Attitude Dynamics of a Spinning Rocket with Internal Fluid Whirling Motion

Marius Ionut MARMUREANU*,[1], Ion FUIOREA[2]

*Corresponding author
*,[1]Military Equipment and Technologies Research Agency
16, Aeroportului Street, Clinceni, Ilfov, 077060
mmarmureanu@acttm.ro
[2]STRAERO – Institute for Theoretical and Experimental Analysis of Aeronautical Structures, 220, Iuliu Maniu Avenue, Bucharest, 061126, Romania
ifuiorea@straero.ro

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Abstract: *This paper evaluates the impact that helical motion of fluid products of combustion within the combustion chamber of a rocket can have on the attitude dynamics of rocket systems. By developing the study presented by Sookgaew (2004), we determined the configuration of the Coriolis moment components, which catch the impact of the combustion product's whirling motion, for the radial and centripetal propellant burn pattern specific to S-5M and S-5K solid rocket motors. We continue the investigation of the effects of internal whirling motion of fluid products of combustion on the attitude behavior of variable mass systems of the rocket type by examining the spin motion and transverse attitude motion of such systems. The results obtained show that internal fluid whirling motion can cause appreciable deviations in spin rate predictions, and also affects the frequencies of the transverse angular velocity components.*

Key Words: *rockets, variable mass systems, attitude dynamics, S-5 solid rocket motors family.*

1. INTRODUCTION

In studying the effect of mass variation on the behavior of rocket systems, the system of interest is often assumed to comprise two phases at any given instant – a solid phase and a fluid phase. The set of assumption that are traditionally made in the study of these systems includes that there is no "whirling motion" of the fluid phase relative to the solid phase. This assumption stipulates that the internal motion of fluid particles relative to the rocket body is symmetric with respect to the rocket axis. In addition, the vectors of relative velocities for these particles are assumed not to have a transverse component.

This seems unreasonable for a spinning rocket, yet it is a very attractive assumption because it simplifies tremendously the equation of motion. Wang and Eke [1] concluded that the neglect of whirling motion does not affect transverse angular velocity, but does affect the frequencies of these quantities. Anyway, in one of the early investigation of the Star 48 problem, Eke [2] suggested that solid rocket motors with larger than normal combustion chambers and propellant grain, had inherent problem with large traverse motion, and that this phenomenon escaped detection because of the drastic assumptions that are made whenever in the equations of motion for these systems. Otherwise, for different propellant burn types, Sookgaew [3] confirms the statement that when the whirling motion is ignored the spin rate is lower than when the whirling motion is taken into account.

2. EQUATIONS OF MOTION

The studied system is shown in Fig. 1, and consists of two main parts – the rigid portion B, whose mass is expected to diminish with time as the propellant burns, and the fluid products of combustion F. B is taken to be rigid and symmetric about the ζ-axis, and it is assumed to remain so as parts of it are depleted by combustion. O^* is the instantaneous mass center of the system, and always lies on the ζ-axis. An imaginary closed shell, C, of constant shape and volume is attached to B, and serves to delimit the system.

