Notes Regarding the Dynamics of an Airplane subjected to Vertical Gusts

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Abstract: *The behavior of the aircraft within turbulent atmosphere is a key aspect of design. Many books and articles deal with this topic. The current paper presents studies related to predicting the responses of aircraft flying through vertical gusts. The equations describing the dynamics of the longitudinal channel of the airplane are written to include the effect of the vertical wind. The paper includes comparisons of results provided by non-linear and linearized equations of motion.*

Key Words: *Flight Dynamics Simulation, Gust, vertical wind, numerical integration*

1. INTRODUCTION

The effects of the wind on airplanes or rockets have been addressed for many years; they have long been included in the FAR and MIL regulations and the main theoretical aspects describing wind effects can be found in many reference textbooks and papers, like, for example, Refs. [1-8]. A comprehensive literature review is beyond the purpose of this paper. Information regarding recent wind effects studies can be found in Refs. [7-12]

Calculating wind effects requires the numerical integration of the airplane's equations of motion, a fundamental aspect in flight dynamics, successfully addressed since the late 1950s.

Fundamentals of the numerical integration are presented, for example, in Refs. [13-15], and improved algorithms are described, for example, in Refs. [16-21].

Literature includes both wind effects obtained from the non-linear equations and gusts effects obtained from the linearized models, however, no comparisons of these two categories of solutions are usually made.

The main purpose of this paper is related to presenting consistent comparisons of results provided by these two types of models (i.e. linear and non-linear).

This study addresses vertical gust effects on airplane in symmetric longitudinal motion. The results presented herein are provided by numerical integrations of the non-linear equations of motion and also by the integration of the linearized model.

2. THE MATHEMATICAL MODELS

The main initial effect of the gusts on modern aircraft is related to the changes in the orientation of the local airflow, changes that vary over the airplane; moreover, aeroelastic effects are also very important, however, adequate descriptions of these two classes of effects require complex models and significant computer power.

Reasonable descriptions of gust effects, sufficient at the early design phases, can, however, be obtained using the rigid airplane as well as the “particle” assumptions, i.e. aeroelastic effects are neglected and the wave-length of the atmospheric turbulence is considered much bigger that the airplane, hence the gust speed is the same over the entire airplane.

Various models for gust effects exist in the literature, e.g. Refs. [1-8]; the nonlinear model discussed below is taken from Ref. [4] and the linearized model is based on equations and assumptions from Ref. [1] (however, the nomenclature used here is slightly different).

2.1 The Non-linear Model

The main changes of the equations of motion are the change in velocity, and angle of attack, the aerodynamic forces being calculated using the relative velocity V_r of the airplane with respect to the wind and the relative angle of attack α_r measured with respect to V_r as showed in Fig.1.

