Scaling Method of Existing Aircraft

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Abstract

This paper presents a method of current aircraft scaling down, on the assumption of the same No. Mach, same force and moment coefficients and same mass ratio.

Nomenclature

z - altitude ($0 \le z \le 11000\ \mathrm{m}$ - troposphere)

ρ $= 1.225(1-\varepsilon z)^{\beta}$ atmospheric density, $\mathrm{kg/m^3}$; $\varepsilon = \dfrac{0.065}{288.15}\ \mathrm{Km^{-1}}$, $\beta = 4.25864$

T $= 288.15(1-\varepsilon z)$ atmospheric temperature , K

m - mass

$\vec{V}$ - velocity vector

$\vec{F}$ - resultant forces

$\vec{M}$ - resultant moment

$\vec{\Omega}$ - resultant angular velocity vector

J - inertia tensor

λ - geometric scale

$s_{(\cdots)}$ - scale factor associated with $(\cdots)$

Introduction

The aircraft equations of motion are derived from the basic Newtonian mechanism. All the basic calculations of atmosphere parameters are performed in International Standard Atmosphere (ISA) conditions[1].

The general force and moment equations for a rigid body in body-axes are the following:

$$\vec{F} = m\left(\frac{d\vec{V}}{dt} + \vec{\Omega} \times \vec{V}\right) \tag{1}$$

$$\vec{M} = J \cdot \frac{d\vec{\Omega}}{dt} + \vec{\Omega} \times \left(J \cdot \vec{\Omega}\right) \tag{2}$$

The two vector-equations describe the motions of aircraft related to the Earth under the folowing restrictive assumption:

1. flat Earth
2. the body is assumed to be rigid during the motion considered

3. the mass of the rigid body is assumed to be constant during the time-interval in wich its motios are studied
4. same Mach number for real and fypothetical aircraft
5. same aerodynamics coefficients
6. same relative density factor
7. identity angles

Scale factors

Applying the similarity equations (1) and (2) leads to the following links between the scale factors:

$$s_m \frac{s_V}{s_t} = s_m = s_\rho s_S s_V^2 = s_m s_V s_\Omega$$

$$s_l\ s_\rho s_V^2 s_S = s_J \frac{s_\Omega}{s_t} = s_J s_\Omega^2$$

for:

- time
- mass
- length ;surfaces
- linear velocity
- angular velocity
- moment of inertia
- air density

Results and discussion

According to the geometric scale, scale factors are obtained as following:

$$s_t = \sqrt{\lambda}$$

$$s_m = \lambda^{3+\beta}$$

$$s_V = \sqrt{\lambda}$$

$$s_\Omega = \frac{1}{\sqrt{\lambda}}$$

$$s_J = \lambda^{5+\beta}$$

$$s_\rho = \lambda^{\beta}$$

For $\lambda \in [0.76, 1.35]$ scale factors can have values:

time	$0.872 \div 1.162$
mass	$0.136 \div 8.832$
linear velocity	$0.872 \div 1.162$
angular velocity	$0.861 \div 1.147$
moment of inertia	$0.079 \div 16.096$

Concluding remarks

Under the considered interpretation a Froude number and a dimensionless inertial moment number can be identified as being identical. By the proposed method a same damping factor, along with the frequency scales ratio as an inverse time scale ratio can be obtained, in case of longitudinal dynamics stability [2]. The report of derivatives altitudes is also verified.

REFERENCES

[1] *ANTONIO FILIPPONE*, ***Flight Performances of Fixed and Rotary Wing***, Elsevier Ltd., 2006.
[2] *BERNARD ETKIN*, ***Dynamics of Flight Stability and Control***, THIRD EDITION.