

# Determining the optimum topology of composites by the flexural stiffness criterion

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**Abstract:** *An important stage in designing of pieces made of composite materials consists of establishing the composite topology in such a way that it has certain properties needed in exploitation. The paper presents the mathematical apparatus and the calculation programme for establishing the optimum thickness of the composite groups so that it should have certain imposed (given) flexural stiffness. The method is applicable to all types of laminate composites, no matter of the cladding or matrix nature. The direct problem consists in determining the thickness of the groups and composite, minimising the bar mass, for an imposed (given) flexural stiffness, knowing the densities and elasticity modules of the groups. The indirect problem consists in determining the maximum stiffness, the thickness of the groups and composite for a given (imposed) mass, knowing the densities and elasticity modules of the groups. The presented programmes offer to the producer of this kind of materials the possibility to quickly establish the optimum topology.*

**Key Words:** *flexural strength, sandwich materials, paper honeycombs, structural element, response surface, design algorithm, optimum structure*

## 1. INTRODUCTION

Composites are heterogeneous and anisotropic materials obtained through macroscopic scale combination of two or more phases, which have a separating interphase or interface. This combination is done in order to obtain a material with certain properties, superior to those of the components. The notion of phase has in this case a strictly descriptive sense of structural homogeneous part of a material system and it does not have a thermodynamic sense. The rules of the phases, true for the phases in equilibrium and which derive from the fact that the chemical potential of one component is the same in each phase, do not apply in the case of composite materials. As the concept of phase equilibrium does not operate for these materials, composites can have supplemental freedom degrees regarding the nature and the quantity of the combined phases. This aspect essentially differentiates composite materials of simple materials (monolithic).

An important category of composite materials is laminate composites, abbreviated LCM (Laminate Composite Materials). They have polymeric matrix (the continuous phase) and

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fibre cladding (the discontinuous phase).

Laminate composites are made of layers, with different fibre thickness and orientation, called lamina groups. Fig. 1 presents the laminate composite with  $(15^0, -15^0, 15^0)$  topology made of three lamina groups.

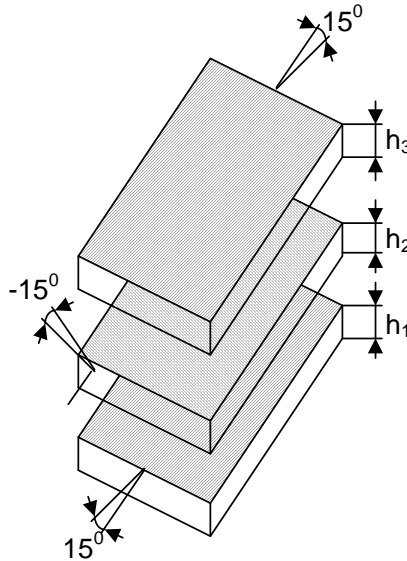


Fig. 1 Composite with  $(15^0, -15^0, 15^0)$  topology

The composite properties are the result of the component phase's properties and also of the thickness and number of groups.

Topological organisation plays an important role in the composite's properties. By an adequate topological organisation, the properties of a composite formed of poorer quality constituents can be superior to those of composites made of constituents with exquisite individual properties, but with the topological organisation inadequate to the application. Therefore, it is very important that we can establish the volume fractions of the fibres ( $V_f$ ), the reinforcement angle ( $\theta$ ), the thickness of the groups ( $h_i$ ) and the thickness of the composite ( $h$ ), so that the composite can have certain/specific mechanical properties. The issue of establishing the optimum values for the volume fractions and reinforcement angles corresponding to certain solicitations has been dealt with in other works (see [1]).

## 2. THE METHOD PRINCIPLE

In order to establish the groups optimum thickness from the flexural stiffness point of view, a composite with rectangular section, made of 5 groups is considered. Due to the technological difficulties that appear in the production of composites with a great number of groups, composites with maximum 5 groups are used in practice. The results obtained for 5-group composites can be easily extended to the 2, 3 or 4 groups by simply considering the last groups as having zero thickness.

The laminate has the shape of a symmetrical rectangular bar made of 5 lamina groups (fig. 2) with height ( $h$ ), length ( $l$ ) and width ( $b$ ) equal to the unit. The composite is made of a median group with thickness ( $h_1$ ), density ( $\rho_1$ ) and elasticity module ( $E_1$ ); two groups situated on the both sides of the median group, marked "2", with thickness ( $h_2$ ), densities ( $\rho_2$ ) and elasticity modules ( $E_2$ ); two external groups, marked "3" with  $h_3$ ,  $\rho_3$  and  $E_3$  features.