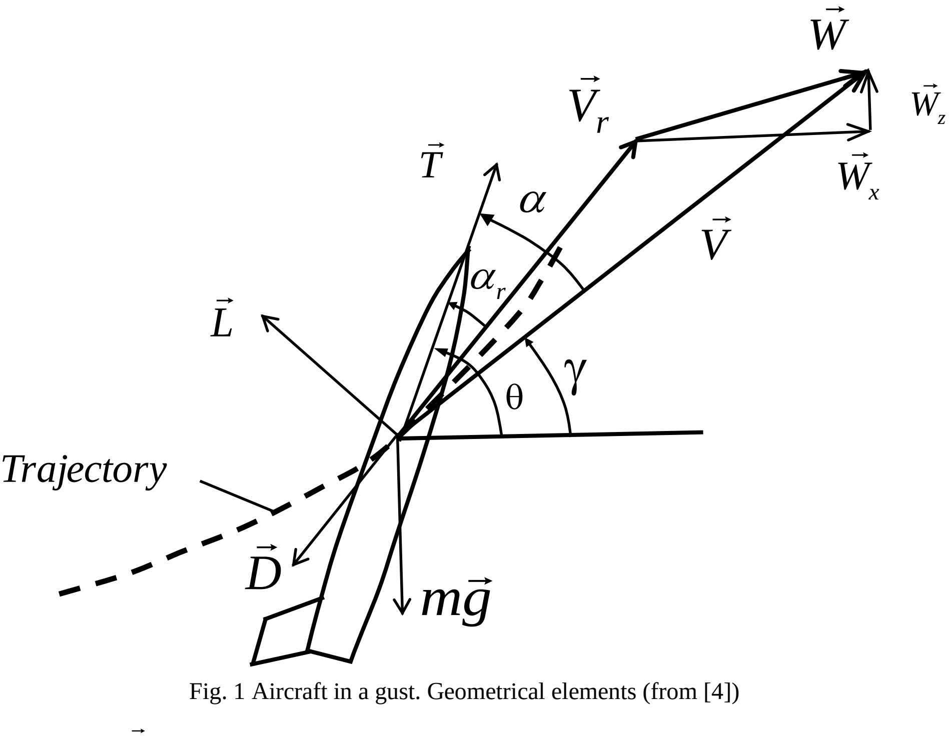

Fig. 1 Aircraft in a gust. Geometrical elements (from [4])

The velocity $\vec{V}$ of the aircraft with respect to the Earth is written as the sum of the velocity $\vec{V}_r$ of the aircraft with respect to the wind and the velocity $\vec{W}$ of the wind with respect to the Earth,

$$\vec{V} = \vec{V}_r + \vec{W} \tag{1}$$

So, the equations of motion for the longitudinal channel become:

$$
\begin{aligned}
& m\frac{dV}{dt} = T\cos(\alpha+\tau) - \frac{\rho}{2}SV_r^2\left[C_D\cos(\alpha-\alpha_r) + C_L\sin(\alpha-\alpha_r)\right] - mg\sin\gamma \\
& mV\,\dot{\gamma} = T\sin(\alpha+\tau) + \frac{\rho}{2}SV_r^2\left[C_L\cos(\alpha-\alpha_r) - C_D\sin(\alpha-\alpha_r)\right] - mg\cos\gamma \\
& J_y\dot{q} = \frac{\rho}{2}SV_r^2\mathrm{c}\mathrm{C}_{\mathrm{m}} \\
& \dot{\alpha} = q - \dot{\gamma} \\
& \dot{z} = V\sin\gamma\ ;\ \ \dot{x} = V\cos\gamma
\end{aligned}
\tag{2}
$$

where

$$V_r^2 = V^2 + W_x^2 + W_z^2 - 2V\left(W_x\cos\gamma + W_z\sin\gamma\right) \tag{3}$$

and

$$\alpha_r = \alpha - \mathrm{atan}\left[\left(W_x\sin\gamma - W_z\cos\gamma\right)\Big/\left(V - W_x\cos\gamma - W_z\sin\gamma\right)\right] \tag{4}$$

The aerodynamic coefficients are written as function of the Mach number, (which is calculated using the relative velocity), the relative angle of attack and its rate of change, the elevator deflection and the pitch rate,

$$
\begin{aligned}
& C_D = C_D\left(M,\alpha_r,\delta_e\right) \\
& C_L = C_L\left(M,\alpha_r,\delta_e,\dot{\alpha}_r,q\right),\ \ C_m = C_m\left(M,\alpha_r,\delta_e,\dot{\alpha}_r,q\right) \\
& M = \frac{V_r}{a}
\end{aligned}
\tag{5}
$$

and further,

$$
\begin{aligned}
& C_L = C_{L0} + C_{L\alpha}\alpha_r + C_{L\delta}\delta_e + C_{Lq}q + C_{L\dot{\alpha}}\dot{\alpha}_r \\
& C_m = C_{m0} + C_{m\alpha}\alpha_r + C_{m\delta}\delta_e + C_{mq}q + C_{m\dot{\alpha}}\dot{\alpha}_r \\
& C_D = C_{D0} + C_D^{\alpha^2}\alpha_r^2 + C_D^{\delta_e^2}\delta_e^2 + C_D^{\alpha\delta_e}\alpha_r\delta_e
\end{aligned}
\tag{6}
$$

2.2 The Linearized Model

The behavior of an airplane in the gust has also been described using linear models and small perturbations assumptions.