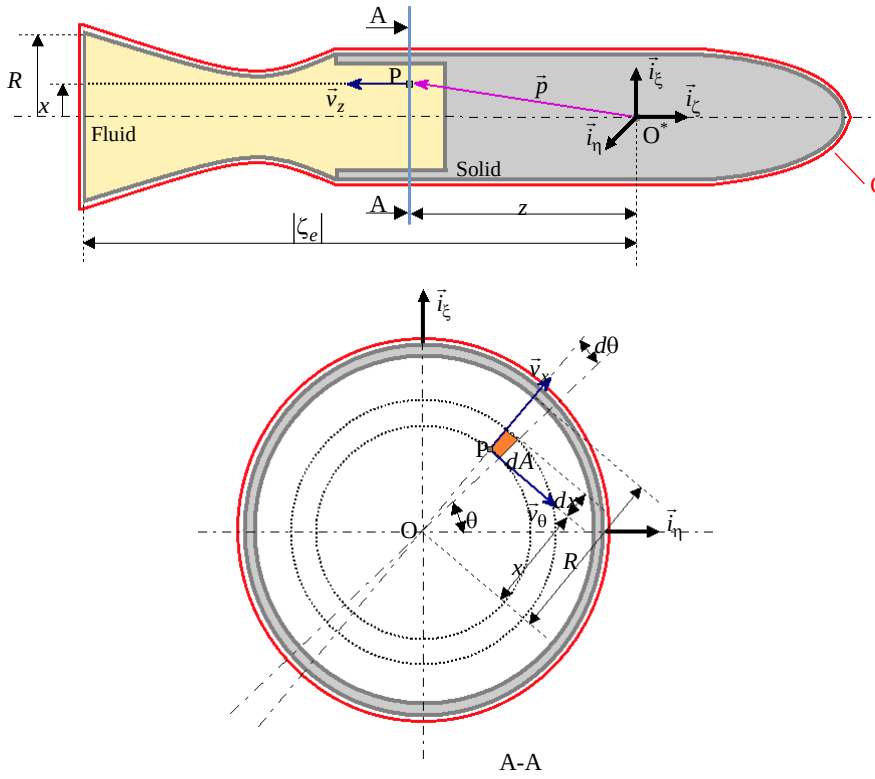

Fig. 1 – Model of variable mass system

One version of the equation of rotational motion for this system is [4], [5], [6], [7]:

$$\tilde{\mathrm{I}}\cdot\vec{\varepsilon}+\vec{\omega}\times\tilde{\mathrm{I}}\cdot\vec{\omega}+\left(\frac{{}^{B}d\tilde{\mathrm{I}}}{dt}\right)\cdot\vec{\omega}+\int_C\rho\left[\vec{p}\times(\vec{\omega}\times\vec{p})\right](\vec{v}_r\cdot\vec{n})dA$$
$$+\frac{{}^{B}d}{dt}\int_C\rho(\vec{p}\times\vec{v}_r)dV+\int_C\rho(\vec{p}\times\vec{v}_r)(\vec{v}_r\cdot\vec{n})dA+\int_C\rho\left[\vec{\omega}\times(\vec{p}\times\vec{v}_r)\right]dV=\vec{M}. \tag{1}$$

In this equation, $\vec{M}$ is the sum of the moments of external forces on the system about the system mass center; $\tilde{\mathrm{I}}$ is the system's instantaneous inertia dyadic, $\vec{p}$ is the position vector from the mass center to a generic particle of the system; ρ is the mass density; $\vec{v}_r$ is the velocity of a generic particle relative to the rigid portion B; $\vec{\omega}$ and $\vec{\varepsilon}$ are the inertial angular velocity and the angular acceleration of the main body B, respectively, and $\vec{n}$ is a unit vector normal to the surface C and pointing outward. The left superscript on the time derivative simply indicates that the derivative is to be taken while the reference frame B is kept fixed.

If we assume that the inertial angular velocity of the main body B has the form,

$$\vec{\omega} = \omega_\xi \vec{i}_\xi + \omega_\eta \vec{i}_\eta + \omega_\zeta \vec{i}_\zeta \tag{2}$$

and that

$$\tilde{\mathrm{I}} = J\left(\vec{i}_\xi \cdot \vec{i}_\xi + \vec{i}_\eta \cdot \vec{i}_\eta\right) + J_\zeta \vec{i}_\zeta \cdot \vec{i}_\zeta \tag{3}$$

where the unit vector basis $\vec{i}_\xi$, $\vec{i}_\eta$ and $\vec{i}_\zeta$ is fixed in B and oriented as in Fig. 1, then, the first three terms of Eq. (1) can be written as

$$\tilde{\mathrm{I}} \cdot \vec{\varepsilon} = J\left(\dot{\omega}_\xi \vec{i}_\xi + \dot{\omega}_\eta \vec{i}_\eta\right) + J_\zeta \dot{\omega}_\zeta \vec{i}_\zeta \tag{4}$$

