rate of twist in each cell and the compatibility of displacement condition that all **n** cells possess the same rate of twist $d\theta / dz$; this arises directly from the assumption of an undistorted cross-section [1].

Consider the i_{th} cell of the wing section shown in Fig.3.

The angle of twist per unit length θ is given by the equation (2) (eq.8-65 from [1])

$$\theta = \frac{1}{2A_i G_1} \oint_i \frac{q\,ds}{t^\bullet} \tag{2}$$

where G_1 is an arbitrary reference modulus (it is possible that the shear module for the wall and the webs G_1, G_2, G_3, G_{12}, etc. will differ if the cells are manufactured of different materials) and $t^\bullet$ is a modulus-weighted thickness defined by

$$t^\bullet = \frac{G}{G_1} t \tag{3}$$

When the cross section is homogeneous, the subscript on **G** and the asterisk on **t** may be dropped in Eq.2.

When only a torque is applied, q is constant in each wall segment and θ (angle of twist per unit length) may be written:

$$\theta = \frac{1}{2A_1 G_1} \left(q_i \oint_i \frac{ds}{t^\bullet} - q_{i-1} \int_{\substack{web \\ i-1,i}} \frac{ds}{t^\bullet} - q_{i+1} \int_{\substack{web \\ i,i+1}} \frac{ds}{t^\bullet} \right) \tag{4}$$

where q_i is the constant shear flow around the ith cell and q_{1-1} and q_{1+1} are the shear flows around $_{\text{the}}$ (i-1) th and (iȘ1) the cells, respectively.

Equation (4) may be applied to each of the **n** cells by equating θ for the first and ith cells. We can write (n-1) equations of the form:

$$\frac{1}{A_1}\left(q_1\oint_1 \frac{ds}{t^*} - q_2\int_{web_{1,2}} \frac{ds}{t^*}\right) = \frac{1}{A_i}\left(q_i\oint_i \frac{ds}{t^*} - q_{i-1}\int_{web_{i-1,i}} \frac{ds}{t^*} - q_{i+1}\int_{web_{i,i+1}} \frac{ds}{t^*}\right) \quad (5)$$

By letting **i** run from **2** to **n**.

The general form of Eq. (5) is applicable to multicell sections in which the cells are connected consecutively.

The simultaneous solution of these equations with Eq (1) gives the n unknown values of q_i.

The value of θ can then be found by evaluating Eq. (4) for any of the cells.

The torsional rigidity $G_1 J^*$ of the nonhomogeneous section can then be found from the defining equation

$$G_1 J^* = \frac{M_i}{\theta} \quad (6)$$

It is possible to apply this approach to sandwich structures with honeycomb core (see Figure 4) and for cheking the rigidity criteria for aileron and tailplane as a preliminary step before the flutter analysis [4].

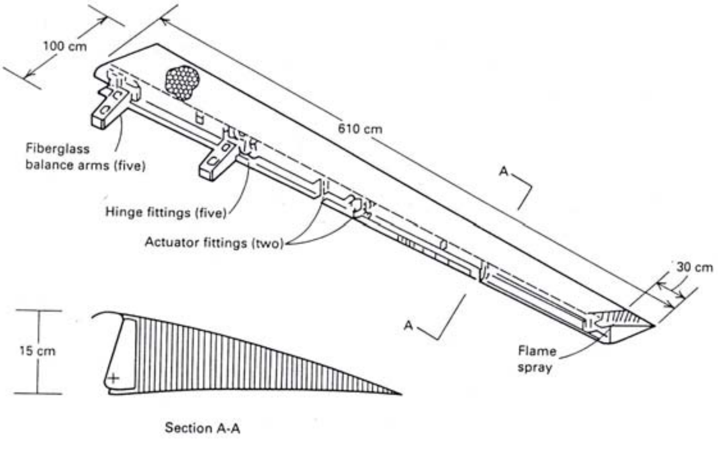

Figure 4 Boeing 767 aileron

3. THE EQUATION SOLUTION

To obtain the solution of the simultaneous algebraic system a lot of methods can be applied. Bruhn [2] used the method of successive approximations. The method provides a simple and rapid approach for finding the shear flow in multiple cells under pure torsion.

The Gauss-Jordan method and / or a special very efficient procedure TRIDIAG [5] are utilized in this work.

4. SOLVING THE PROBLEM

Flow chart of the program

Read input data

↓

Print input data

↓

Determination of the geometrical parameters

↓

Generation of the algebraic equations PX= ACEL

↓

Solving the equation system

↓

Calculus of the shear flows distribution

↓

Calculus of the angle of twist **TETA**

↓

Torsional rigidity **GJ_t**

↓

Calculus of the shear stresses

↓

Printing the results

Figure 5 Flow chart for the software

5. NUMERICAL APPLICATION

Example 1 [1]

Calculate the shear stress distribution in the walls of the three-cell wing section shown in Fig.6, when it is subjected to an anticlockwise torque of 11.3 kNm.

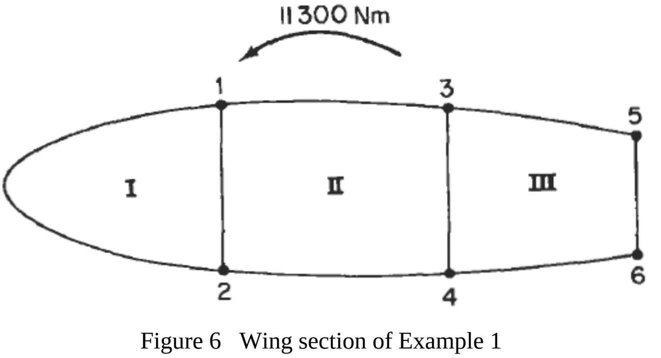

Figure 6 Wing section of Example 1

Input data:

Wall	Length (mm)	Thickness (mm)	G (N/mm^2)
12^0	1650	1.22	27600
12^i	508	2.03	27600
13,24	775	1.22	27600
34	380	1.63	27600
35,46	508	0.92	27600
56	254	0.92	27600

Cell area (mm^2) : A_I = 258000 ; A_{II} = 355000 ; A_{III} = 161000

Output data:

The torsional constant J = .64707641E+05 cm^4
The torsional rigidity GJ = .17859309E+10 daNcm^2

The shear flows in the skin :

QMT(1) = 6.9764 daN/cm.
QMT(2) = 8.6951 daN/cm
QMT(3) = 4.7412 daN/cm.

Shear flows in the spar webs :
Web no.1 6.9764 daN/cm.
Web no.2 1.7186 daN/cm.
Web no.3 -3.9539 daN/cm.
Web no.4 4.7412 daN/cm.

The shear flows and the shear stresses from torque:

No. Cell.	QMT daN/cm	QMTIN daN/cm	TAUE daN/cm^2	TAUI daN/cm^2	TAUIN daN/cm^2
1	6.98	6.98	57.184	57.184	57.184
2	8.70	1.72	71.271	71.271	8.466
3	4.74	-3.95	51.535	51.535	-24.257
4	0.00	0.00	.000	.000	51.535

6. CONCLUSIONS

In spite of its inexactness, this simple and classical method enjoys a considerable popularity even in the present finite-element age.

The shearing stresses in the skins and webs are required to determine whether these components buckle and fail in diagonal tension.

Also, the forces on the fasteners of a joint can be determined once the shear flow that is transmitted through the joint is known. The shear flow gives the force that is transmitted across a unit length of the joint, so that:

$$P_{riv} = \frac{q}{n} \tag{7}$$

where $\mathbf{P_{riv}}$ is the shear force per rivet and **n** is the number of rivets per unit length of the joint.

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