The Design Algorithm of the Optimum Structure from the Elasticity Module Point of View for Composite Materials

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DOI: 10.13111/2066-8201.2012.4.3.2

Abstract*: Unlike other researches in the field, the simultaneous influence of all structural elements on the elasticity module is studied in this paper. The obtained physical – mathematical model is in good accordance with the experimental results and it can be used in practice by calculating, prior to the production, the values the structural elements must have so that the composite has a certain elasticity module. The model gives the opportunity of quick and exact determination of the values for the structural elements without experimental determinations and successive difficult calculations.*
The method of establishing the optimum structure presented here can be also used for other composites and other mechanical properties.

Key Words: *laminate composite materials, elasticity module, mathematical model, response surface, design algorithm, optimum structure*

1. INTRODUCTION

An important category of composite materials is the laminate ones (L.C.M.). Laminate composites are made of several layers called plies or laminas. Laminas are made of a single row of fibres bound in a matrix material.

The thickness of the laminas does not usually exceed 0, 2 [mm]. Several adjoining laminas with the same fibre orientation form a lamina group. The laminate is made of one or more lamina groups. When the laminate has only one lamina group, it is called unidirectional composite.

The mechanical properties of the laminate are the result of the lamina groups' properties and of the sequence of their orientation inside the composite, as well.

The structural elements of a laminate composite are: the volume fractions of the fibres and of the matrix (V_f ; V_m); fibres orientation in the lamina group (the angle θ); number of lamina groups (N); thickness of the lamina groups (h_i); sequence of the lamina groups orientation.

The first two elements characterise the lamina group, while the last three characterise the composite ensemble, being also called topological elements.

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Some of the obstacles that prevent laminate composite materials from being more frequently used in different areas are these materials structural complexity and the difficulty of predicting with accuracy the properties of a L.C.M. piece with a certain structure. In other words, it is still hard to accurately say which are the volume fractions of the constituents, the reinforcement directions, the thickness of the lamina groups, etc, so that the laminate has certain values of the physical – mechanical properties, values imposed by the practical necessities.

The optimum structural organisation, corresponding to a certain performance wanted for the composite, is a very important stage in designing L.C.M. items, because through an appropriate structural organisation, the properties of a composite made of poorer quality constituents can be superior to the properties of composites made of constituents with exquisite individual properties but with a structural organisation inadequate for the application.

Therefore, the research in the structure – properties interdependence area is very important from this point of view.

By analysing the current researches in the structure – properties interdependence area, one can see that they rely on the following theories: the theory of the macromechanics of linear elastic anisotropic bodies, the theory of micromechanics and the theory of elasticity under contignity and noncontignity circumstances. These researches were focused on getting valid mathematical models for a wide range of composite materials, disregarding the constituents' nature and the coupling mechanisms between them. These global approaches could only be made in certain simplifying hypotheses, which led to results that are not entirely consistent with the real facts.

Therefore, the predictions resulted from these researches have only an indicative meaning for the manufacturers of pieces made of composite materials.

In the case of the theory of macromechanics, the results are only acceptable for orthotropic (orthothropic, ortotropic, ortotrope, orthotrope, orthothrope) laminates with symmetrical and simple topologies. For orthotropic composites with asymmetrical and no symmetrical topologies, or totally anisotropic, the calculated values of the properties significantly differ from the ones obtained experimentally. (see [1] pages 208 – 209 and [3] pages 79 – 80).

Also, determining the engineering constants from the compliance matrix is extremely exacting and it has a high level of uncertainty, because it implies mechanical tests on samples made of one lamina with very thin thickness, tests whose results are hard to reproduce and often incorrect.

Such small determinations cannot be technical arguments with authority, because the international standards in the area of composite materials tests require much thicker thickness for the test samples and certain test conditions that cannot be fulfilled by determinations on samples at lamina level.

Another major boundary of the theory of macromechanics is that the resulted relations do not explicitly take into account the reinforcement volume fractions. Because of this, the manufacturer of pieces made of composite materials is forced to choose initially a volume fraction of the fibres based on intuition or on its own experience. The correctness or incorrectness of this choice only results after the experimental determination of the engineering constants from the compliance matrix and after calculating the values of the composite properties. The stage is repeated if the choice was incorrect. The computing programmes based on these theories and developed by different companies have the same inconveniences.