The composite can be associated with a non-homogeneous section bar made of several elements with distinct properties, which do not have longitudinal sliding between one another. These elements are, in the case of the composite, the lamina groups. Consequently, the shearing efforts and the transversal ones, resulted from the variation of the Poisson coefficients, are negligible.

The fixing of the optimum topology by the flexural stiffness criterion has two problems.

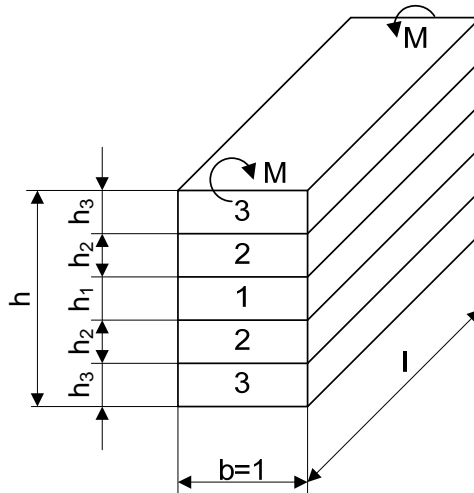


Fig. 2 The 5-group laminate solicited at flexure

The direct problem consists in determining the thickness of the groups and composite, minimising the bar's mass, for an imposed (given) flexural stiffness, knowing the densities and elasticity modules of the groups.

The indirect problem consists in determining the maximum stiffness, the thickness of the groups and composite for a given (imposed) mass, knowing the densities and elasticity modules of the groups.

The elasticity modules of the groups are determined (see [1]) depending on the volume fraction of the fibres and on the reinforcement angle (the two structural elements of the group), with the formula:

$$E = 19,4 - 11,4V_f - 0,3\theta + 16,5V_f^2 + 0,002\theta^2 \quad (1)$$

where:

- $E$  – the elasticity module of the group (GPa);
- $V_f$  – the volume fraction of the fibres (%);
- $\theta$  – the reinforcement angle (degrees).

The flexural stiffness of the laminate, according to the theory of the non-homogeneous section bars has the expression:

$$D = E_1I_1 + 2E_2I_2 + 2E_3I_3 \quad (2)$$

where:

- $D$  – the flexural stiffness of the laminate;
- $E_1, E_2, E_3$  – the elasticity modules of the groups;
- $I_1, I_2, I_3$  – the inertia moments of the groups related to the central axis of the bar's transversal section.

Writing the inertia moments depending on the sections dimensions, after making the calculations, the following expression of the stiffness ( $D$ ) is obtained:

$$D = \frac{1}{12} \left\{ E_1 h_1^3 + E_2 [2h_2^3 + 6h_2(h_1 + h_2)^2] \right\} + \frac{1}{12} E_3 [2h_3^3 + 6h_3(h_1 + h_3 + 2h_2)^2] \quad (3)$$

To simplify the expression we note:

$$V = h_1/h; U = (h_1 + 2h_2)/h \quad (4)$$

The “ $h$ ” thickness of the laminate has the expression:

$$h = h_1 + 2h_2 + 2h_3 \quad (5)$$

The thickness of the lamina groups has the following expressions, depending on  $h$ ,  $V$ ,  $U$ :

$$h_1 = Vh; \quad h_2 = (U - V)h/2; \quad h_3 = (1 - U)h/2. \quad (6)$$

By introducing the relations (4) into expression (3), the relation for the calculation of the stiffness ( $D$ ) depending on the  $h$ ,  $V$  and  $U$  dimensions, is obtained as:

$$D = \frac{h^3}{12} [E_1 V^3 + E_2 (U^3 - V^3) + E_3 (1 - U^3)] \quad (7)$$

The mass per the area unit of the laminate ( $W$ ) is given by the relation:

$$W = \rho_1 h_1 + 2\rho_2 h_2 + 2\rho_3 h_3 \quad (8)$$

Where:  $\rho_1, \rho_2, \rho_3$  are the densities of the lamina groups.

With the notations (4), the mass per the area unit of the laminate becomes:

$$W = h[\rho_1 V + \rho_2 (U - V) + \rho_3 (1 - U)] \quad (9)$$

The external layers, usually being finishing layers, made of tixotropically modified resins, like gel-coat, have limited thickness due to technological reasons. Usually, the ratio between the thickness of the external layers and the total thickness of the composite is less than 0,2. The most used value for this ratio is 0,1.

