Using PID Controller and SDRE methods for tracking control of Spacecrafts in Closed-Rendezvous Process

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Abstract: *Rendezvous process of spacecrafts is one of major issues in study of aerospace engineering. Tracking control in Rendezvous process is very complex due to the need to fulfill the conditions and constraints to determine the control forces to bring Chaser toward Target. The paper will implement the control of the trajectory of the relative translation movement between Chaser and Target in the Closed-Rendezvous stage through two different approaches: the first using PID Controllers, and the second using the SDRE (State Dependent Riccati Equation) technique. Then based on obtained results, a comparison between two methods is carried out.*

Key Words: *Closed-Rendezvous, trajectory control, PID controller, SDRE*

1. INTRODUCTION

Nowadays, with the desire to further explore space, studies on aerospace missions are becoming more and more attractive to researchers. Rendezvous and Docking operations will be concentrated to study and improve, especially in the field of control.

There are some authors with remarkable achievements that have applied in their works various advanced methods of control such as: Guillermo Ortega who used in [9]. Fuzzy logic the techniques for rendezvous and docking of two geostationary satellites; I. Lopez and C. R. McInnes who used the artificial potential function for autonomous Rendezvous, [8]; P. Singla, K. Subbarao, and J. K. Junkins who used the Adaptive Control for their study of Rendezvous and Docking, [7]; With robust parametric method, Dake Gu and Yindong Liu solved spacecraft rendezvous problem, [6]. In aerospace field, the State-Dependent Riccati Equation technique (SDRE) is widely applied in designing controller for nonlinear systems thanks to its simplicity and effectiveness. The use of the State-Dependent Riccati Equation (SDRE) is to provide feedback control for nonlinear systems by allowing nonlinearities in the system state. Specifically, by imitating the Linear Quadratic Regulator (LQR) for linear systems, SDRE allows computing a sub-optimal solution of nonlinear control problem by solving online the Algebraic Riccati Equation (ARE).

Then this method is used in lots of works related to Rendezvous problem to solve particular aims, [1-5].

This paper also uses SDRE approach for tracking control of spacecrafts in closed-Rendezvous range and applies it in the specific example.

Besides, as one of the most popular control method, PID control method is employed here for solving an identical issue. When implementing Rendezvous operation, the V-bar approach is considered as approach strategy.

The work does not intend to establish a hierarchy of methods, to determine if a method is better than the other.

However, based on the obtained results we will have an overview about performance of each method. Additionally, depending on the criteria of specified task such as: energy consumption or issue relating to time for steady state, we can choose suitable control method for desired purposes.

2. RELATIVE TRANSLATIONAL MOTION DYNAMICS

Commonly, in Rendezvous and Docking missions, local orbital frame centered on the Chaser, and the Target namely Local Vertical Local Horizontal (LVLH) frame, and an Earth – Centered Inertial frame are used to compute position vectors of spacecrafts. They are depicted in Figure 1.

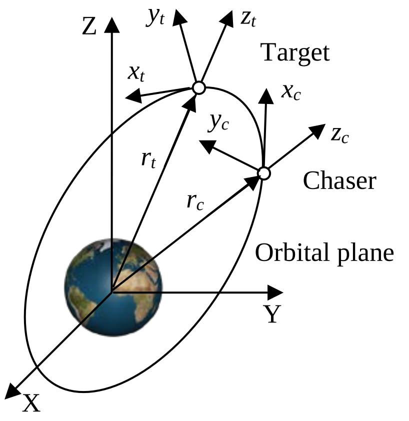

Figure 1. Reference frames

x – axis in the same direction and orientation as the orbital velocity vector (V-bar), **y** – axis normal to the orbit, with opposite direction of the orbital angular momentum vector (H-bar), **z** – axis completes the system, oriented in the radial direction, perpendicular to the plane of horizon, nadir direction (R-bar).