$$\vec{\omega} \times \tilde{\mathrm{I}} \cdot \vec{\omega} = \left(J_\zeta - J\right)\omega_\zeta\left(\omega_\eta \omega_\xi \vec{i}_\xi - \omega_\xi \vec{i}_\eta\right) \tag{5}$$

and

$$\left(\frac{{}^B d\tilde{\mathrm{I}}}{dt}\right) \cdot \vec{\omega} = \dot{J}\left(\omega_\xi \vec{i}_\xi + \omega_\eta \vec{i}_\eta\right) + \dot{J}_\zeta \omega_\zeta \vec{i}_\zeta \tag{6}$$

The fourth term of Eq. (1) has been evaluated by Wang and Eke [1] and shown a dependence on the velocity field of exhaust gas particles as they cross the nozzle exit plane. For uniform velocity profile with constant exhaust gas velocity u, the fourth term of Eq. (1) can be expressed as

$$\int_C \rho\left[\vec{p} \times \left(\vec{\omega} \times \vec{p}\right)\right]\left(\vec{v}_r \cdot \vec{n}\right) dA = -\dot{m}\left[\left(\zeta_e^2 + \frac{R^2}{4}\right)\left(\omega_\xi \vec{i}_\xi + \omega_\eta \vec{i}_\eta\right) + \frac{R^2}{2}\omega_\zeta \vec{i}_\zeta\right] \tag{7}$$

where the distances ζ_e and R are as shown in Fig. 1. None of these first four terms is affected by the introduction of fluid whirling motion. This is so because the first three terms do not contain the fluid velocity vector at all, and the fourth term only involves the axial component of this velocity. Next, it is assumed that fluid flow within the system is steady. Because the above assumption, the fifth term of Eq. (1) is negligible

$$\frac{{}^B d}{dt}\int_C \rho\left(\vec{p} \times \vec{v}_r\right) dV \cong 0 \tag{8}$$

Each of the last two terms on the left hand side of Eq. (1) contains the vector $\vec{v}_r$, which represents the velocity vector of a generic fluid particle relative to the rocket's main body. Spin motion of the rocket body introduces helical or whirling motion of the fluid particles. By assuming a uniform distribution for the axial exhaust gas velocity over the nozzle exit plane, the sixth term of Eq. (1) becomes [3]

$$\int_C \rho\left(\vec{p} \times \vec{v}_r\right)\left(\vec{v}_r \cdot \vec{n}\right) dA = \dot{m}\frac{R^2}{10}\omega_\zeta \vec{i}_\zeta. \tag{9}$$

Clearly, this term will have some influence on the spin rate but will not affect the transverse components of the rocket's angular velocity.

In this chapter, we determine an explicit expression for the seventh term of Eq. (1). Expressions for $\vec{p}$ and $\vec{v}_r$ are needed for the remaining term of Eq. (1). The position vector, $\vec{p}$, measured from the system mass, can be written as

$$\vec{p} = x\sin\theta\vec{i}_\xi + x\cos\theta\vec{i}_\eta - z\vec{i}_\zeta \tag{10}$$

Hence, the relative velocity of P is (see Fig. 1)

$$\vec{v}_r = (v_x \sin\theta - v_\theta \cos\theta)\vec{i}_\xi + (v_x \cos\theta + v_\theta \sin\theta)\vec{i}_\eta - v_z\vec{i}_\zeta \tag{11}$$

And, if we assume that the fluid density ρ is constant at steady state, then the last term of Eq. (1) becomes

$$T_7 = \int_C \rho[\vec{\omega}\times(\vec{p}\times\vec{v}_r)]dV = \rho\vec{\omega}\times\int_C \left\{\begin{array}{l}\left[zv_\theta\vec{i}_\xi - (zv_x - xv_z)\vec{i}_\eta\right]\sin\theta \\ +\left[(zv_x - xv_z)\vec{i}_\xi + zv_\theta\vec{i}_\eta\right]\cos\theta + xv_\theta\vec{i}_\zeta\end{array}\right\}dV. \tag{12}$$