2. EXPERIMENTAL RESULTS

Through the conducted researches, we wanted to ascertain through theoretical – experimental way, some mathematical models usable in practice, that would include explicitly and simultaneously the influence of all structural elements on the elasticity module of glass E/epoxy laminates. Based on mathematical models we also wanted to create an algorithm and a computing programme regarding the designing of L.C.M. optimum structure, so they should have certain imposed properties. It has to be noticed that this algorithm and the study method presented below are applicable to all composite materials and physical – mechanical properties.

The elasticity module of the composite is the result of the elasticity modules of the composing lamina groups, arranged in a certain order, with certain orientations and thickness. Consequently, initially there was studied the structural elements (V_f ; θ) influence on the elasticity module of the lamina group using the regression analysis of the active experiment and the optimisation without restrictions. Knowing the elasticity module of the group, a theoretical model to determine the elasticity module of the composite was defined, in which some other structural factors occurred (N, h_i). The theoretical model obtained in this way was experimentally tested. The experiments were conducted on the lamina group and not on the individual lamina, which allowed the use of samples and testing methodology in accordance with international standards. Thus, there were obtained correct and reproducible results that make up technical arguments with authority, thus eliminating the inconveniences that occur in tests conducted at lamina level.

The experiments were organised on the active experiment principle, using the second-degree orthogonal compositional central programme (PCCO2) with three variable levels. The “star” points were established through previous experiments. In order to exclude the appearance of some non-random links between determinations, these were randomised in time based on the random numbers string (see[2]). The abnormal results were eliminated on the Q criterion.

The values presented in table 1 were obtained for the glass E/epoxy laminate composite. The glass E armour is of Roving type with the finish Z6040. The used epoxy resin is of DGEBA standard type. The plates from which the test-pieces were drawn were made in steriliser, using the solidifying and thermal treatment diagrams recommended by the resin manufacturer. The resin solidifying was made with TETA solidifying agent.

Table.1 The experimental values obtained for the elasticity module of the lamina group.

No	Volume fractions of the fibres (V_f) (%)	Fibres orientation-angle θ (grade)	Elasticity module E (GPa)
1.	30	0	17,22
2.	30	45	8,66
3.	30	90	4,53
4.	70	90	11,87
5.	70	0	19,72
6.	70	45	12,23
7.	50	90	7,52
8.	50	0	17,92
9.	50	45	9,69
10.	50	45	9,84
11.	50	45	9,32
12.	70	0	19,6

13.	11	100,3	2,83
14.	30	90	4,7
15.	40	15	14,23
16.	60	15	15,82
17.	40	30	11,46
18.	60	30	12,34

3. THE MATHEMATICAL MODEL OF THE ELASTICITY MODULE

The linear model does not lead to a good approximation of the dependence of the lamina group's elasticity module on the volume fraction and the reinforcement angle. In return, the second order model proved to be harmonious.

This has the general form:

$$E = \beta_0 + \beta_1 V_f + \beta_2 \theta + \beta_{12} V_f \theta + \beta_{11} V_f^{\,2} + \beta_{21} \theta^2 \tag{1}$$

where: E – the elasticity module of the lamina group – the objective function (GPa);
V_f – the volume fraction of the fibres – independent variable (%);
θ – the reinforcement angle (the angle between the direction of the fibres and the direction of the solicitation) – independent variable (degrees);
$\beta_0 \ldots \beta_{22}$ – unknown coefficients.

After passing to encoded variables and the change of variable necessary to realise the orthogonality of the matrix PCCO2, equation (1) becomes:

$$Y = b_0 + b_1 x_1 + b_2 x_2 + b_{12} x_1 x_2 + b_{11} x_1^{\,2} + b_{22} x_2^{\,2} \tag{2}$$

where: Y – the objective function;
x_1, x_2 – encoded variables corresponding to the V_f and θ variables;
$b_0 \ldots b_{22}$ – unknown coefficients.

The experimental matrix PCCO2 is presented in table 2.

Table 2. PCCO2 matrix.