In the approximately circular orbit, Hill equations derived by Clohessy and Wiltshire (CW) have been widely used to describe the relative motion between spacecrafts, [10]. Then the guidance of Chaser vehicle during Rendezvous and Docking maneuvers in the case without disturbing acceleration are expressed as:

$$\begin{aligned}\ddot{x}-2\omega\dot{z}&=\frac{F_x}{m_c}\\ \ddot{y}+\omega^2 y&=\frac{F_y}{m_c}\\ \ddot{z}+2\omega\dot{x}-3\omega^2 z&=\frac{F_z}{m_c}\end{aligned}\tag{1}$$

assuming the case in which the target is in a low circular orbit (LEO) with the position vector r_t and ω constant, according to:

$$\omega=\sqrt{\frac{\mu}{r_t^3}}\tag{2}$$

where μ is Earth's gravitational constant (m^3/sec^2),

F_x, F_y, F_z are forces exerted by Chaser to control position and velocity, and x, y, z are the coordinates of the chaser in the local vertical/ local horizontal (LVLH) coordinated system attached to the Target.

In case the perturbation is taken into account, the equations of relative motion between Chaser and Target are expressed as:

$$\begin{aligned}\ddot{x}-2\omega\dot{z}+\Delta a_x&=\frac{F_x}{m_c(t)}\\ \ddot{y}+\omega^2 y+\Delta a_y&=\frac{F_y}{m_c(t)}\\ \ddot{z}+2\omega\dot{x}-3\omega^2 z+\Delta a_z&=\frac{F_z}{m_c(t)}\end{aligned}\tag{3}$$

where $\Delta a_x, \Delta a_y, \Delta a_z$ are the perturbing acceleration components due to J_2 in local orbital frame. They are computed as following:

$$a_{J_2}=-\frac{3\mu J_2 R_e}{r^4}\left[\left(\sin^2 i.\sin\theta.\cos\theta\right)e_x+\left(\sin i.\sin\theta.\cos i\right)e_y+\frac{1}{2}\left(1-3\sin^2 i.\sin\theta\right)e_z\right]\tag{4}$$

in there, e_x, e_y, e_z are unit vector in the LVLH frame of the Target,

R_e is the Earth's radius,

i, θ are inclination and argument of latitude of spacecraft, respectively.

Then relative effect of the Earth oblateness due to J_2 becomes:

$$\Delta a=a_{J_2}(r_c, i_c, \theta_c)-a_{J_2}(r_t, i_t, \theta_t)\tag{5}$$

In the equation above, inclination i, argument of latitude θ, and position vector r are assumed to be known for Target and Chaser.

From Equation (1), the general solution can be expressed conveniently in terms of state vector as, [10]:

$$[x,\, y,\, z,\, \dot{x},\, \dot{y},\, \dot{z}]^T = H(t)[x_0,\, y_0,\, z_0,\, \dot{x}_0,\, \dot{y}_0,\, \dot{z}_0]^T \tag{6}$$

where $H(t)$ is the state transition matrix for CW equation, and is expressed as:

$$H(t) = \begin{bmatrix} M_{rr}(t) & N_{rv}(t) \\ S_{vr}(t) & T_{vv}(t) \end{bmatrix} \tag{7}$$

in there, its components representing for position vector and velocity vector are written in more detail as:

$$M_{rr}(t) = \begin{bmatrix} 1 & 0 & 6(\omega t - \sin\omega t) \\ 0 & \cos\omega t & 0 \\ 0 & 0 & 4 - 3\cos\omega t \end{bmatrix},$$

$$N_{rv}(t) = \begin{bmatrix} \frac{4}{\omega}\sin\omega t - 3t & 0 & \frac{2}{\omega}(1-\cos\omega t) \\ 0 & \frac{1}{\omega}\sin\omega t & 0 \\ \frac{2}{\omega}(\cos\omega t - 1) & 0 & \frac{1}{\omega}\sin\omega t \end{bmatrix} \tag{8}$$

$$S_{vr}(t) = \begin{bmatrix} 0 & 0 & 6\omega(1-\cos\omega t) \\ 0 & -\omega\sin\omega t & 0 \\ 0 & 0 & 3\omega\sin\omega t \end{bmatrix},\ T_{vv}(t) = \begin{bmatrix} 4\cos\omega t - 3 & 0 & 2\sin\omega t \\ 0 & \cos\omega t & 0 \\ -2\sin\omega t & 0 & \cos\omega t \end{bmatrix}$$

Let denote t_f, r_f be the final time and final relative position of two spacecrafts, then the required initial velocity in LVLH frame of Target is:

$$V_{h_0} = -N_{rv}^{-1}(t_f).M_{rr}(t_f)r_0 + N_{rv}^{-1}(t_f)r_f \tag{9}$$

Then the first velocity impulse in the LVLH frame of Target is:

$$\Delta V_{t_0} = V_{h_0} - V_0 \tag{10}$$

where V_0 is the initial relative velocity between two spacecrafts.