The subsequent development is inspired by the method from Ref. [1] where the gust effects are introduced in the linearized equations in a manner similar to controls, i.e. a term that includes a gust-induced increase of the angle of attack is added to the equations.

As is often done in literature, terms describing small effects are neglected, so that linearized equations of motion, with gust effects included, become:

$$\begin{aligned}
&\frac{d}{dt}\delta V = A_V\delta V + A_\gamma\delta\gamma + A_\alpha\delta\alpha + A_\delta\delta\delta_e + A_\alpha\left(\delta\alpha\right)_{GUST}\\
&\frac{d}{dt}\delta\gamma = B_V\delta V + B_\gamma\delta\gamma + B_\alpha\delta\alpha + B_q\delta q + B_\delta\delta\delta_e + B_\alpha\left(\delta\alpha\right)_{GUST}\\
&\frac{d}{dt}\delta q = C_V\delta V + C_\alpha\delta\alpha + C_q\delta q + C_{\dot\alpha}\delta\dot\alpha + C_\delta\delta\delta_e + C_\alpha\left(\delta\alpha\right)_{GUST}\\
&\frac{d}{dt}\delta\theta = \delta q = \frac{d}{dt}\delta\alpha + \frac{d}{dt}\delta\gamma\\
&\frac{d}{dt}\delta Z = \delta V\sin\gamma + V\cos\gamma\delta\gamma\\
&\frac{d}{dt}\delta X = \delta V\cos\gamma - V\sin\gamma\delta\gamma
\end{aligned} \tag{7}$$

where, for a vertical gust,

$$\left(\delta\alpha\right)_{GUST} \approx \frac{W_z}{V} \tag{8}$$

The equations of the stability and control derivatives are widely available in the literature, see for example Ref. [1]

3. RESULTS AND DISCUSSIONS

A set of numerical integrations of the equations of motion, both for the non-linear and linearized models is shown below.

The initial conditions correspond to the airplane in level flight (constant speed, horizontal, straight and balanced), at an altitude of 10km.

Three velocities were considered, that is 200m/s, 250m/s and 300m/s. The angle ot attack, thrust and elevator deflection correspond to this flight.

The airplane is next subjected to a 5 *m/s* abrupt vertical ascending gust, step-function type, with no attenuation factor and it is assumed that the controls remain constant (both stick and throttle).

As stated before, the gust effects were assumed to be identical over the entire airplane.

The aerodynamic data correspond to a hypothetical fighter, with the mass of about 10 tons, Ref. [22].

Both the non-linear and the linearized equations were integrated using 4th order Runge-Kutta algorithms.

The data for the first 5 seconds after the plane enters the burst are presented.

Figure 2 shows the velocity with respect to Earth as provided by the nonlinear model (referred to as “exact velocity”) and by the linear equations, together with the relative velocity, i.e. the velocity with respect to the air.

The results are extremely closed and they indicate an extremely small change in velocity.