As shown in Fig. 1, the elementary volume dV can be written as

$$dV = dAdz = xd\theta dxdz \tag{13}$$

Thus Eq. (12) becomes

$$T_7 = \rho\vec{\omega}\times\int_{z_1}^{z_2}\int_x\int_0^{2\pi}\left\{\begin{array}{l}\left[zv_\theta\vec{i}_\xi - (zv_x - xv_z)\vec{i}_\eta\right]\sin\theta \\ +\left[(zv_x - xv_z)\vec{i}_\xi + zv_\theta\vec{i}_\eta\right]\cos\theta + xv_\theta\vec{i}_\zeta\end{array}\right\}xd\theta dxdz \tag{14}$$

Recalling that v_θ is a function of x, but not of θ and z, T_7 becomes, after some simplifications

$$T_7 = 2\pi\rho\left(\int_{z_1}^{z_2}\int_x x^2 v_\theta\, dxdz\right)\left(\vec{\omega}\times\vec{i}_\zeta\right) = 2\pi\rho\left(\int_{z_1}^{z_2}\int_x x^2 v_\theta\, dxdz\right)\left(\omega_\eta\vec{i}_\xi - \omega_\xi\vec{i}_\eta\right) \tag{15}$$

Eq. (15) indicates that T_7 has no $\vec{i}_\zeta$ component; hence, it has no influence on the spin rate.

In the case of radial and centripetal simultaneously burn, as appropriate S-5 solid rocket motors family, the combustion chamber is a cylinder of length L, whose parameter x varies from 0 to r_i and r_e to R_c, as shown in Fig. 2.

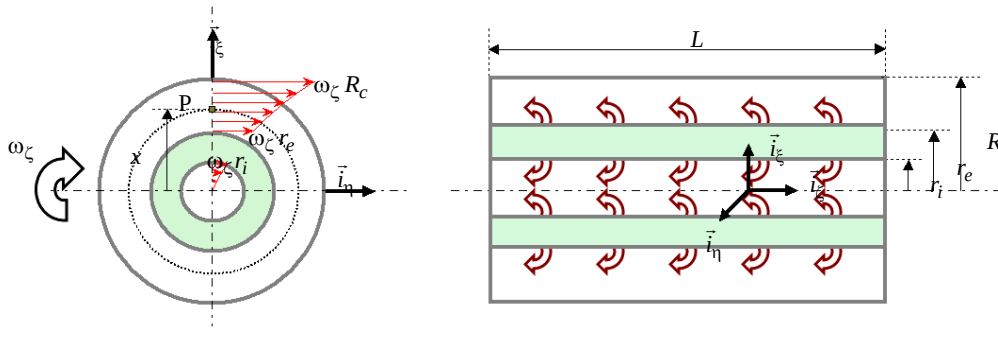

Fig. 2 – The cylinder in radial and centripetal burn

In this case, T_7 can be written as

$$T_7 = 2\pi\rho\left[\int_0^L\left(\int_0^{r_i} x^2 v_\theta^i\, dx + \int_{r_e}^{R_c} x^2 v_\theta^e\, dx\right)dz\right]\left(\omega_\eta\vec{i}_\xi - \omega_\xi\vec{i}_\eta\right) \tag{16}$$