From that, the final relative velocity upon successful rendezvous is determined as:

$$V_f = S_{vr}(t_f)r_0 + T_{vv}(t_f)\Delta V_{t_0} \tag{11}$$

And the velocity required for second impulse is:

$$\Delta V = -V_f \tag{12}$$

To nullify the approach velocity the exponential braking law characterized by an exponential change of velocity with time may be given, [11].

3. USING SDRE FORMULATION FOR CONTROLLING RELATIVE TRANSLATIONAL MOTION

Let consider the autonomous system that is full-state observable, nonlinear in state and affine in the input given by:

$$\dot{x}(t) = f(x) + B(x).u(t) \tag{13}$$

where $x \in R^n$ is the state vector, $u \in R^m$ is the input vector, and it is assumed that $f : R^n \to R^n$, $B : R^n \to R^{n\times m}$.

Through the state-dependent coefficient (SDC) factorization, the nonlinear equations can be represented as linear structures with state-dependent coefficients. Thus, this procedure is similar to the LQR method except that all matrices may depend on the state.

Based on this concept, the sate-space equations for nonlinear system in Equation (13) can be expressed as a linear state-space equation using direct factorization as:

$$\dot{x}(t) = A(x).x + B(x).u \tag{14}$$

where the factorization for $f(x) = A(x).x$ with $A(x) \in R^{n\times n}$ is possible if and only if $f(0) = 0$, and $f(x)$ is continuously differentiable.

The state-dependent dynamic matrix *A(x)* is non-unique where $n > 1$, [15]. The optimal control problem mentioned above is to find a state-feedback control law $u(x)$, which minimizes the cost functional:

$$J = \frac{1}{2}\int_0^{\infty}\left(\left[x^T(t) - x_r^T(t)\right]Q(x)\left[x(t) - x_r(t)\right] + u^T(x)R(x)u(x)\right)dt \tag{15}$$

where $x_r(t)$ is the reference or desired state vector provided by the guidance scheme,

$Q(x) \in R^{n\times n}$ is the state weighting matrix satisfying $Q(x) = Q^T(x) \geq 0$,

$R(x) \in R^{m\times m}$ is the input weighting matrix satisfying $R(x) = R^T(x) > 0$.

for all *x* in order to ensure the local stability, [16]. It should be noted that $Q(x)$ and $R(x)$ are not only allowed to be constant, but can also be varied as functions of states. And it is assumed that $f(0) = 0$ and $B(x) \neq 0$. For a valid solution to the SDRE, the pair $\{A(x), B(x)\}$ must be wise-point stabilizable in the linear sense for all *x* in the domain interests.

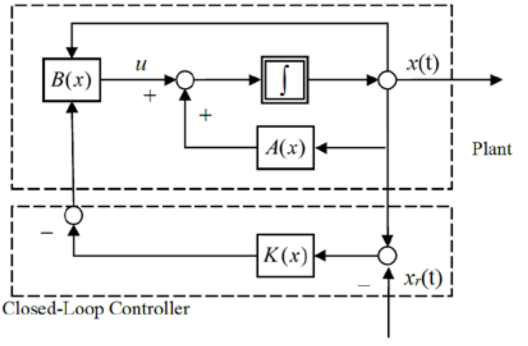

Figure 2. Diagram of the SDRE control in closed-loop

As shown in Figure 2 the SDRE method for obtaining sub-optimal solution problem can be summarized, [13]:

- Through the SDC factorization, transforming the nonlinear equation of Equation (13) into the linear-like structure of Equation (14),
- Solving the SDRE of the form:

$$P(x).A(x)+A^T(x).P(x)-P(x).B(x).R^{-1}(x).B^T(x).P(x)+Q(x)=0 \tag{16}$$

Provided $P(x)$ exists, the nonlinear feedback control law is then:

$$u(x)=K(x).\left[x(t)-x_r(t)\right] \tag{17}$$

where $K(x)$ is denoted as the state feedback gain for minimizing Equation (15), and is expressed

$$K(x)=-R^{-1}(x).B^T(x).P(x) \tag{18}$$

Applying this control law, results in the closed-loop system dynamics being given by:

$$\dot{x}(t)=A(x).x(t)-B(x).K(x).\left[x(t)-x_r(t)\right] \tag{19}$$

For the case of relative translational motion between two spacecrafts Chaser and Target, by following the instruction mentioned above, the system matrix *A(x)* is:

$$A(x)=\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -\dfrac{\Delta a_x}{z} & 0 & 0 & 2\omega \\ 0 & -\omega^2 & -\dfrac{\Delta a_y}{z} & 0 & 0 & 0 \\ 0 & 0 & 3\omega^2-\dfrac{\Delta a_z}{z} & -2\omega & 0 & 0 \end{bmatrix} \tag{20}$$

in there, the denominator *z* must not be allowed to be zero to avoid a singularity.

Without loss of generality, in our study it is supposed that mass of the Chaser is constant during simulation time. Then the control matrix is:

$$B=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \dfrac{1}{m_c} & 0 & 0 \\ 0 & \dfrac{1}{m_c} & 0 \\ 0 & 0 & \dfrac{1}{m_c} \end{bmatrix} \tag{21}$$

Since Equation (16) should be determined at each instant time, so the state weighting matrix Q and the input weighting matrix R are also considered to be constant. Then they are respectively expressed as:

$$Q = 10^{p}.I(6\times 6) \quad (22)$$

$$R = 10^{q}.I(3\times 3) \quad (23)$$

The properly chosen initial matrices without causing the thruster saturation are required. If larger Q and smaller R weighting matrices are chosen initially, the controller may become saturated then consequently resulting in control commands that cannot be executed by the thruster.

When the weighting matrices are adjusted at steady state, the control forces are modified and tracking is then reduced to the desired value without thruster saturation.

This adjustment of the weighting matrices is very important in order to generate approximately control forces.

4. USING PID CONTROLLERS FOR CONTROLLING RELATIVE TRANSLATIONAL MOTION

In this section, PID controller, one of the most popular control methods, will be applied for the trajectory control for the same problem mentioned above.

Similar to any control method, the system will become closed-loop control system when using PID controller.

Based on updating current state of the system through the feedback signal, the controlling forces are executed correspondingly to perform given task thanks to PID controllers.

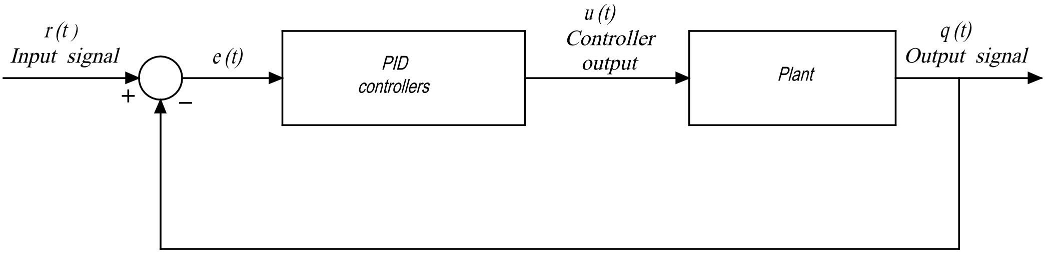

Figure 3. Diagram of closed-loop control system using PID controller

With a closed-loop system as we can see in Figure 3, the tracking error *e(t)* is sent to the PID controller, then the controller output *u(t)* is computed as, [12]:

$$u(t) = K_P.e(t) + K_I \int e(t).dt + K_D.\frac{de(t)}{dt} \quad (24)$$

in there, K_P, K_I, K_D are proportional, integral, and derivative gains, respectively, $u(t)$ is force for controlling the Chaser as in Equation (3),

$$e(t) = r(t) - q(t) \quad (25)$$

with $r(t)$ is reference state of Chaser given by homogenous solution of CW equations, $q(t)$ is current state of the Chaser.