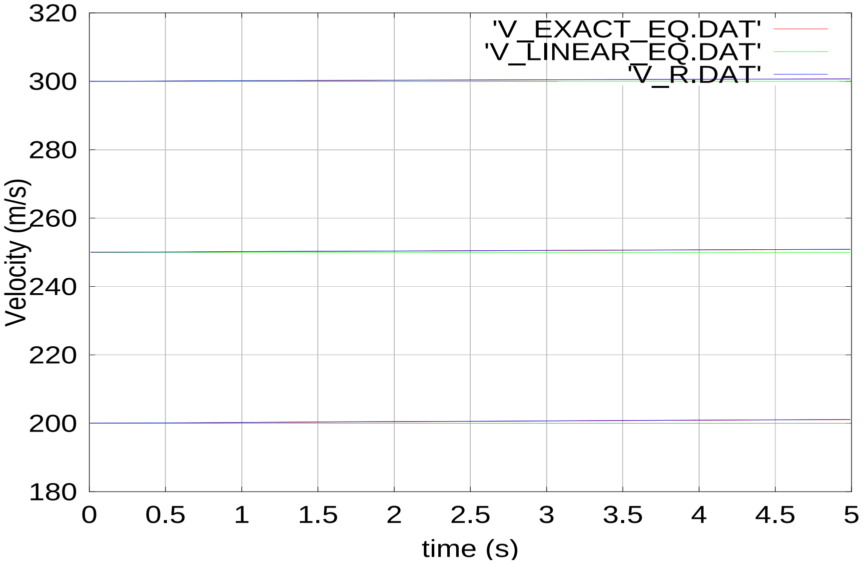

Fig. 2 Velocity of the airplane in the uniform vertical gust

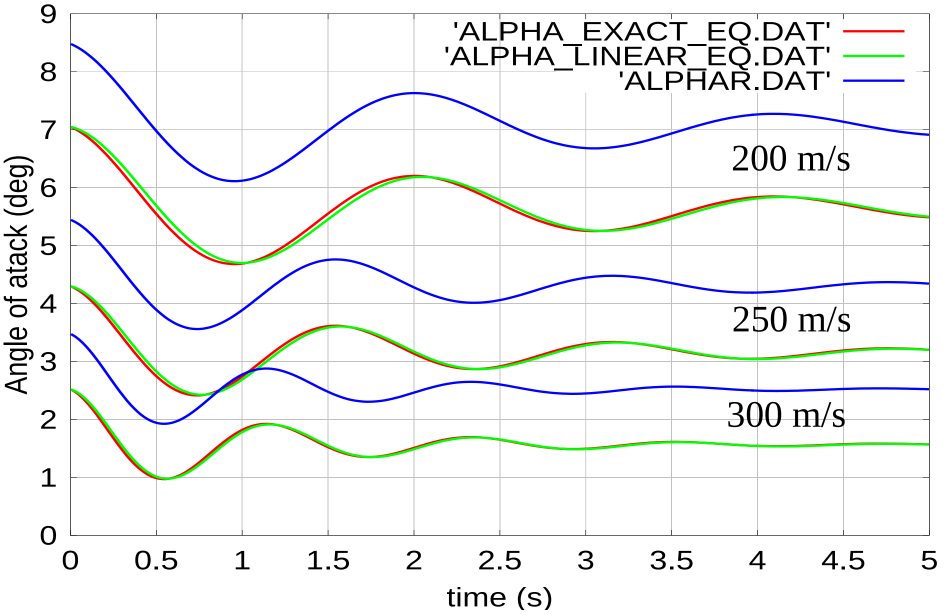

Fig. 3 Angle of attack of the airplane in the uniform vertical gust

Figure 3 shows the angle α (measured with respect to V) obtained from the nonlinear model (called “exact”) and from the linear equations, as well as the angle α_r (measured with respect to V_r.) that was obtained from the nonlinear model. The values of α provided by the non-linear and the linearized equations are almost identical and they indicate a fast damping of the perturbation. This was expected, as the airplane was known to be stable. Also, the nonlinear model showed that the angle α_r is higher than α (as expected for an airplane in a vertical ascending gust) and the difference $\left(\alpha_r-\alpha\right)$ is very closed to the $\left(\Delta\alpha\right)_{GUST}$ that was utilized to model the gust effect in the linearized model.