No. exp.	x_0	x_1	x_2	$x_1\,x_2$	$x_1' = x_1^2 - 2/3$	$x_2' = x_2^2 - 2/3$	Y (GPa)
1.	+1	-1	-1	+1	+1/3	+1/3	17.22
2.	+1	-1	+1	-1	+1/3	+1/3	4.53
3.	+1	+1	+1	+1	+1/3	+1/3	11.87
4.	+1	+1	-1	-1	+1/3	+1/3	19.72
5.	+1	+1	0	0	+1/3	-2/3	12.23
6.	+1	-1	0	0	+1/3	-2/3	8.66
7.	+1	0	+1	0	-2/3	+1/3	7.52
8.	+1	0	-1	0	-2/3	+1/3	17.92
9.	+1	0	0	0	-2/3	-2/3	9.69

The regression coefficients were determined with the calculation relations of PCCO2 (see [2]), getting the following values:

$b_o = 9.75$; $b_1 = 2.23$; $b_2 = -5.16$; (3)
$b_o = 9.75$; $b_1 = 2.23$; $b_2 = -5.16$;

The verification of the significance degree of the coefficients was made through the Student criterion for the significance limit of 0,05.

There was ascertained that all coefficients have the absolute value superior to the reliance interval.

It results that all coefficients are significant. The following mathematical model was thus obtained:

$$Y = 9.75 + 2.23x_1 - 5.16x_2 + 1.21x_1x_2 + 0.66x_1^2 + 2.94x_2^2 \quad (4)$$

The concordance of the model was verified through the Fischer criterion for the significance limit of 0,05.

There was ascertained that the mathematical model is in good accordance with the experimental data.

After passing to real variables, the model of the dependence of the lamina group's elasticity module on the structural elements V_f and θ has the expression:

$$E = 19.4 - 11.4V_f - 0.3\theta + 0.1V_f\theta + 16.5V_f^2 + 0.002\theta^2 \text{ (GPa)} \quad (5)$$

Fig. 1 and fig. 2 show the dependence of elasticity module on the fibres orientation and on the volume fractions of the fibres.

Fig. 3 shows the response surface.