To obtain the transverse inertial velocity for the general fluid particle P, we assume a quadratic function of the distance x from the axis:

$$v_P = ax^2 + bx + c \tag{17}$$

Then, using the boundary condition shown in Fig. 2, we find that

$$\begin{aligned} x = 0 \quad &\Rightarrow \quad v_P^i = 0, \ \frac{dv_P^i}{dx} = 0 \\ x = r_i \quad &\Rightarrow \quad v_P^i = \omega_\zeta r \end{aligned} \tag{18′}$$

$$\begin{aligned} x = r_e \quad &\Rightarrow \quad v_P^e = \omega_\zeta r_e, \ \frac{dv_P^e}{dx} = 0 \\ x = R_c \quad &\Rightarrow \quad v_P^e = \omega_\zeta R_c \end{aligned} \tag{18″}$$

so that $a = \dfrac{\omega_\zeta}{r_i}$, and $b = c = 0$ for v_P^i or $a = \dfrac{\omega_\zeta}{R_c - r_e}$, $b = -\dfrac{2r_e}{R_c - r_e}\omega_\zeta$, and $b = \dfrac{r_e R_c}{R_c - r_e}\omega_\zeta$ for v_P^e. Substituting a, b and c back into Eq. (17), we have

$$v_P^i = \frac{\omega_\zeta}{r_i} x^2 \tag{19}$$

and

$$v_P^e = \frac{\omega_\zeta}{R_c - r_e}\left(x^2 - 2r_e x + r_e R_c\right) \tag{20}$$

hence, the transverse speeds of the general fluid particle P relative to the spinning body B becomes

$$v_\theta^i = \omega_\zeta x\left(\frac{x}{r_i} - 1\right) \tag{21}$$

and

$$v_\theta^e = \frac{\omega_\zeta}{R_c - r_e}\left[x^2 - \left(r_e + R_c\right)x + r_e R_c\right] \tag{22}$$

For uniform exit velocity profile, $U_e = ct$, T_7 then becomes

$$T_7 = -\frac{1}{10}\pi\rho\left[r_i^4 + \left(R_c - r_e\right)^2\left(R_c^2 + \frac{4}{3}r_e R_c - 9r_e^2\right)\right]L\omega_\zeta\left(\omega_\eta \vec{i}_\xi - \omega_\xi \vec{i}_\eta\right) \tag{23}$$

or

$$T_7 = \frac{1}{10}\frac{\dot{m}}{U_e R^2}\left[r_i^4 + \left(R_c - r_e\right)^2\left(R_c^2 + \frac{4}{3}r_e R_c - 9r_e^2\right)\right]L\omega_\zeta\left(\omega_\eta \vec{i}_\xi - \omega_\xi \vec{i}_\eta\right) \tag{24}$$

As the explicit expression for each term of Eq. (1) has been determined, including those terms resulted from fluid whirling motion, we are in a position to write the complete scalar

equations of rotational motion. Using equations (4), (5), (6), (7), (9), and (24), and assuming that the external moment $\vec{M}$ is zero, Eq. (1) can be broken into its scalar components

$$J\dot{\omega}_\xi + \left[\dot{J} - \dot{m}\left(\zeta_e^2 + \frac{R^2}{4}\right)\right]\omega_\xi + \left[\left(J_\zeta - J\right)\omega_\zeta + \Delta\omega_\zeta\right]\omega_\eta = 0 \tag{25}$$

$$J\dot{\omega}_\eta + \left[\dot{J} - \dot{m}\left(\zeta_e^2 + \frac{R^2}{4}\right)\right]\omega_\eta - \left[\left(J_\zeta - J\right)\omega_\zeta + \Delta\omega_\zeta\right]\omega_\xi = 0 \tag{26}$$

and

$$J_\zeta\dot{\omega}_\zeta + \left(\dot{J}_\zeta - \dot{m}\frac{R^2}{2}\right)\omega_\zeta + \dot{m}\frac{R^2}{10}\omega_\zeta = 0 \tag{27}$$

where $\Delta = \frac{1}{10}\frac{\dot{m}}{U_e R^2}\left[r_i^4 + \left(R_c - r_e\right)^2\left(R_c^2 + \frac{4}{3}r_e R_c - 9r_e^2\right)\right]L$.

3. RESULTS AND ANALYSIS

The attitude behavior of a variable mass system for the S-5M rocket in a force free environment is solved using Eq. (25), (26), and (27) in a fourth-order Runge-Kutta time scheme. Within Figs. 4 to 6 there are presented in a comparative manner results with fluid whirling motion and results that ignore the whirling motion.