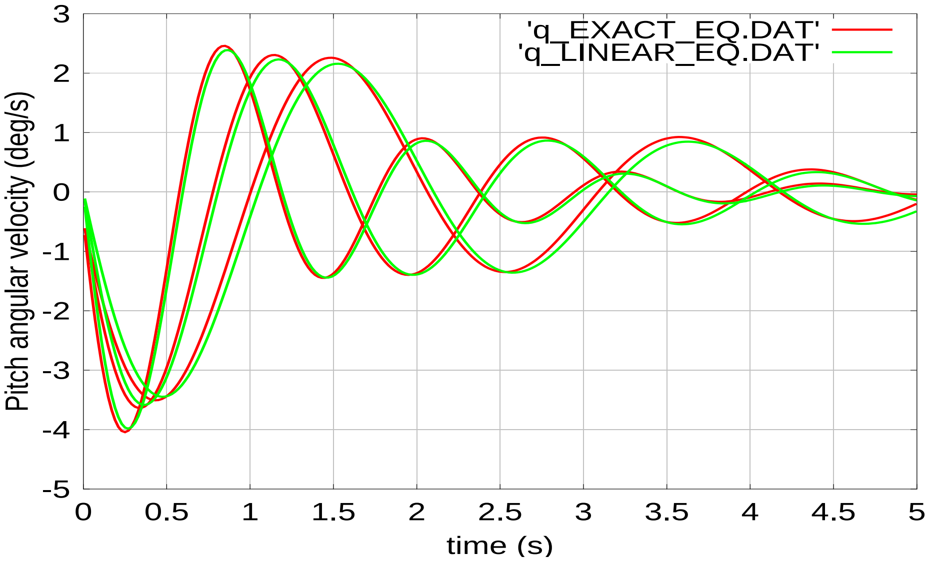

Fig. 4 Pitch angular velocity of the airplane in the uniform vertical gust

Figure 4 displays the pitch angular velocity, *q*, obtained from the nonlinear model (in the graphs, these values were called “exact”) and from the linear equations. Same as before, differenced between the results provided by the non-linear and the linearized model are small and both models indicate (as expected) a fast damping of the perturbations.

Figure 5 depicts the slope of the airplane, again, obtained from the nonlinear model (called “exact”, same as before) and from the linear equations. The results are closed, the average value of the slope obtained form the nonlinear model being about 0.75^0 to 1.25^0 higher that the values obtained from the linearized equations.

Figure 6 shows trajectories of the airplane in the vertical gust, obtained from the nonlinear model, for a time of 180 seconds.