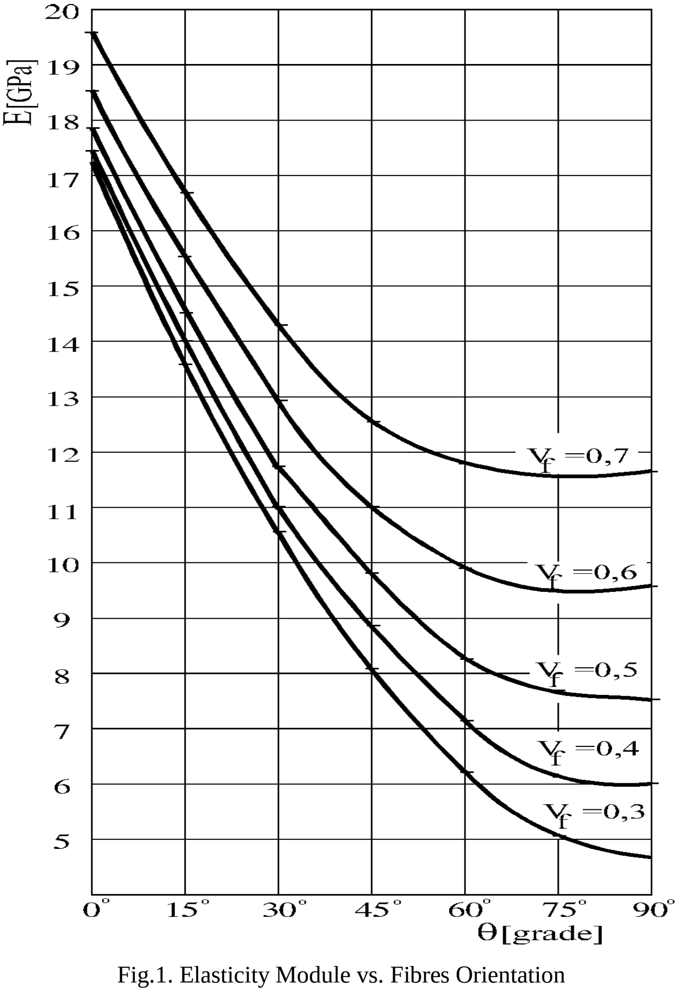

Fig.1. Elasticity Module vs. Fibres Orientation

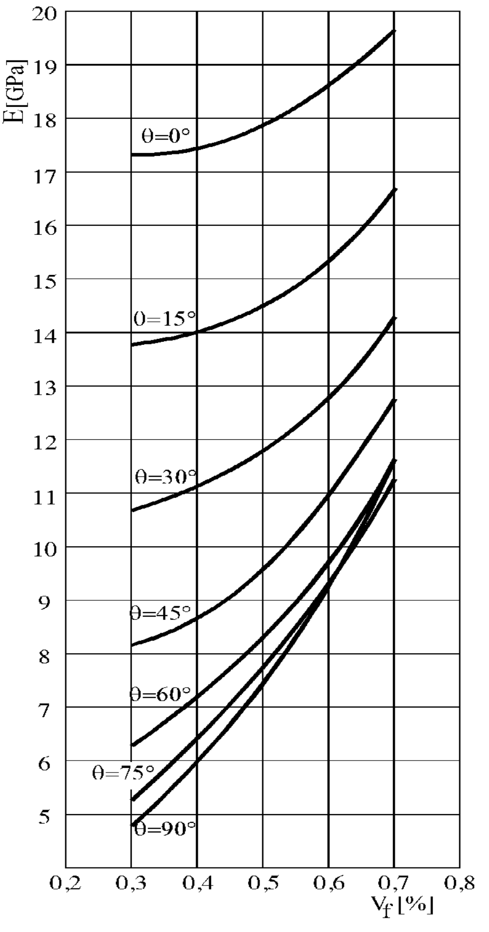

Fig.2. Elasticity Module vs. Volume Fractions of the Fibres

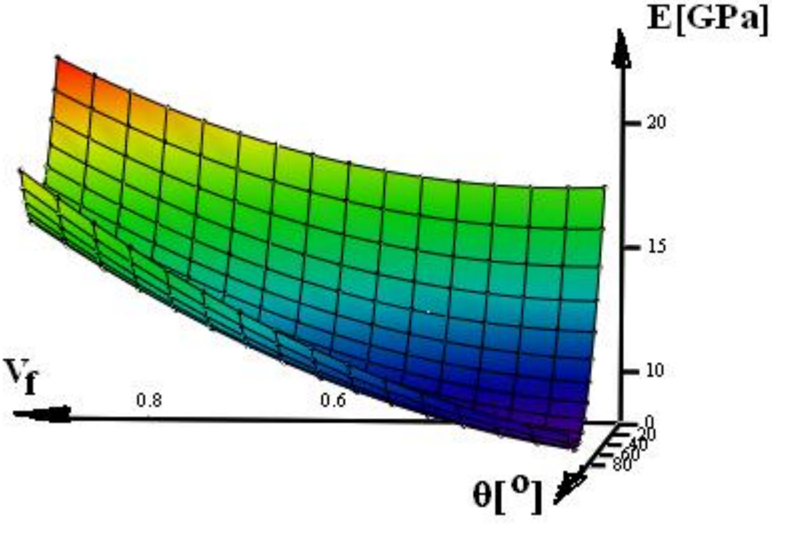

Fig.3. The response surface

4. THE EQUAL SIGNIFICANCE CURVES

For an easier geometrical interpretation, the non-linear model of the elasticity module must be transformed from form (4) to the standard form. The transformation is made by choosing a new reference system, with the origin in the centre of the answering area. The transformation actually reduces itself to a translation through which the first degree terms disappear, and a rotation through which the binary interaction term $b_{12}x_1x_2$ disappears. The equation gets the following form:

$$Y = Y_s + B_{11}\overline{X}_1^2 + B_{22}\overline{X}_{22}^2 \tag{6}$$

where: Y – the value of the elasticity module in the x_1, x_2 coordinates system;

Y_s – the value of the elasticity module in the centre of the answering area;

$\overline{X}_1, \overline{X}_2$ - the variables corresponding to the new coordinates system;

B_{11}, B_{22} – standard coefficients.