The main data and firing conditions of the studied S-5M rocket are summarized below:

- overall length l_R = 825 mm;
- total mass m = 3,88 kg;
- initial center of mass from the nose tip x_{CM} = 431 mm;
- final center of mass from the nose tip x_{CM} = 417 mm;
- initial lateral moment of inertia J = 0,1536 kg·m^2;
- final lateral moment of inertia J = 0,1326 kg·m^2;
- initial axial moment of inertia J_ζ = 0,0018 kg·m^2;
- final axial moment of inertia J_ζ = 0,0015 kg·m^2;
- PRTF 121 propellant mass m_c = 1,13 kg;
- propellant grain dimensions (r_i = 4,05 mm, r_e = 22,8 mm, and L = 450 mm);
- internal radius of the combustion chamber R_c = 25,1 mm;
- radius of the nozzle throat r_{cr} = 7 mm;
- radius of the nozzle exit plane R = 17,5 mm;
- firing condition (ω_ξ = 0, ω_η = 5 s^{-1}, and ω_ζ = 32 s^{-1}).

To obtain the pressure-time characteristic, the author tested statically the S-5M rocket motor for conditioned temperature of 293 K at S.C. Electromecanica Ploieşti S.A. (Fig. 3). Acquiring accurate chamber pressure readings over the duration of the motor burn is an important aspect of the static testing. From the pressure data, it is possible to derive the performance parameter such as the mass flow rate, needed for numerical calculations. The data acquisition system consists of one HBM P3MB pressure transducer and for this static testing the recording rate is of 1000 samples per second.

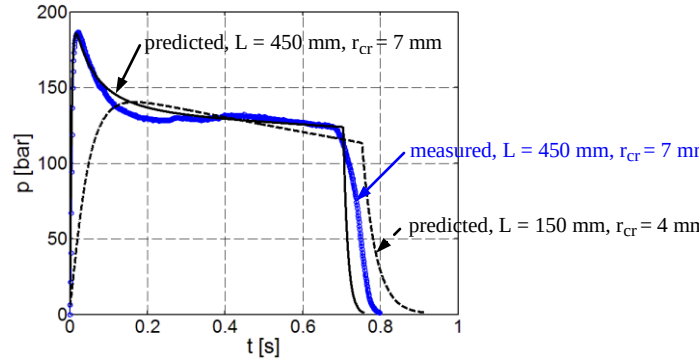

Fig. 3 – Pressure-time curves for the S-5M rocket motor and for modified S-5M rocket motor

For a prolate model ($J_\zeta < J$), the frequency of oscillations for ω_ξ and ω_η transverse angular velocities when whirling motion is added will always be higher than the one considered without taking into account the whirling motion (see Fig. 4). Hence, we can state that internal fluid whirling motion has no effect on the amplitude of the transverse angular velocity vectors.

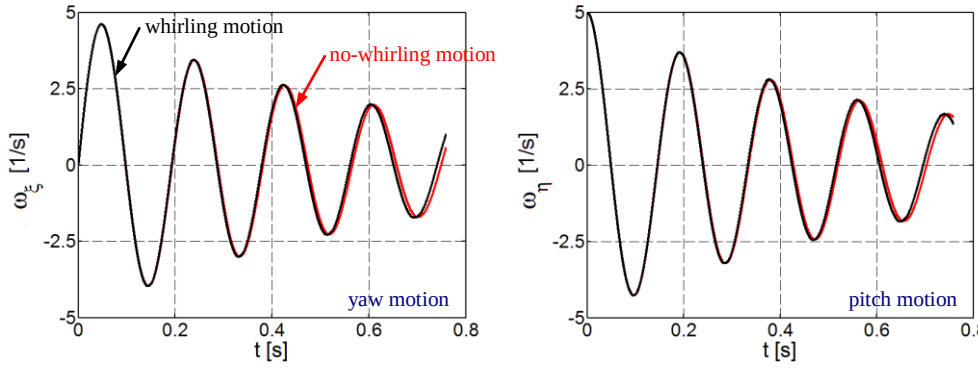

Fig. 4 – Variation of transverse angular velocities for S-5M rocket system (L = 450 mm, r_{cr} = 7 mm)

Artificially decreasing the length of cylindrical propellant grain and contracting the nozzle throat diameter, in order to maintain the constant burn time result in decreasing of the damping coefficient of the oscillation for both the transverse angular velocities and the angular frequency.