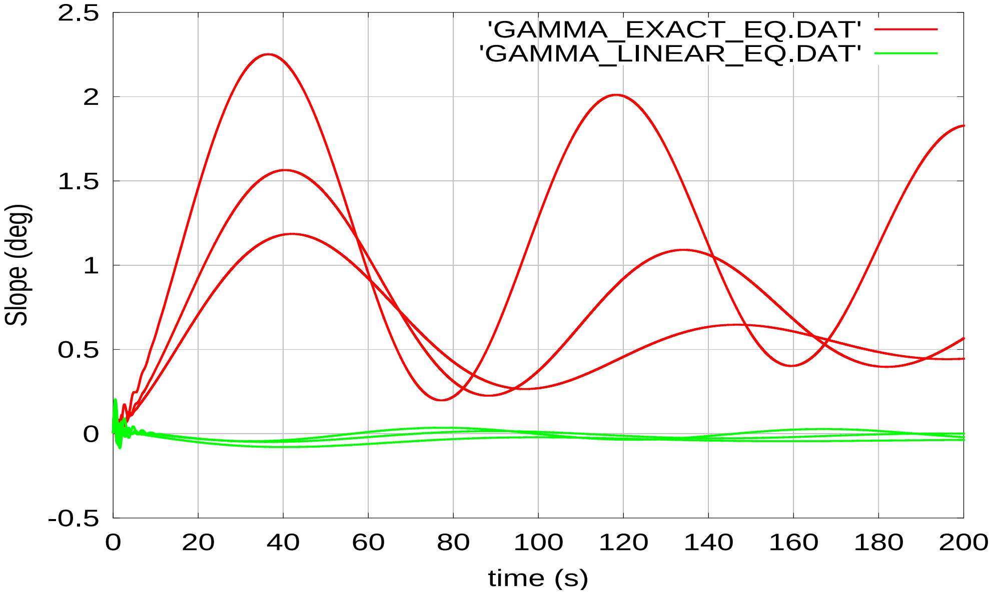

Fig. 5 Slope of the airplane in the uniform vertical gust

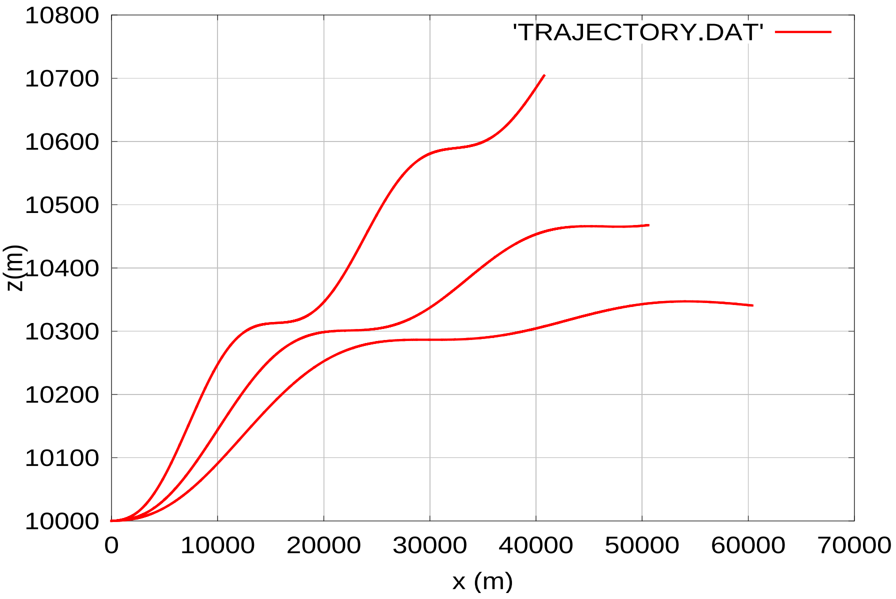

Fig. 6 Trajectories of the airplane in uniform vertical gust

4. CONCLUSIONS

The current paper presents studies on predicting the responses of aircraft flying through vertical gusts. The equations describing the dynamics of the longitudinal channel of the airplane are written to include the effect of the vertical wind. The paper includes comparisons of results provided by non-linear and linearized equations of motion. The numerical results show that, within the limit of this paper, gust behavior predicted by the linear model is close to the results obtained from the non-linear equations, so the data from the nonlinear analysis confirm the assumptions embedded in the linearized gust modeling.

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REFERENCES

[1] B. Etkin, *Dynamics of Flight*, John Wiley and Sons, New York, 1972.
[2] L.V. Schmidt, *Introduction to Aircraft Flight Dynamics (AIAA Education Series),* American Institute of Aeronautics and Astronautics, 1998.
[3] B. Etkin, Turbulent wind and its effect on flight, *J. Aircraft*, **18**: 327-345, 1981.
[4] F. Moraru, Regarding the Computation of the Wind Effects on the Motion of Rockets with Empennage, *Military Academy Bulletin*, no. 4 (in Romanian), 1969.
[5] H. S. Choi, S. Lee, H. Ryu and C. Ha, Dynamics and Simulation of the Effects of Wind on UAVs and Airborne Wind Measurement, *Trans. Japan Soc. Aero. Space Sci*., Vol. **58**, No. 4, pp. 187–192, 2015.
[6] E. N. Abdulwahab and C. Hongquaan, Aircraft Response to Atmospheric Turbulence at Various Types of the Input Excitation, *Space Research Journal*, 1: 17-28.