Considering the ratio  $h_3/h = (1 - U)/2$  as constant and known, mathematically the problem reduces itself, to determine the dimensions ( $V$ ) and ( $h$ ), so that they minimise the mass ( $W$ ) for an imposed (given) stiffness ( $D$ ) and a fixed ratio ( $U$ ).

The optimum values for  $V$  and  $h$  are those which minimise the Lagrange-type function obtained as a connection between stiffness and mass. Considering the (7) and (9) expressions of the stiffness and mass, the Lagrange function ( $L$ ) will have the following form:

$$L = h[\rho_1 V + \rho_2 (U - V) + \rho_3 (1 - U)] + \lambda \left\{ D - \frac{h^3}{12} [E_1 V^3 + E_2 (U^3 - V^3) + E_3 (1 - U^3)] \right\} \quad (10)$$

where:  $\lambda$  - is the Lagrange multiplier.

The values ( $V$ ) and ( $h$ ), which minimise the Lagrange function, and the multiplier ( $\lambda$ ) are determined in the case of direct problem in the following three-equation system:

$$\begin{cases} \frac{\partial L}{\partial V} = 0 \\ \frac{\partial L}{\partial h} = 0 \\ D = \frac{h^3}{12} [E_1 V^3 + E_2 (U^3 - V^3) + E_3 (1 - U^3)] \end{cases} \quad (11)$$

By introducing the Lagrange multiplier's expression, obtained from the first equation (11), into the second equation (11), it results the following expression for  $V$ :

$$V = \left[ \frac{(\rho_2 - \rho_1)[E_3 + U^3(E_2 - E_3)]}{(E_2 - E_1)[\rho_3 + U^3(\rho_2 - \rho_3)]} \right]^{\frac{1}{2}} \tag{12}$$

From the third equation (11) the optimum  $h$  is obtained as being:

$$h = \left[ \frac{12D}{E_1V^3 + E_2(U^3 - V^3) + E_2(1 - U^3)} \right]^{\frac{1}{3}} \tag{13}$$

In the case of the indirect problem, the third equation of the system (11) will be the equation (9). In this case, the optimum ratio ( $V$ ) will have the same expression (12), while the optimum ( $h$ ) will be given by the following relation:

$$h = \frac{W}{\rho_1V + \rho_2(U - V) + \rho_2(1 - V)} \tag{14}$$

The nomogram in fig. 3 provides the values of the optimum ratio ( $V$ ) in the case of  $h_3/h=0,1$ ;  $E_1=E_3$  and  $\rho_1=\rho_3$  depending on  $\rho_1/\rho_2$  and  $E_1/E_2$ .

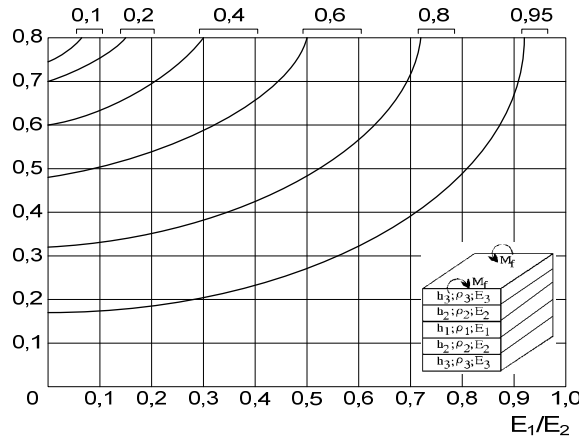


Fig. 3 Nomogram of the optimum ratio ( $V$ )/values

### 3. THE ALGORITHM FOR SETTING UP THE OPTIMUM TOPOLOGY

To determine the optimum thickness with the minimisation of the composite’s mass so that the laminate should have the imposed (given) flexural stiffness, the following stages are passed through:

1. The value for the ratio ( $V$ ) that minimises the Lagrange function is determined from relation (12);
2. The thickness of the laminate is determined from relation (13) with the imposed ( $D$ ) and the calculated ( $V$ );
3. The thickness of the lamina groups ( $h_1, h_2, h_3$ ) are calculated using relation (6) for the value imposed to ( $U$ ) by the technology (usually  $U=0,8$ , value corresponding to a ratio  $h_3/h=0,1$ );
4. The minimum mass is determined using relation (9).