For our study case, the working process can be interpreted as follows: the control signal-controlling forces $u(t)$ will be sent to the Chaser needed to control, then the new signal output $q(t)$ will be obtained.

Subsequently, the feedback is needed to take in period of time to find the new error signal.

Based on this new error, the PID controller computes to give the new control signal and the process keeps going on to control the Chaser approaching the Target.

5. APPLIED SIMULATION AND RESULTS

For simulating relative translational motion as expressed in Equation (3), MATLAB Simulink is the useful tool for that purpose. Let's use some given data as:

$$m_c = 150\,(kg);\ \mu_0 = 398600.6\ (m^3/s^2);\ R_e = 6378\ (km);$$

$$r_c = 400\ (km); \mu = 10^{-5};\ g = 9.81\,(m/s^2);\ J_2 = 1082.6*10^{(-6)};$$

$$i_c = \frac{30\pi}{180};\ i_t = \frac{51.6\pi}{180};\ \theta_c = \frac{150\pi}{180};\ \theta_t = \frac{135\pi}{180}.$$

in there, m_c is mass of the Chaser, and is considered constant during simulation time.

The model will simulate closed-rendezvous stage in 900 seconds (corresponding to S31-S32 proximity operation).

Based on that, some initial state conditions of system are used as:

$$x(t_0) = -8\ (m);\ \dot{x}(t_0) = 0.0064\ (m/s);$$

$$y(t_0) = 0.001\,(m);\ \dot{y}(t_0) = 0\ (m/s);$$

$$z(t_0) = 0.001\,(m);\ \ \dot{z}(t_0) = 0.0072\ (m/s).$$

By applying the Genetic Algorithm as in [14] for tuning PID parameters in our system, the values of K_{Pi}, K_{Ii}, K_{Di} (i=1, 2, 3) are determined as:

$$K_{P1} = 2.7100;\ K_{I1} = 0.0220;\ K_{D1} = 7.6800;$$

$$K_{P2} = 5.3100;\ K_{I2} = 0.1170;\ K_{D2} = 6.9400;$$

$$K_{P3} = 5.5800;\ K_{I3} = \text{-}0.0100;\ K_{D3} = 7.6100.$$

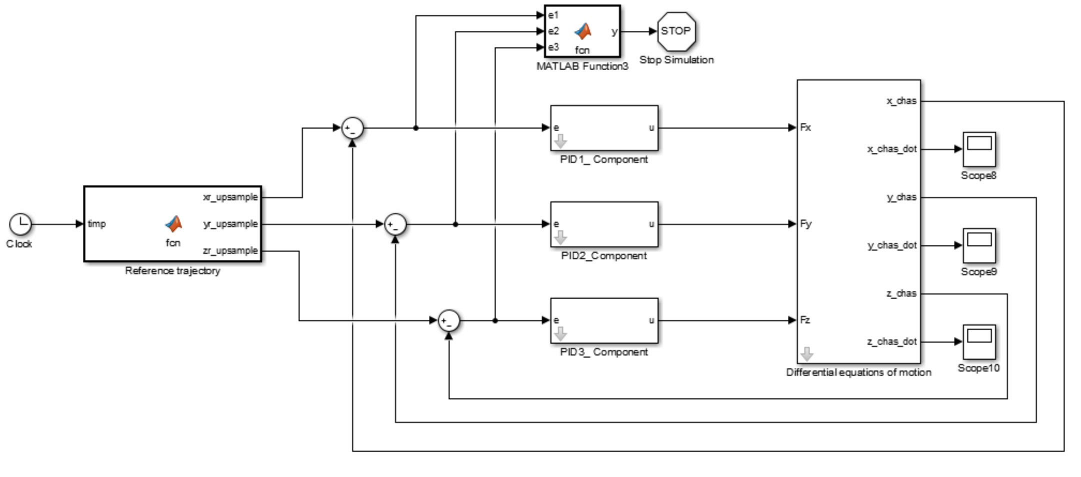

Figure 4. Simulink model for PID controller's method

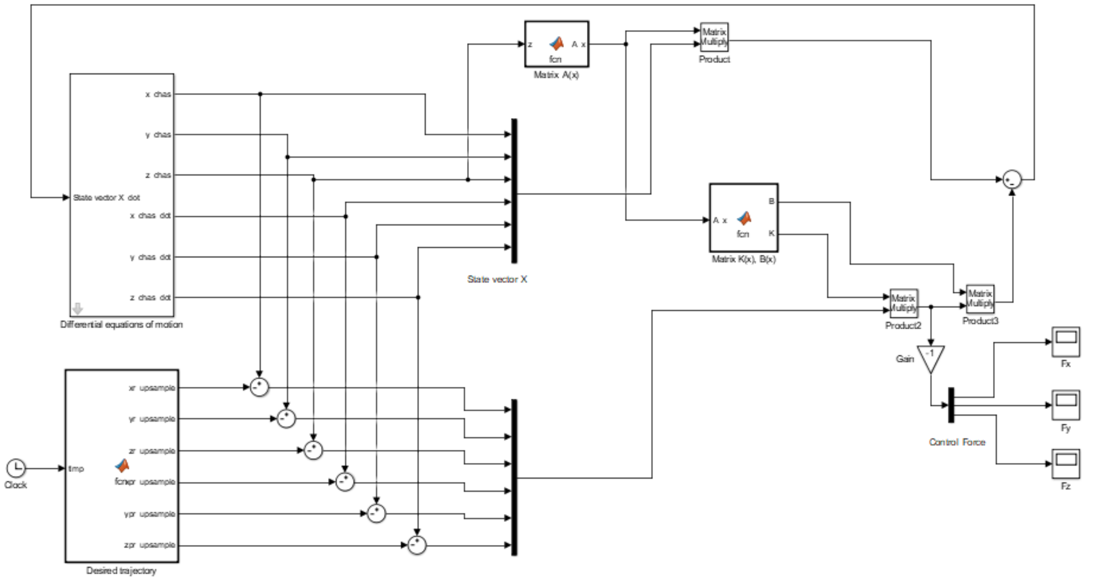

Figure 5. Simulink model for controlling SDRE method

And the results obtained from Simulink model are shown in Figures as following:

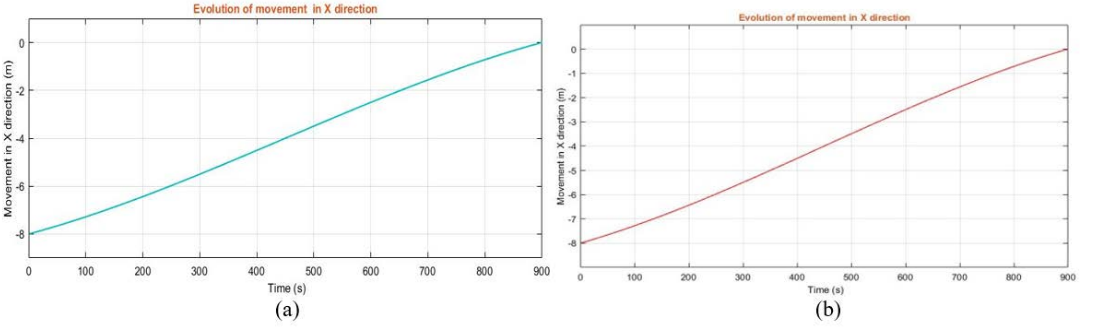

Figure 6. Movements in X direction versus time by using PID controllers (a), SDRE (b)

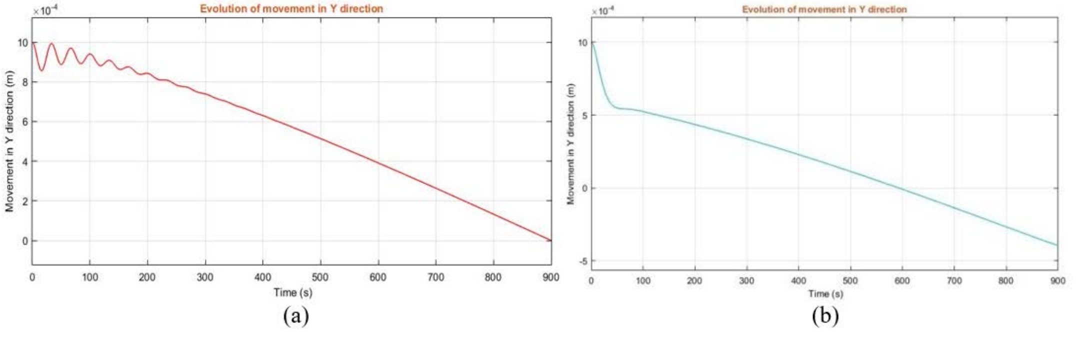

Figure 7. Movements in Y direction versus time by using PID controllers (a), SDRE (b)