[7] J. Etele, *Overview of Wind Gust Modelling with Application to Autonomous Low-Level UAV Control*, Defence R&D Canada – Ottawa CONTRACT REPORT DRDC Ottawa CR 2006-221 November 2006.

[8] S. Wang, X. Song and Y. Wang, Modeling and Simulation of China C Series Large Aircraft with Microburst, *Mathematical Problems in Engineering*, Hindawi Publishing Corporation, Article ID 2371594, Volume 2016.

[9] S. Berbente, *Automatic Flight Control Systems: Methods for Synthesis of Algorithms for Automatic Landing Control Systems*, PhD Thesis, The University Politehnica of Bucharest, (in Romanian), 2007.

[10] A. I. Afloare, *Detection and Prevention of Oscillating Effects Generated by Aircraft Pilot Couplings*, PhD Thesis, The University Politehnica of Bucharest, (in Romanian) 2019.

[11] I. B. Ştefănescu, A.M. Stoica, *A Turbulence Model for the PUMA SA 330 Helicopter*, *University Politehnica of Bucharest Scientific Bulletin, Series D*, Vol. **80**, Issue 4, 2018.

[12] X. Wang, E. Van Kampen, Q. P. Chu and Roeland De Breuker, Flexible Aircraft Gust Load Alleviation with Incremental Nonlinear Dynamic Inversion, *Journal of Guidance, Control, and Dynamics*, Volume **42**, Issue 7, https://doi.org/10.2514/1.G003980, 2019.

[13] F. Moraru, *Chapters of Numerical Analysis. Applications in Ballistics and Flight Dynamics*, The Publishing House of The Technical Militery Academy, Bucharest (in Romanian), 2002.

[14] C. Berbente, S. Mitran, S. Zancu, *Numerical Methods*, The Technical Publishing House, Bucharest, Romania (in Romanian), 1995.

[15] S. Danaila and C. Berbente, *Numerical Methods in Fluid Dynamics*, ISBN 973-27-0958-8, The Romanian Academy (in Romanian) , 2003.

[16] S. Radnef, *Differential Modeling of DAE's Solutions for System's Control*, Proceedings of the 10th WSEAS International Conference on Mathematical and Computational Methods in Science and Engineering", pp. 432-437, WOS: 000262436800092, 2008.

[17] R. A. Oprea, A Constrained Motion Perspective of Railway Vehicles Collision, *Multybody System Dynamics*, Vol. **30**, Issue 1, pp. 101-116, DOI: 10.1007/s11044-013-9368-5, WOS number 000318800900006, 2013.

[18] R. M. Botez, T. V. Chelaru, P. Parvu and C. Gheorghe, Calculus Model for a Rolling Guided missile, *Journal of Vibration and Control*, Vol. **7**, No. 6, pp. 863-869, 2001.

[19] J. Betts, Survey of Numerical Methods for Trajectory optimization, *Journal of Guidance Control and Dynamics*, Vol. **21**, No. 2, pp. 193-207, http://dx.doi.org/10.2514/2.4231, 1998.

[20] N. Yokoyama and S. Suzuki, Modified Genetic Algorithm for Constrained Trajectory Optimization, *Journal of Guidance, Control, and Dynamics*, Vol. **28**, No. 1, pp. 139-144, http://dx.doi.org/10.2514/1.3042, 2005.

[21] K. Subbarao and B. M. Shippey, Hybrid Gen(2005)etic Algorithm Collocation Method for Trajectory Optimization, *Journal of Guidance, Control, and Dynamics*, Vol. **32**, No. 4, pp. 1396-1403, http://dx.doi.org/10.2514/1.41449, 2009.

[22] L. Moraru, An Expedite Approximate Algorithm for Calculating the Controls in the Longitudinal Maneuver, *INCAS Bulletin*, (online) ISSN 2247–4528, (print) ISSN 2066–8201, ISSN–L 2066–8201, vol **7** (3), DOI: 10.13111/2066-8201.2015.7.3.11, 121-128, 2015.