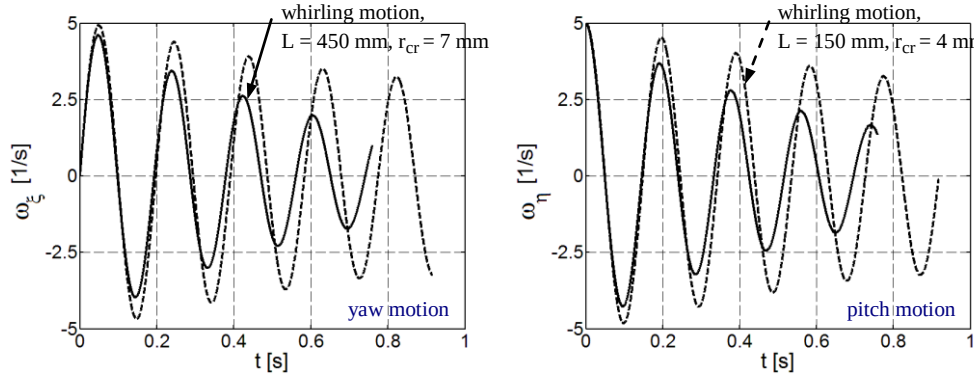

Fig. 5 – Variation of transverse angular velocities for S-5M rocket system and for modified S-5M rocket motor

Analysis performed using a S-5M rocket system shows that spin rate predictions made with the no-whirling motion assumption will be less than those which would have been predicted if whirling motion were properly accounted for (see Fig. 6). For oblate systems, the frequency of rocket transverse motion will always be less than the frequency for a prolate system.

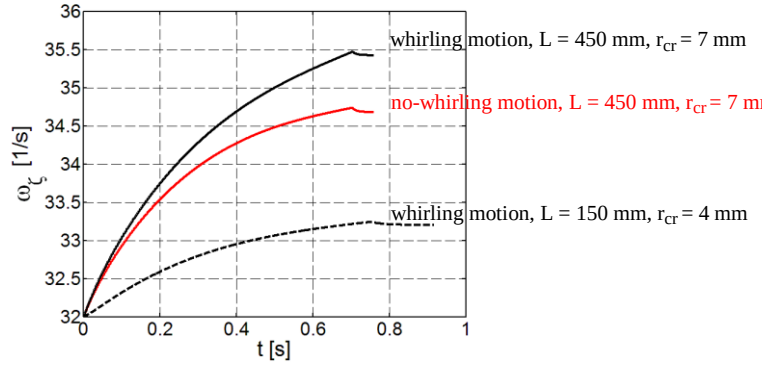

Fig. 6 – Variation of spin rate for S-5M rocket system and for modified S-5M rocket motor

4. CONCLUSIONS

This paper presents a study of the attitude behavior of a variable mass system for the S-5M rocket in a force free environment. The results indicate that mass variation can have a major impact on the attitude behavior of the system.

The results obtained indicate that the spin predictions made with the no-whirling motion assumption will always be in error, unless the spin rate is zero. The deviation in spin rate increases with rocket propellant burn, and can reach a value as high as 3% for S-5M system [8].

On the other hand, the impact of internal fluid whirling motion on system transverse motion is not as dramatic as that on the spin motion. The amplitude of the system's transverse angular velocity is unaffected by the fluid whirling motion. However, the frequencies of the transverse angular velocity components are affected, but only marginally.

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