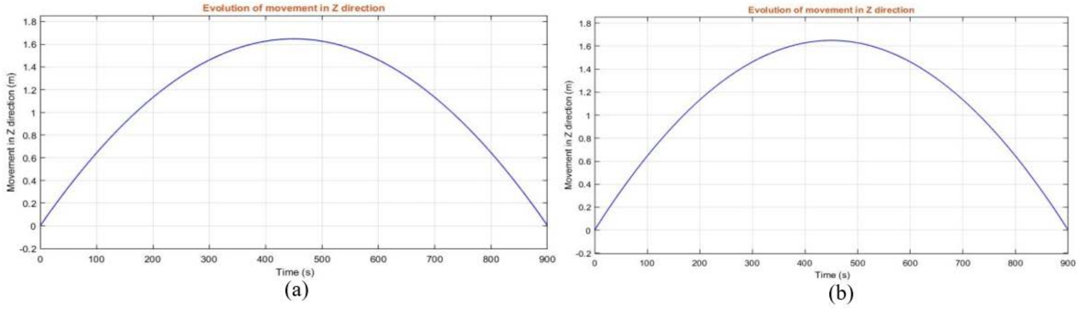

Figure 8. Movements in Z direction versus time by using PID controllers (a), SDRE (b)

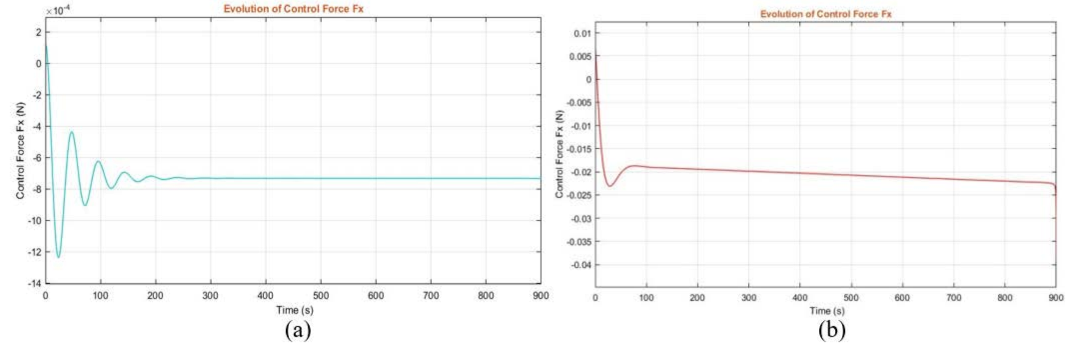

Figure 9. Variation of Control Forces in X direction using PID controllers (a), SDRE (b)

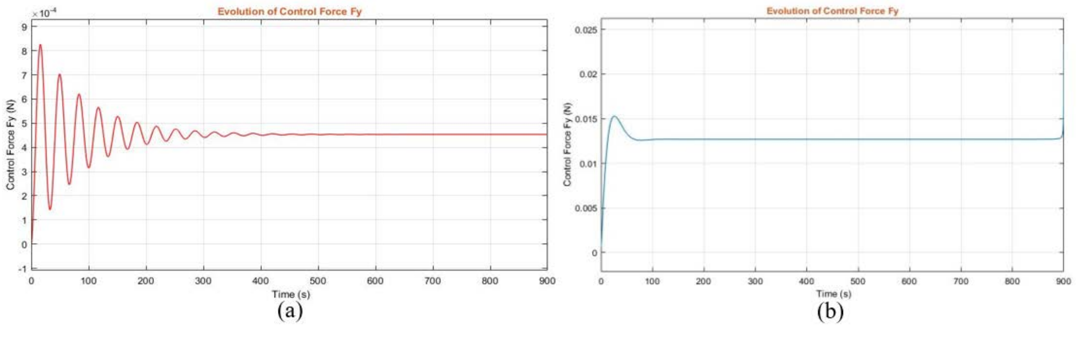

Figure 10. Variation of Control Forces in Y direction using PID controllers (a), SDRE (b)