The solution of the equations system obtained by equalising with zero the partial derivatives with x_1 and x_2 of the mathematical model (4) represents the coordinates of the symmetry centre of the answering area.

$$\frac{\partial Y}{\partial x_1} = 0 \qquad \text{and} \qquad \frac{\partial Y}{\partial x_2} = 0 \tag{7}$$

The coordinates of the centre of the answering area have the following values in encoded and real dimensions:

$$\begin{aligned} x_{1s} &= -1.95; x_{2s} = 1.23 \\ V_{fs} &= 0.11; \theta = 79.25^o \end{aligned} \tag{8}$$

The value of the elasticity module in the centre of the area is Y_s=2,43 GPa.

The rotation angle (φ) of the coordinate axes is:

$$tg2\varphi = \frac{b_{12}}{b_{11} - b_{22}} = -0.53 \Rightarrow \varphi = -13.98^o \tag{9}$$

The standard equation for the elasticity module gets that way its final form:

$$Y = 2.43 + 0.51\overline{X}_1^2 + 3.09\overline{X}_2^2 \tag{10}$$

So the answering area is an elliptical paraboloid whose symmetry centre represents the minimum value of the elasticity module (B_{11}>0, B_{22}>0).

To highlight the shape of the equal significance lines (the same elasticity module), equation (10) can be written in the following form:

$$\frac{\overline{X}_1^2}{\frac{Y - 2.43}{0.51}} + \frac{\overline{X}_2^2}{\frac{Y - 2.43}{3.09}} - 1 = 0 \tag{11}$$

Equation (11) represents a family of ellipses in $\overline{X}_1, \overline{X}_2$ coordinates (fig.4). The ellipses are lengthened in the x_1 direction (B_{11}<B_{22}).

We call the centre of the answering area marginal point (P_M) and its coordinates' marginal values (V_M) of the structural elements.

The marginal volume is V_{fM}=11 % and the marginal reinforcement angle $\theta_M = 79{,}25^o$.

When the volume fraction of the armour is smaller than the critical volume fraction and the reinforcement angle is greater than θ_M, the elasticity module of the composite is controlled by the matrix and it is smaller than the elasticity module of the matrix. In this case, the armour does not fulfil its reinforcement role; on the contrary it behaves like heterogeneity, diminishing the elasticity module of the matrix.

For volume fractions greater than the marginal volume fraction (V_{fM}) and reinforcement angles smaller than the marginal reinforcement angle (θ_M), the elasticity module of the composite is controlled by the armour and it is greater than that of the matrix, so the reinforcement is efficient. The elasticity module increases by moving from the marginal point in the $\overline{X}_1, \overline{X}_2$ directions.

The marginal values highlighted here accurately limit the real variation domain of structural elements for an efficient reinforcement from the elasticity module point of view. That is why knowing these values practical and theoretical importance. These marginal values have been confirmed by numerous other experimental determinations conducted by us or by our co-workers. Considering that the notion of composite material must be correlated with the property or properties for which it was created, the coordinates of the marginal point (P_M) are an answer to this question: “Under what quantitative and topological circumstances the introduction of a fibrous heterogeneity transforms the matrix material from monolithic material into composite material”.

A certain value of the elasticity module can be obtained in several points of the structural factors space that is for several values of the corresponding volume fraction and reinforcement angle. An example, the elasticity module of 19 GPa can be that of any composite with the volume fraction included between 65 % and 70 % and the corresponding reinforcement angle between 0^o and 4.5^o (fig.4).