To determine the optimum topology of the laminate composites, there has been developed a computer calculation programme whose logical diagram, for the direct problem, is presented in fig.4.

The determination of the optimum thickness of the groups and of the composite so that the latter should have a maximum stiffness and an imposed (given) mass requires passing through the following stages:

1. The ratio ( $V$ ) that minimises the Lagrange function is determined from relation (12);
2. Depending on the imposed mass ( $W$ ) and the calculated ( $V$ ), the thickness ( $h$ ) of the composite is determined with relation (14);
3. The groups' thickness ( $h_1, h_2, h_3$ ) are calculated from relation(6);
4. The value for the maximum stiffness results from relation (7).

The calculation programme that solves the indirect problem has its logical diagram presented in fig.5.

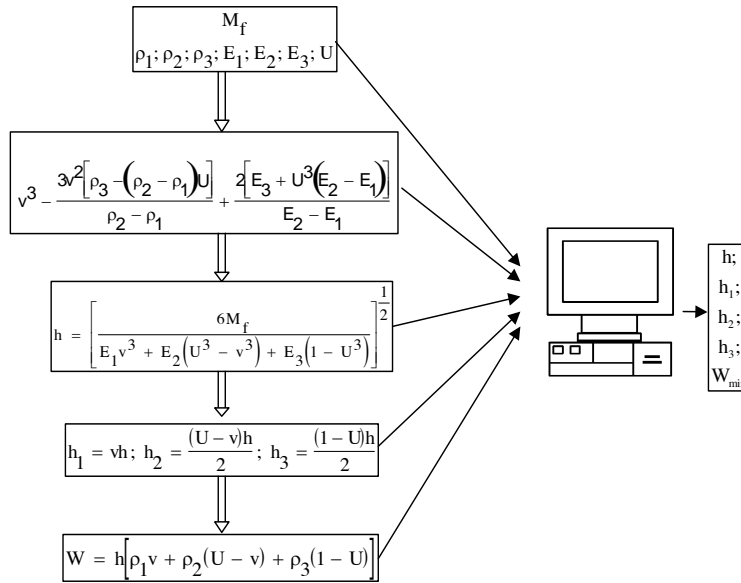


Fig. 4 Programme for optimum typology determination. The direct problem

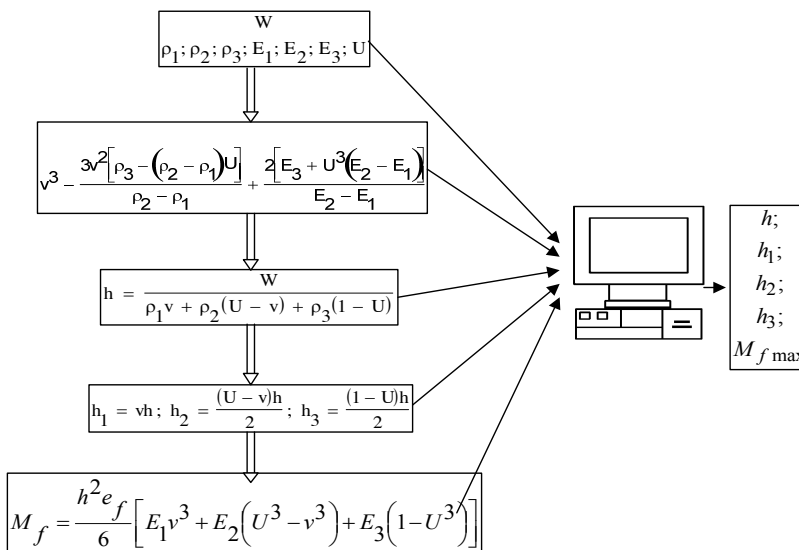


Fig. 5 Programme for optimum typology determination. The indirect problem

#### 4. CONCLUSIONS

The method sets the groundwork for designing the optimum topology of laminate composites, so they should have a certain imposed (given) flexural stiffness. On the basis of the mathematical apparatus presented above, the optimum topology for composites with 5, 4, 3 or 2 lamina groups can be determined. Due to the technological complications, composites with less than 5 groups are used in practice.

The method is applicable to all types of laminate composites, no matter of the cladding or matrix nature.

The presented programmes offer to the producer of this kind of materials the possibility to quickly establish the optimum topology.

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