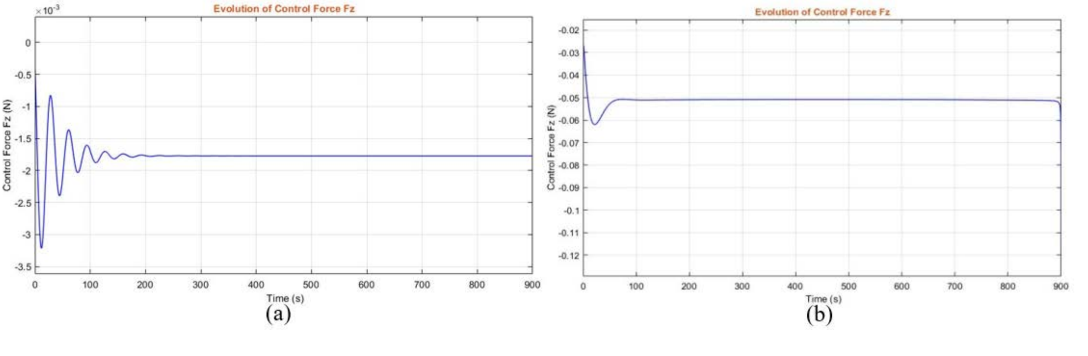

Figure 11. Variation of Control Forces in Z direction using PID controllers (a), SDRE (b)

With the above control forces, we now observe the total translation error to see how well they performe the given task, which is computed in the expression as:

$$e_{tot}(t) = \sqrt{\left(x_{des}(t) - x(t)\right)^2 + \left(y_{des}(t) - y(t)\right)^2 + \left(z_{des}(t) - z(t)\right)^2} \quad (26)$$

where $x_{des}(t), y_{des}(t), z_{des}(t)$ are desirable coordinates, and $x(t), y(t), z(t)$ are actual coordinates of the Chaser, respectively.

Then the result is shown in Figures below:

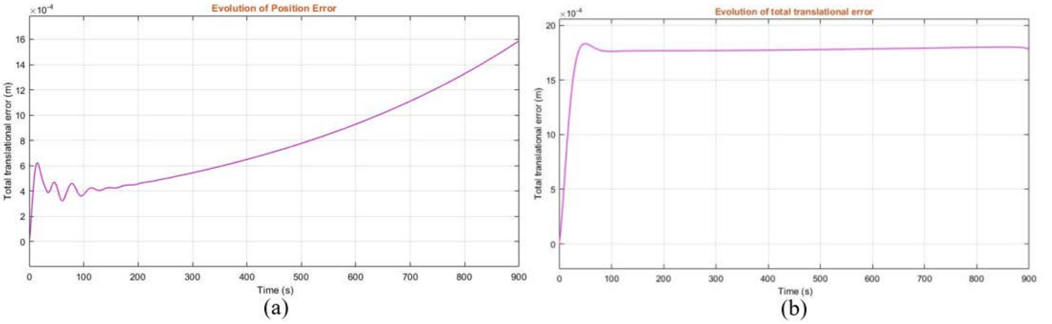

Figure 12. Variations of total error using PID controllers (a), SDRE (b)

6. CONCLUSIONS

Based on the obtained results, one can observe that both two methods performed very well the given task, i.e the actual relative trajectory tracks tightly the commanded relative trajectory. Speciffically, with the control method using PID controllers, the absolute values of control forces when the Chaser is in steady state is smaller than if using the SDRE method. Consequently, the total translational error when using PID controllers is also smaller than the other one. However, when using the SDRE method, the system reached steady state faster than using PID controllers. Therefore, as mentioned previously, that does not mean that the PID control approach is more advantageous than the SDRE one, due to existing difficulty: with PID controllers, we have to determine PID parameters: proportional, integral, and derivative gains; this is complicated because of their interaction. Then, in practice, we will choose the appropriate control method, according to the specific control requirement.

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