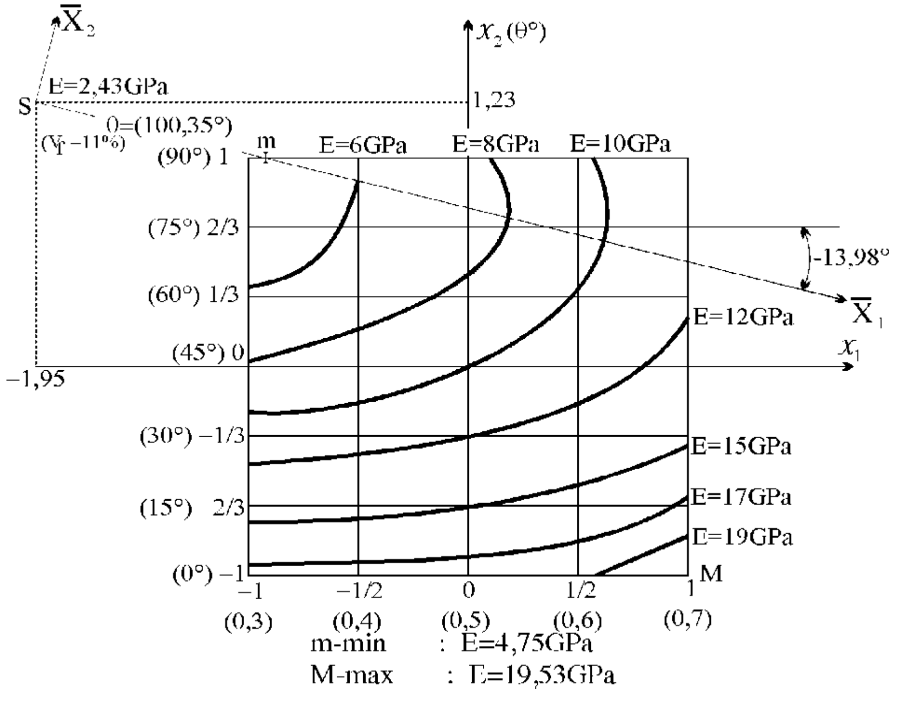

Fig. 4. The nomogram of the equal significance lines for the elasticity module of the unidirectional composite

5. THE ELASTICITY MODULE OF THE LAMINATE

The laminate can be considered, from the solicitation at traction point of view, a bar with non-homogeneous section made of several elements with different properties and the same deformation.

The composing elements are the lamina groups that make up the composite. Consequently, the traction stiffness of the laminate is the sum of the traction stiffness of the composing lamina groups. So there can be written:

$$E_c.A = \sum_{i=1}^{N} E_i.A_i \tag{12}$$

where: E_c – the elasticity module of the composite;
E_i – the elasticity modules of the composing lamina groups;
A – the transversal area of the laminate;
A_i – the transversal areas of the lamina groups;
N – the number of lamina groups.

The width of the composite is equal to the width of the groups and so relation (12) becomes:

$$E_c = \sum_{i=1}^{N} E_c p_i \tag{13}$$

where: $p_i = h_i/h$ – the thickness fractions of the groups
h – the thickness of the composite;
h_i – the thickness of the groups;

Considering expression (5) for E_i, equation (13) gains its final form.

$$E_c = \sum_{i=1}^{N} E_c (19.4 - 11.4V_{fi} - 0,3\theta_i + 0,1V_{fi}\theta_i + 16.5V_{fi}^2 0,002\theta_i^2) p_i \quad \text{(GPa)} \tag{14}$$

The relation (14) allows the calculation of the elasticity module of any glass E/epoxy composite depending on the volume fractions, the reinforcement angles, the thickness and the number of the lamina groups. Values calculated with relation (14) are in a good accordance with the experimental data.

Thus, for composites that have less than 12 lamina groups the differences between the calculated values and the experimental ones are under 10%. In the case of composites with the number of groups between 13 and 20, these differences are under 14%. It must be stipulated that in practical applications the probability of necessary orientation sequences that need a number of groups greater than 8 is almost zero, so we can consider that relation (14) meets the practical needs entirely, also knowing the meaning of the difference between the calculated values and the real ones. For situations where more than 10 % accuracy is needed, the relation that calculates the elasticity module depending on the structural elements can be rectified with one coefficient (K_E).

6. THE DESIGN ALGORITHM OF THE OPTIMUM STRUCTURE FROM THE ELASTICITY MODULE POINT OF VIEW

The following stages are passed through in order to establish the optimum structure corresponding to a certain elasticity module of the imposed laminate:

1) From geometrical, functional and technological conditions the thickness (*h*) of the laminate and the number (*N*) of the lamina groups are established.
2) The values established for E_c, *h* and *N* are introduced in relation (13), obtaining a second degree equation with the real and positive unknowns (E_i) and (h_i). The solutions of the equation are obtained using common programmes that solve second degree equations with "*n*" unknowns. The solutions are the elasticity modules and the lamina thickness, corresponding to the elasticity module of the imposed composite.
3) With equation (5), the volume fraction (V_f) and the reinforcement angle (θ) are determined for each lamina group. The values (V_{fi}) and (θ_i) are obtained from the graphic (see the nomogram in fig.4), or with the help of a computer using common programmes that solve second degree equations with two unknowns.

Fig. 5 presents the logical diagram of a programme for designing the optimum structure of the composite from the elasticity module point of view, programme that also allows solving the indirect problem, the calculation of the elasticity module depending on the structural elements.

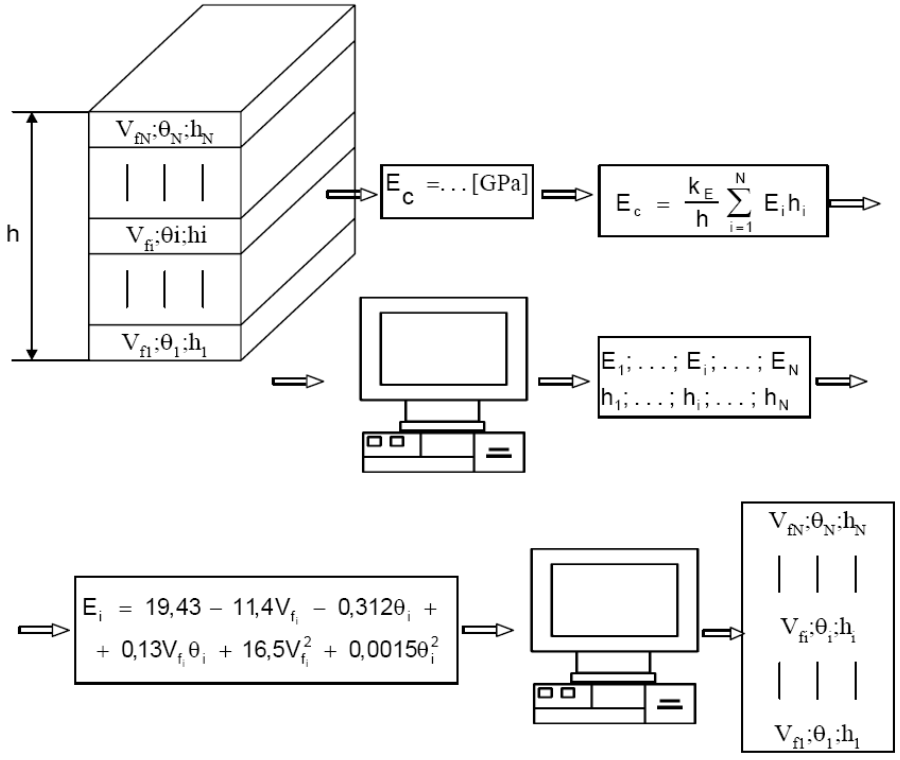

Fig. 5 The logical diagram of the programme for designing the optimum structure

7. CONCLUSIONS

1. Unlike other researches in the field, this paper studies the simultaneous influence of all structural elements on the elasticity module.
2. The obtained physical – mathematical model is in good accordance with the experimental results and it can be used in practice by calculating, prior to the production,

the values the structural elements must have so that the composite has a certain elasticity module. The calculation relation contains the structural elements explicitly, unlike the constitutive equation from macromechanics analyses.

3. The model gives the opportunity of quick and exact determination of the values for the structural elements without experimental determinations and successive difficult calculations.
4. The method of establishing the optimum structure presented here can be also used for other composites and other mechanical properties.
5. The experimentally confirmed finding that the properties of the laminate depend on the properties, sequence, thickness and number of lamina groups that compose it, made it possible to conduct experiments on the lamina group and not on the individual lamina as in previous researches. This is an essential aspect because, in this way, the difficulties and imperfections that appear in experiments conducted on the individual lamina are avoided.
6. The coordinates of the marginal point (P_M), called marginal values (V_M) are limit values that structural elements can have so that the reinforcement be efficient. Knowing these values has a theoretical and practical importance because they limit the real domain of structural factors variation, in which the composite fulfils the purpose it was created for.
7. The paper also presents the logical diagram of a programme for designing the optimum structure from the elasticity module point of view.

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